QUBITS : 1) NUCLEAR SPIN → not real interest in technological development BUT fundamental for understanding decoherence modeling and states manipulation

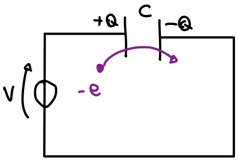
Lecture 11 - 14/11/23

2) ELECTRON SPIN → Introduced in 1998 by Di Vincenzo. ↳ T_1, T_2^*, T_2

The qubit can be represented by a single electron with its spin. The advantages are: compatibility with CMOS implementation. Then we have SCALING → the qubit is on average 100 nm × 100 nm. The drawback is the COHERENCE time → having an electron confined in quantum dot, it will be subject to many disturbances. For $\frac{T_c}{T_2^*} \simeq 10^3$ → QEC (Quantum error correction) and the achievement of such ratio are necessary in order to get a properly working quantum circuit.

COULOMB BLOCKADE

Minimum amount of potential to block a charge. Let's assume the following situation:



$$E_{ini} = \int V I \, dt = \int V \frac{C\,dV}{dt} dt = C \int V dV = \frac{CV^2}{2} = \frac{Q^2}{2C} \quad (Q = C \cdot V)$$ → energy stored in the capacitance.

I want to check if there's the possibility for a leakage current to flow across the capacitance (we want to see if an electron is able to jump from one side to the other.) If I move a negative charge from one side to the other it is like increasing of e the initial charge stored in the capacitance.

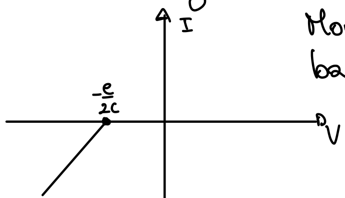
$E_{fin} = \frac{(Q+e)^2}{2C}$

↳ $\Delta E = E_f - E_i = \frac{(Q+e)^2 - Q^2}{2C} = \frac{e}{c}\left(Q + \frac{e}{2}\right)$ → The energy is changing and I need $\Delta E < 0$ in order to have the spontaneous transfer of these electrons.

$\Delta E < 0$ → $Q < -\frac{e}{2}$

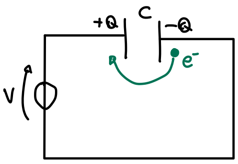
↓

$\boxed{V < -\frac{e}{2C}}$ → This means that we have a minimum voltage able to start a negative current that promotes the electrons jump.



Moving one electron means creating a potential barrier → the electron itself must face an intrinsic potential barrier → this potential self-barrier is the COULOMB BLOCKADE.

Now let's assume this situation:



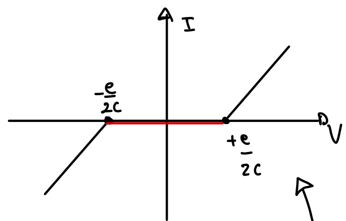
In this case we are moving the electron from the opposite side to the other. The initial energy is equal to the one before. The final energy instead is going to decrease $E_f = \frac{(Q-e)^2}{2C}$ → $\Delta E = E_f - E_i = \frac{(Q-e)^2 - Q^2}{2C} = \frac{(Q-e+Q)(Q-e-Q)}{2C} =$

$= \frac{e}{c}\left(Q - \frac{e}{2}\right)$

↓

This must be negative in order to have the spontaneous transfer

$Q > \frac{e}{2}$ → $\boxed{V > \frac{e}{2C}}$

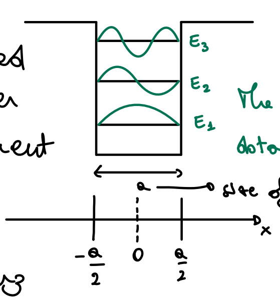


Why don't we usually see the Coulomb Blockade effect? Let's consider a $C = 5\ fF$ → $V = \frac{e}{2C} \simeq \frac{1{,}6 \cdot 10^{-19}}{10 fF} = 16\ \mu V$ → → as we see it is not so small, we could even measure it, but the noise is stronger and therefore such effect is buried into the noise. It is very small WRT thermal energy → the electrons are thus able to break the Coulomb blockade thanks to the thermal energy. $E_{th} = 25 meV$ (@ ROOM T.)

We have to operate at cryostate conditions (mK temperature).

QUANTUM DOTS

The quantum well was defined as follows: we have different potential levels organized in a square shape. We studied the Schrödinger equation from which we extracted the different quantized levels at which the electrons behave like a stationary wave with different number of half waves, depending on the quantized level at which they are.

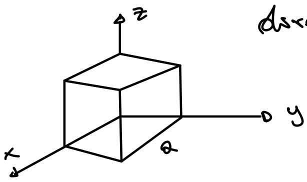


a → size of the quantum well

$E_n = \frac{h^2}{8 m a^2} n^2$ — Plank constant

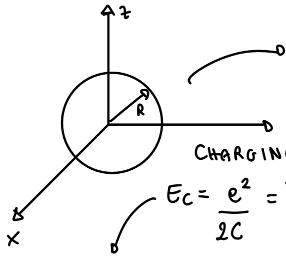
The wave front are just distorted by De Broglie

The quantum well is in one dimension. The quantum dot is 3D and we confine the electron in all the directions.



So for the expression of quantum dot energy we end up with $E = \frac{h^2}{8 m a^2} (n_x^2 + n_y^2 + n_z^2)$ we have 3 quantum numbers. We assume that the electron will be in the state of minimum energy → $n_x = n_y = n_z = 1$ (can't be 0 as in an harmonic oscillator, this quantum number establishes the number of half waves).

The q-dot is capacitively coupled to the gate and drain of a single electron transistor → the q-dot represents the floating island implented in this kind of transistor. More than a cube we can assume the q-dot to be a sphere.



$C = 4\pi\varepsilon_0 R$ → $R \approx 10$ nm (assumption) → we need very little dimensions in order to be able to distinguish among the different energy levels. Taking the example of the 1-D quantum well, since $E \propto \frac{1}{a}$ → in order to have well defined and displaced energy levels we must implement small a.

$C \approx 1$ aF

CHARGING ENERGY

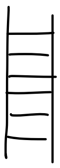
$E_C = \frac{e^2}{2C} = 70$ meV

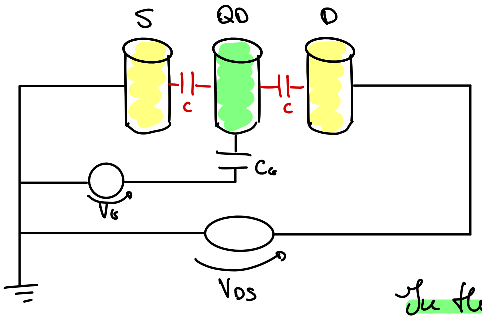
energy needed to allow a single electron to jump in the quantum dot.

we can compute the spacing of the energy levels that will be around 0,1 - 0,5 eV

In typical conditions of quantum computing $T = 3$ mK → $E_{th} = 0{,}25\ \mu$eV (@ ROOM T it is 25 meV) 10000 times lower than room T

As we see we are in a safe range in which we are able to confine an electron. We have to be able to bias the transistor and move the electron among different energy levels → such levels are not the quantized level, but just 1 quantized level that results in a LADDER (related to Coulomb blockade).

The basis  I have only one quantized energy BUT the energy steps increase on of how many electrons are confined in my island. To observe such effect we have to consider the SINGLE ELECTRON TRANSISTOR:



RESERVOIRS

Source and Drain act as an infinite source of electrons (sea of electrons)

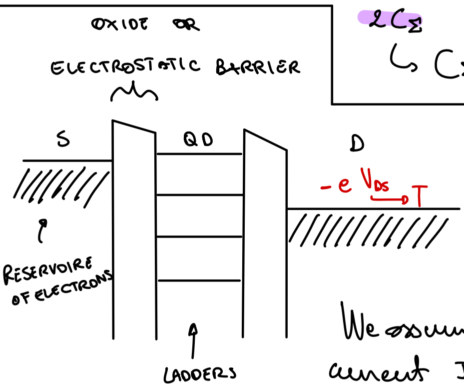
In the middle we have the ISLAND/QUANTUM DOT

QD will have very small capacitance. We can now introduce the CHEMICAL POTENTIAL → it is like the Fermi Level for the quantum dot and it establishes the confinement of the RESERVOIRES from the island. I have my seas of electrons and there will be a tendency for them to pass from the higher potential area at Fermi level, to the lower one. BUT, we have a capacitively coupled area called ISLAND

The initial energy can be estimated by the electrostatic potential that depends on the number of electrons: $E(N) = \frac{(C_G V_G - Ne)^2}{2 C_\Sigma}$

↳ $C_\Sigma = C_G + 2C$

The CHEMICAL POTENTIAL is given by the energy difference when an electron gets added to the island:



$$\mu(N) = E(N) - E(N-1) = \frac{(C_G V_G - Ne)^2 - (C_G V_G - Ne + e)^2}{2 C_\Sigma} =$$

$$= \frac{-e\left(C_G V_G - Ne + \frac{1}{2} e\right)}{C_\Sigma} =$$

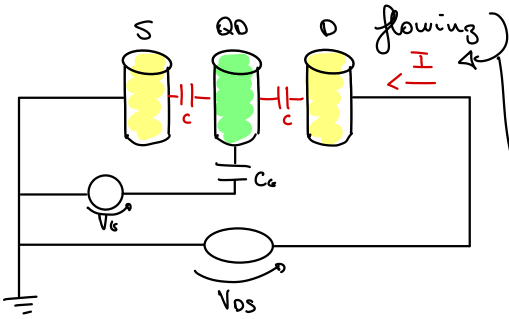
$$= \frac{e}{C_\Sigma}\left[\left(N - \frac{1}{2}\right)e - \boxed{C_G V_G}\right]$$

It is the ADDITION ENERGY $\Delta E = \frac{e^2}{2 C_\Sigma}$

energy that we must supply in order to add an electron in the island.

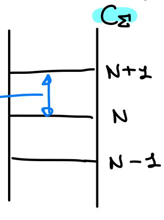
↳ proportional to the number of electrons

We assume a current I flowing



Can I observe any current?

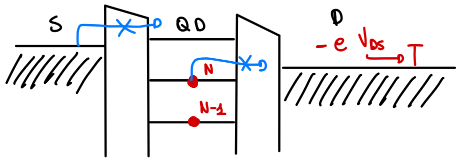
SEE NEXT PAGE



The space represents the Coulomb Blockade (of Q-DOT level)

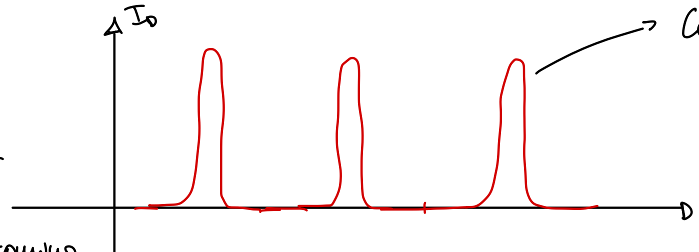
I don't care too much about $C_G V_G$, it will act as a sort of continuous biasing. We use it as a controller.

To answer the question, let's assume to have different electrons in the QDOT



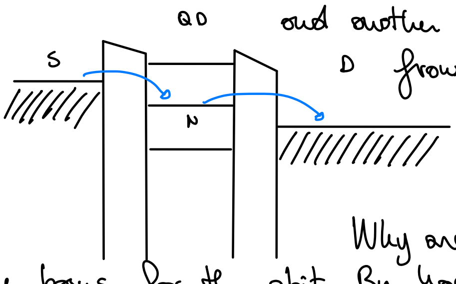
We'll have a tendency of the electrons in the source, to jump into the quantum dot. The electron at LADDER N would like to go toward the DRAIN BUT IT CAN'T because of the energy barrier. The electron from the source instead would like to jump at N+1 but it is at higher energy WRT the one at which the source is. So I have COULOMB BLOCKADE → NO CURRENT CAN FLOW.

If I plot my I-V curve I see:



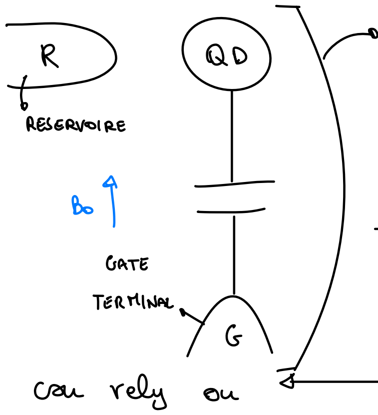
Current RESONANCE ; corresponds to the interval in which the energy level is comprised between the two Fermi levels.

If I increase the LADDER energy, I can raise the N level to a point in which electron is able to escape and another coming from the source

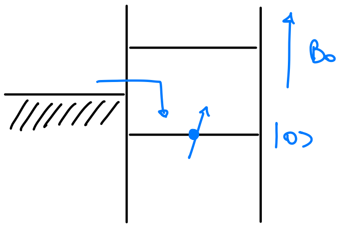


can enter. If I further increase, I may end up going under the Fermi level of the drain and I have again the Coulomb blockade

Why are we interested in SINGLE ELECTRON TRANSISTOR? Because it is the basys for the qbit. By having a single electron stored with its SPIN constitutes a Q-BIT.



Very simplified model for a single Q-BIT.



I can impose my qdot LADDER to be able to host 1 single electron in the LADDER, in order to don't mess up too much the computation with many electrons in the ladder.

I can initialize my qdot in order to start the algorithm from the preferred basys state

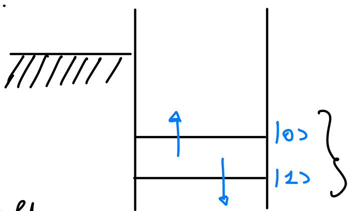
I can manipulate my electron with ESR

For 2 q-bit operations we need some COUPLING.

READ OPERATION: how to read the SPIN? It consists of a magnetic dipole moment we need to sense. In principle it is difficult or almost impossible to apply a magnetic measurement.

We can rely on SPIN TO CHARGE CONVERSION:

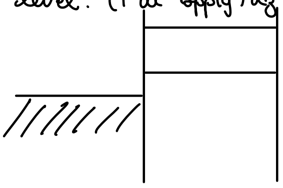
I apply a constant magnetic field and I get the Zeeman Splitting:



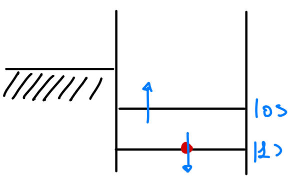
splitting for Bo pointing ↑ UP → remember that the magnetic dipole moment of the electron is ANTIPARALLEL to its SPIN. By applying a magnetic field Bo, we saw that it is possible to generate an energy splitting larger than the thermal energy one. It could be possible to overcome the higher energy ladder step, but we don't care too much since we are dealing with a single electron

We have many phases for the SPIN TO CHARGE conversion:

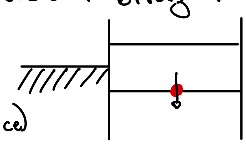
1) We start by applying a voltage V_G that puts the two energy levels of the first ladder step ABOVE the SOURCE Fermi level. (I'm applying a negative voltage in order to discharge the island and get an empty quantum dot



2) I inject one electron in the q-dot. I'm storing a random electron that can be either with SPIN UP (50%) or SPIN DOWN (50%). We now assume to inject an electron with SPIN DOWN.

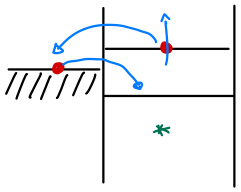


3) I pass to the READ phase in which I bring the two levels middle point to the source Fermi level (Before arriving at (...) this point many operation have taken place)

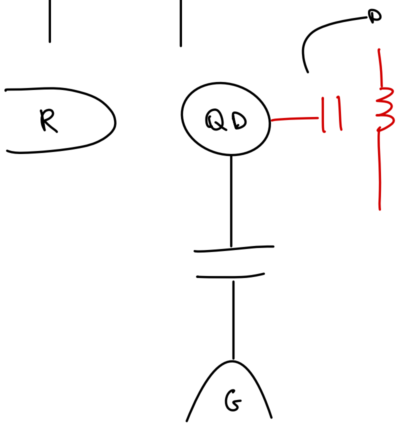


At this point this electron can't escape to the reservoire. I need to sense the charge.

I can sense the charge and this measurement will tell me that the state was $|1\rangle$. In case I had previously stored a $|0\rangle$ state electron, then the situation would have been different:

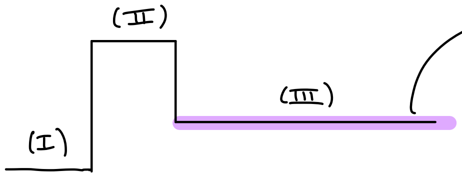


The $|0\rangle$ state electron can escape into the reservoire but from the reservoire another electron can jump into the $|1\rangle$ state, so the probe of charge will show something like $-e \rightarrow 0 \rightarrow -e$ (instead of constant $-e$ charge)



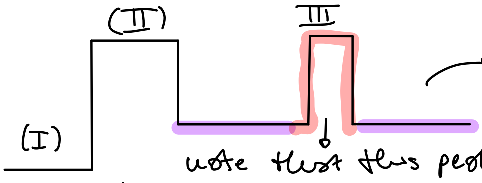
In the end it is like having a resistor capacitively coupled to the quantum dot (it constitutes by itself a sort of transistor) that shows a current depending on the SPIN of the electron stored. We define the QUANTUM POINT CONTACT (QPC) that allows to see, during the three phases, how the current changes and, depending on the behaviour, we understand what SPIN we have.

CASE $|1\rangle$



current remains constant then a $|1\rangle$ is stored

CASE $|0\rangle$



current peaks, then a $|0\rangle$ is stored

note that this peaking given by the electron tunneling is RANDOM due to its nature.

These phases can be used also to initialize our q-dots since as we see, during the READ phase, in the end we always impose a $|1\rangle$ state.

$|1\rangle \longrightarrow |1\rangle$

$|0\rangle \longrightarrow |1\rangle$

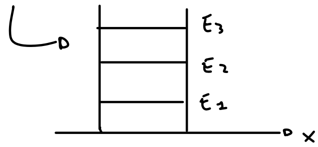
* after we inject the electron to the $|1\rangle$ state, it cannot move due to the coulomb blockade.

For this reason we have that this read operation is usually performed by RF, analyzing the change of RF response.

A quantum dot requires confinement (in 3D) → we need first to create a quantum well (confinement in 1D) where I have my quantized levels.

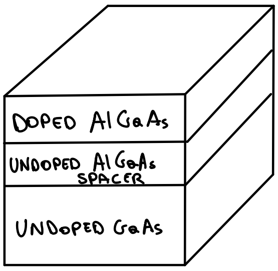
Lecture 12 - 16/11/23

QUESTA LEZIONE È STATA SPIEGATA PRINCIPALMENTE CON LE SLIDE → VEDI DA P. 14 IN POI



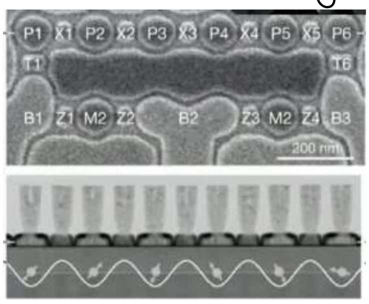
2DEG (2D electron gas)

A typical material used to implement the structure is the GaAs and AlGaAs: GaAs has smaller Bandgap WRT AlGaAs I get a conduction band offset and valence band offset in the transition from GaAs to AlGaAs. This offset, combined with the bands bending, creates a WELL. Such well takes the name of 2DEG and consists of a 2D structure filled with FREE electrons → so it can be assumed to be a gas where the particles can move freely with kinetic energy in x and y directions. Note that this is just a way to confine electrons and not to create a quantum dot.



A similar result can be achieved thanks to other materials like Si and Ge. Si and SiGe are put in contact and the upper Si develops a tensile stress given by the fact that the bottom layer composed by Si-Ge has larger atoms (due to germanium). Such tensile stress promotes the establishment of a conduction band offset → thus implementing a well. If we replicate the structure, at the interface SiGe and Si, it is possible to implement a well in the valence band, thus accumulating holes! There are some advantages in the use of holes

holes accumulation due to valence band offset



X GATES → we apply a negative voltage.

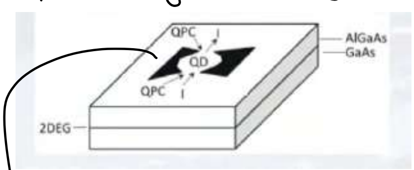
P GATES → PLUNGERS (STANTUFFI) we apply a positive voltage. The plungers are needed in order to attract and create an electrons well.

This alternance establishes a modulation of the potential and the creation of different wells

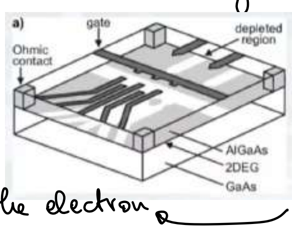
HEMT → High Electron Mobility Transistors

The structures we are analyzing are composed by monocrystalline blocks, almost perfectly matched and defect free, in order to ensure the creation of fast channels in which carriers mobility is very high. For Q-BITS we don't really benefit from such high mobility. Obviously less doping translates in less scattering and so longer decoherence time. The electrons are confined and I don't need high currents.

The Q-DOT is obtained by using some gates electrodes → thanks to such electrodes we can apply a negative voltage that will establish a barrier (looks like a "volcano" with a "VALLEY" where I host my electron) The dot is implemented just with electrodes. We have then some INLETS and OUTLET structures that allow electrons to escape from the potential valley or enter inside of the confinement region.



The shape of the gates allows the creation of a SADDLE point where I can inject my electron, and this can also go out (depending on the applied voltage). This point where I can measure a current is called Quantum Point Contact (QPC) → it is a kind of tunneling region. It is useful for injecting the electron or for sensing the charge.



The grey area is the one where the electrons are present.

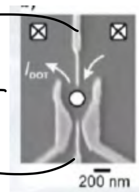
The plumbers are necessary in order to switch the different phases and move the potential of the electron

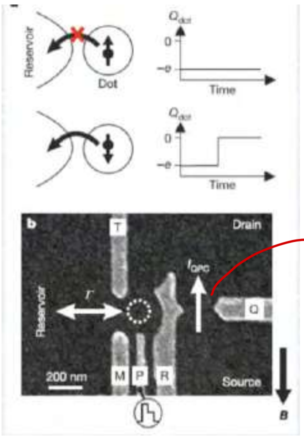
In the ECT I need a gate to move the channel potential up and down for Coulomb Bl.

PLUNGER

SINGLE Q-DOT

PLUNGER

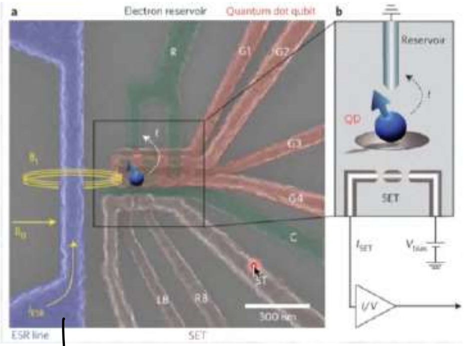




By reading the QPC I'm able to read the SPIN of the electron.
As we have seen in the previous lesson, the reading process can also be used as an initialization step.
The READ operation can be induced in error → due to thermal broadening (TRAD: AMPLIAMENTO) we may have enough energy to let the electron in state $|1\rangle$ jump again into the reservoir. FIDELITY is the measure of a quantum circuit that expresses the ACCURACY with which we READ a Q-DOT.

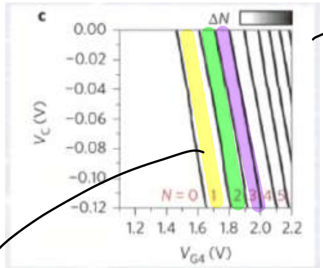
Here I have a QPC
It is like a saddle point where I can sense a current (acts like a sensor)

HOW DO WE CONTROL THE QBIT? With ESR (electron speed resonance). In the structure below (note it has 4 gate electrodes that are thought to host 2 Q-DOTS) we have a readout performed by means of SET (single electron transistor) instead of QPC.



B_0 is 1 T to 10 T and determines the Zeeman splitting and Larmor precession. On top of DC B_0, we apply B_1 AC field.
In this case the island is sensitive to the potential of the quantum dot → it is capacitively coupled to the quantum dot and depending on the potential it accumulates positive or negative charges, thus inducing a movement of the ladder potential energy and promoting (or not) a current from DRAIN to SOURCE

TRANSMISSION LINE → kind of ohmic resistance, used at high frequencies in order to induce a local magnetic field. We have an AC current that generates an AC field.
Such AC field is resonant with Larmor frequency.



This is a stability diagram where we can observe a 3D Plot: white means small current and black large current. The intended current is the one from the RESERVOIR to the QDOT (or out of the QDOT)
V_{G4} is the gate voltage responsible for Q-DOT potential. The V_c potential is operated as the DRAIN TERMINAL of SET. The black lines represent the Coulomb peaks → conditions for which I have a current (remind the concepts of previous lesson where we use gate potential to move the ladder potential WRT the Fermi Level of the RESERVOIR.

In this region I see one electron inside my Q-dot that cannot tunnel out. Then we have the region with 2 electrons, 3 electrons and so on. I want to stay in region 1 → one electron with one spin. WHY DO I HAVE LINES? The Q-DOT is capacitively coupled to all the terminals → such capacitance coupling with the gate, is much stronger WRT R and C terminals. At this point the potential of my Q-DOT will be given by:

↳ R (Reservoir), C (collector) and G_4 (Gate) → this latter one is right on TOP of Q DOT while R and C are on the sides.

$$\Phi = -\frac{N}{C_\Sigma} + V_G \cdot \frac{C_G}{C_\Sigma} + V_C \cdot \frac{C}{C_\Sigma}$$

potential of QDOT

because we deal with electrons

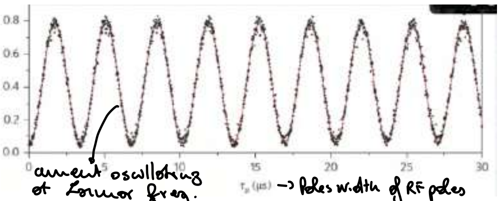
sort of V partition

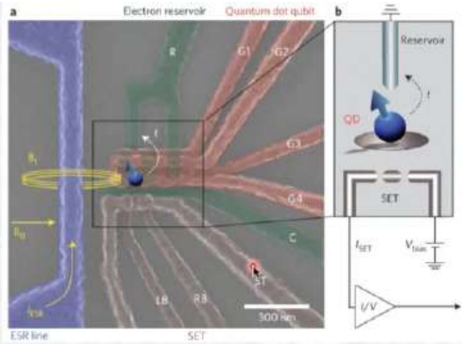
(R is grounded)
$C_\Sigma = C_G + 2C$ = total capacitance
The larger C_G, the larger its impact on the q-dot potential.

I evaluate the potential by superposition, summing up the different contributions: charge dependent potential, potential due to capacitive coupling with gate terminal, contribution due to collector. R is not present since it is grounded.
From the plot of the currents we see the points for which V_G and V_C make my QDOT potential aligned with R and C in order to guarantee an electrons flux. The potential is affected in a very weak way from V_C. By changing V_G we instead have the possibility to pass from N=1 to N=2 and so on.
The current in the SET is independent from the one in the Q-DOT.

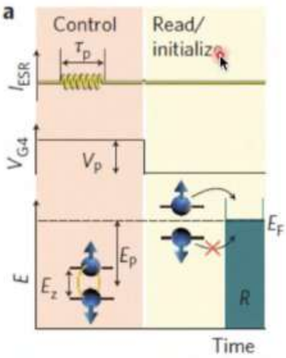
SPIN CONTROL

We control the SPIN by running RABI oscillations. On the plot below we observe the probability to have SPIN UP as a function of time. I assume to have initialized my SPIN (so I have |1>). NOTE: The SPIN direction is defined by the B_0 field so, in the case we are observing, we can have SPIN toward right (higher energy - SPIN UP) or left (lower energy - SPIN DOWN).



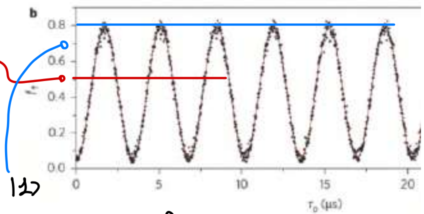


- Probability to measure a SPIN UP. The SPIN is read out by means of SPIN TO CHARGE CONVERSION. In the CONTROL PHASE I apply an ESR field with a certain poles width τ_p (poles width of the RF poles). After this phase I pass to the READ one → my qubit will for sure having been affected by some rotations given by the ESR. The larger the poles width the larger the rotation along x (anti clockwise rotation).



Looking at the probability graph, we see that we don't go from 0 to 1. This is due to the error given by the RESERVOIR BROADENING. The difference between MAX and MIN probability is called VISIBILITY. It can also depend on some error in the gate that makes the rotation happening in a TILTED direction.

Let's assume to apply a $\frac{\pi}{2}$ rotation. I'm at the point highlighted in the graph and it corresponds to have 50% probability to have SPIN UP or SPIN DOWN. With π rotation I see MAX probability to see |1>

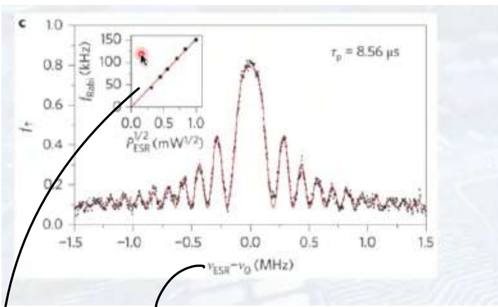


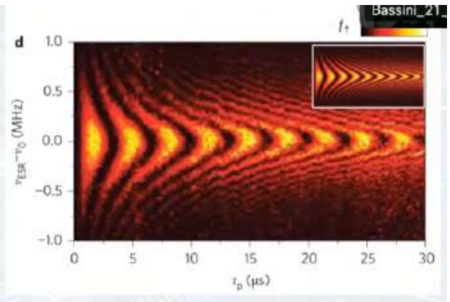
All the points have been obtained by repeating many measurements for different rotations.

We calculate the Larmor frequency as $\nu_0 = g^* \frac{\mu_B B_0}{h}$ g^* is a correction factor.

The following plot expresses the $f_\uparrow$ probability in terms of colors instead of currents.

If we measure the angular frequency we put $\hbar$. $\hbar = 2\pi h$





In the previous case we had ν_{ESR} matched to ν_0. We can opt for detuning our ν_{ESR} WRT Larmor frequency. At detuning, the RABI (apparent) frequency increases.

The effective Rabi frequency goes as:

$\sqrt{\omega_R^2 + \Delta\omega^2}$ → detuning frequency

$\nu_{ESR} - \nu_0$ = DETUNING FREQUENCY

↳ frequency of our signal inside the transmission line

- It is interesting to notice the dependence of Rabi frequency from the power of ESR. It linearly increases with $\sqrt{P_{ESR}}$

QUALITY FACTOR AND FIDELITY

We have already seen that the visibility of the system is not 100%. There are problems with quality factor and fidelity. Quality factor expresses a measure of decoherence time.

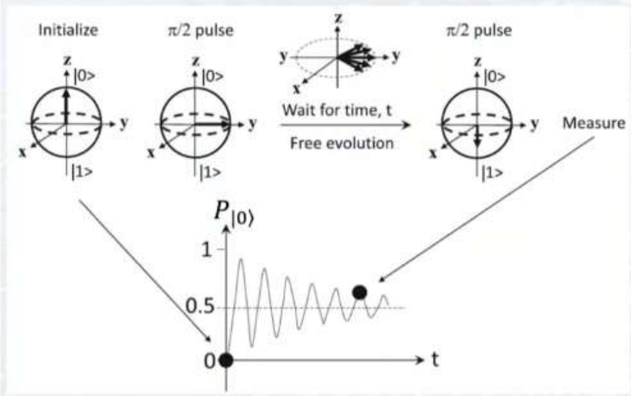
The QUALITY FACTOR expresses the number of RABI rotations I can apply before coherence time ends: typically $Q \sim 10^3$

Q is evaluated by means of T_2^*.

↳ If I repeat my rotation and extend my Rabi oscillation to several time intervals, I'll observe my signal decay → such decay means that there will be errors.

The FIDELITY instead states the accuracy of a quantum gate and how precise I am operating my q-bit. During time we may be expecting to be applying a rotation of $\frac{\pi}{2}$ but we may not be very precise. The FIDELITY is found with RANDOMIZED BENCHMARKING.

RAMSEY EXPERIMENT → It consists of an ensemble measurement in order to evaluate T_2^*



- Ramsey experiment:
  - Initialize to $|0\rangle$ → SPIN UP
  - Rotate $\frac{\pi}{2}(x)$ → Clockwise rotation
  - Wait for time t → During this time there will be some free precession that will be affected by T_2^* → the vector components will start to do precession at different speeds. What usually happens is that the Bo field is affected by oscillations due to local screening. To find T_2^* it is necessary to run the measurement for hours. By measuring just a single Q-BIT we would not be able to extract the ensemble effect.
  - Rotate $\frac{\pi}{2}(x)$
  - Measure the probability $P_{|0\rangle}$
- Due to precession, $P_{|0\rangle}$ oscillates with the Larmor frequency
- $P_{|0\rangle}$ also decays due to the dephasing time T_2^*

We may have Q-BITS which are screened by other Q-BITS with respect to the Bo source and this brings variations from Q-BIT to QBIT.

The T_2^* measures the broadening of the vector components precessions.

After having waited enough we apply $\frac{\pi}{2}(x)$ rotation and we measure again $P_{|0\rangle}$ → in this measurement I have a different situation. We will have that the probability will be more and more close to zero.

→ The time to wait corresponds to the one needed in order to have a 2π precession.

HAHN ECHO → kind of SPIN REFOCUSING. Improves T_2. With Hahn Echo I can evaluate T_2 instead of T_2^*. The T_2 is related to the real decoherence, not related to the precession broadening but to the random scattering behaviour.

The two most explored qubits from industries are SPIN QUBIT and SUPERCONDUCTIVE QUBIT. The SPIN one is implemented by means of GaAs and AlGaAs or Si and SiGe. Into the superconductive implementation we don't find semicon. but metals, oxides and insulating material (Josephson Junction → INSULATOR-METAL-INSULATOR) — Lecture 13 - 23/11/23

One issue of SPIN QUBIT, as we saw, is the COHERENCE → this is one of the Figures of Merit in order to choose the QUBIT technology.

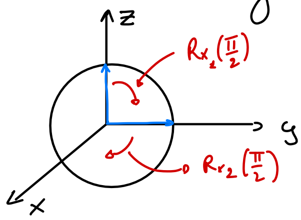
COHERENCE $\simeq \frac{T_{COHERENCE}}{T_G} \geq 1000$ is what we generally need.

T_G → gate time.

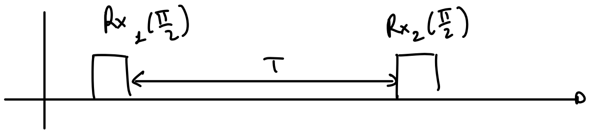
We introduced the Ramsey experiment in order to assess the exact Larmor frequency (w_0) of my system and to assess the T_2^* time.

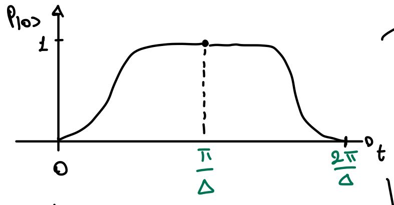
In RAMSEY we first apply a $\frac{\pi}{2}$ rotation along x axis. 1)

Then I wait for a time T and I apply another $R_x(\frac{\pi}{2})$ rotation. Such experiment is a sort of INTERFERENCE. Similar to Mach Zehnder in optics.



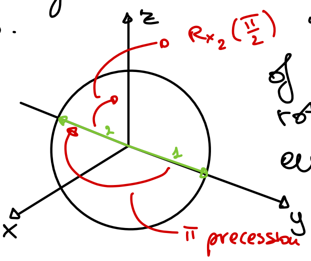
The $R_x(\frac{\pi}{2})$ rotation is operated at a slightly different w WRT w_0 (Larmor frequency) $\Delta = w - w_0$ where such Δ is the detuning frequency.

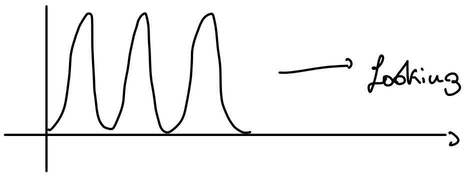




The probability to have $|0\rangle$ after the first rotation is 0. We assume to wait for time T and I will have some precession and with the second rotation I'll get something between $|0\rangle$ and $|1\rangle$.

The time T will be equal to the time to get a π precession along z, such that after a second rotation we should end again in state $|0\rangle$. If I wait for 2π I should be again in the point of before and for this reason the probability to have $|0\rangle$ after the rotation comes back to zero. If I repeat such measure more times, I will end up with an interference pattern whose freq. is given by Δ

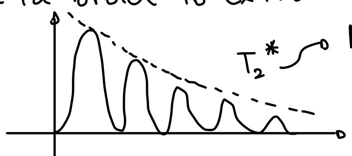




Looking at this plot I can understand which is the detuning frequency and then be able to tune properly our signal.

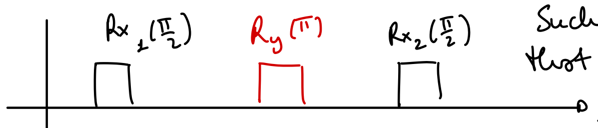
The plot is also useful in order to extract the T_2^* parameter.

In fact, due to dephasing we'll experience a decay



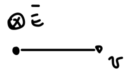
Note that it is referred to more qubits measures or more measurements taken from the same qubit.

A method to improve the T_2^*, as we already mentioned is the HAHN ECHO (and CPMG technique in particular). The HAHN ECHO consists of a RAMSEY EXPERIMENT in which we add an additional pole, $R_y(\pi)$, between the two x rotations.

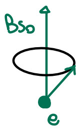


Such rotation compensates for the broadening of the SPIN COMPONENTS that precess at differents speeds. By implementing such technique we get T_2^* approaching T_2. We define T_2^H the T_2 time obtained by applying the HAHN ECHO principle ($R_y(\pi)$ a single time). While by means of CPMG technique we are able to get T_2^{CPMG} which is higher (in CPMG we apply the rotation $R_y(\pi)$ several times - 100 ∼ 500 times).

It is important to understand why we have such decoherence: 1) SPIN ORBIT INTERACTION: According to the special relativity if we have an electron moving at a certain velocity and it interacts with a perpendicular electric field, a SPIN ORBIT MAGNETIC FIELD develops: $B_{SO} \sim \underline{v} \wedge \underline{E}$

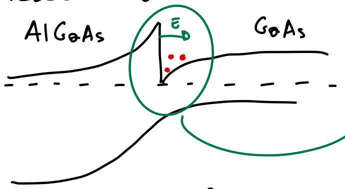


So we'll have an electron spin within a DC field → this determines a precession.



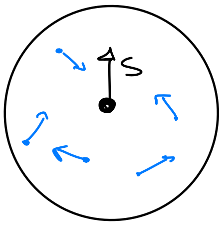
This effect is a DISTURB → it represents an additional and random fluctuation of the electrons' spin. Although such magnetic field is low, it will introduce a RANDOM WALK inside my bloch sphere. Usually I can make sure no electric field is present, but it is impossible to avoid the local electric field → this is because of ions. In GaAs for example we have Ga and

As that are polar. In the case of the 2DEG we remember the 2D structure that presents a band structure as follows



such bending translates in an electric field where the electrons are confined in the well

This determines the presence of a local electric field at the qubit level.
The second DEPHASING responsible is the 2) HYPERFINE INTERACTION: it consists of the interaction the electron spin with the "bath" of nuclear spins.
Let's suppose to have an island that represents my Q-DOT with some SPIN. My electron is orbiting inside my island → this island is composed by atoms and their nucleons will have a SPIN which is randomly oriented. Each nuclear SPIN is in the order of $\frac{\hbar}{2}$ but the magnetic dipole moment will be much different



$$\underline{\mu} = \gamma \cdot \underline{S} = g \cdot \frac{q}{2m} \cdot S$$

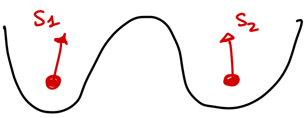
such mass is much larger for the nucleons, this makes their magnetic dipole moment very low and therefore the SPIN is weaker than electron one. BUT, we have 10^6 nucleons in the island we are observing → the contribution is not negligible.

Each nucleous represents a local micro magnetic field that interacts with the electron spin and induces fluctuations.
In order to get rid of this effect we have to avoid elements with nuclear spin.
For example we can use ^{28}Si → It presents same number of protons and neutrons whose spins will mostly arrange themselves in antiparallel way, thus determining NO NUCLEAR SPIN. In nature such ^{28}Si represents the 92% of all the Si isotopes. This means that a lower percentage of disturbing isotopes is still present. It is however possible to purify an Silicon and therefore employ ^{28}Si at 100% in order to implement our devices.

	p	n	I	
^{28}Si	14	14	0	→ 92%
^{29}Si	14	15	1/2	→ 5%
^{30}Si	14	16	0	→ 3%

GaAs SPIN QUBITS are influenced by hyperfine interaction due to the nature of the elements employed. They belong to 4th and 6th group and will present a nuclear spin of 1/2 → such effect cannot be compensated in GaAs.

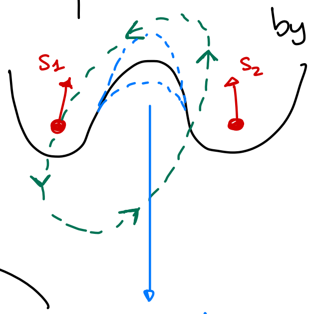
HOW ABOUT THE DOUBLE SPIN QUBIT? Let's suppose to have two spin qubits close to each other and they can interact. In the CLIFFORD group of quantum gates we have H, S, CNOT.



↳ we have the potential wells where the two electrons are hosted.

Supposing that we want to apply a CNOT, we will have to put in contact the two qubits. After putting the electrons are close to each other we can have the EXCHANGE INTERACTION: taking in consideration the situation below, we have that being the S_1 SPIN pointing up, a magnetic dipole moment will be present in the opposite direction and determines a fringing magnetic field that interacts with the other qubit. The second qubit will be characterized by an energy which is influenced by such magnetic field.

The lower the barrier the higher the coupling. The barrier level is established on the basis of the operation we want to perform. In the case of a CNOT we would like for example to FLIP the spin.



J depends on how much I couple the two qubits and therefore on how I impose the potential barrier.

$$\underbrace{U_2}_{\text{potential energy}} = -\underbrace{\mu_2}_{\text{magnetic dipole moment of the second qubit spin}} \cdot B \longrightarrow \text{determined by the first qubit}$$

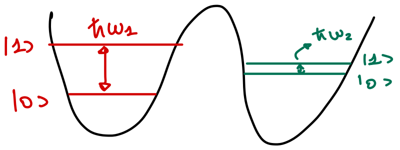
$\mu_2 = \gamma S_2$

We can define an interaction energy $\hat{U} = J \hat{\sigma}_{z_1} \otimes \hat{\sigma}_{z_2}$ that depends on a tunable parameter J

$\hat{\sigma}_{z_1}$ → Pauli operator of first qubit

Now we write the Hamiltonian: $\hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{H}_{int} =$

$\hookrightarrow$ Hamiltonian of the interaction, it corresponds to $\hat{U} = \hat{H}_{int}$



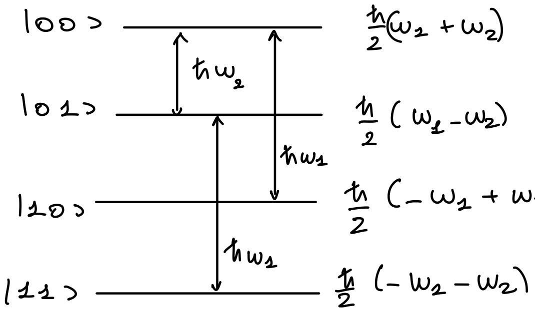
$$= \frac{\hbar\omega_1}{2}\hat{\sigma}_{z_1}\otimes\hat{I} + \quad (\hat{H}_1)$$
$$+ \frac{\hbar\omega_2}{2}\hat{I}\otimes\hat{\sigma}_{z_2} + \quad (\hat{H}_2)$$
$$+ J\hat{\sigma}_{z_1}\otimes\hat{\sigma}_{z_2} \quad (\hat{U} = \hat{H}_{int})$$

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

This represents where I'm moving the original Eigenvalue due to interaction

$$\hat{H} = \frac{\hbar}{2}\begin{pmatrix} \omega_1 & 0 & 0 & 0 \\ 0 & \omega_1 & 0 & 0 \\ 0 & 0 & -\omega_1 & 0 \\ 0 & 0 & 0 & -\omega_1 \end{pmatrix} + \frac{\hbar}{2}\begin{pmatrix} \omega_2 & 0 & 0 & 0 \\ 0 & -\omega_2 & 0 & 0 \\ 0 & 0 & \omega_2 & 0 \\ 0 & 0 & 0 & -\omega_2 \end{pmatrix} + J\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} \omega_1+\omega_2+\frac{2J}{\hbar} & 0 & 0 & 0 \\ 0 & \omega_1-\omega_2-\frac{2J}{\hbar} & 0 & 0 \\ 0 & 0 & -\omega_1+\omega_2-\frac{2J}{\hbar} & 0 \\ 0 & 0 & 0 & -\omega_1-\omega_2+\frac{2J}{\hbar} \end{pmatrix}$$

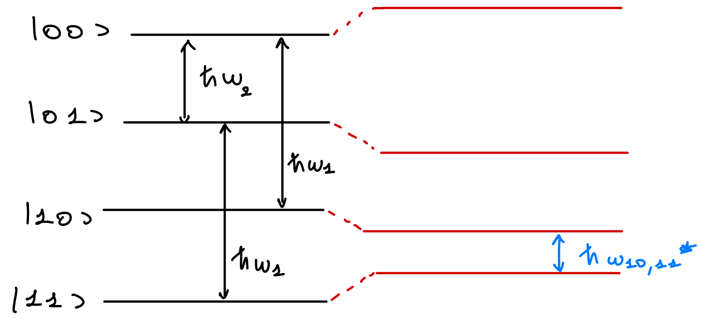
Let's assume $J=0$ and so No interaction: as we see below all the spacings between the different energy levels are equal.



If I want to transition from $|10\rangle$ to $|11\rangle$, it is difficult to handle it if we don't do the first two transitions.

Now $J>0 \to$ positive J introduces a positive term for $|00\rangle$ and $|11\rangle$ (pushing up the energy levels) and a negative one for $|01\rangle$ and $|10\rangle$ (pushing down the energy levels).

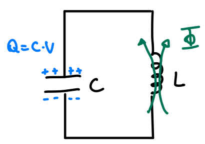
As we can see it is possible to tune the energy levels differences in order to promote a certain switch. Thanks to the coupling I'm able to move the energy levels and obtain different and unique splitting $\to$ these splittings allow to individuate a unique ω_u^* at which I'll tune my system in order to apply a RABI OSCILLATION and cause a ROTATION.



Let's assume that I want to apply a π rotation. I apply RABI poles tuned to $\omega_{10,11} \to$ ONLY the states $|10\rangle$ and $|11\rangle$ will be affected by such rotation. If I apply a π rotation to $|10\rangle$ it means that I switch to $|11\rangle$. If I apply it to $|11\rangle$ I obtain $|10\rangle$. What are these switches equal to? $\to$ CNOT $\begin{cases} |10\rangle \to |11\rangle \\ |11\rangle \to |10\rangle \end{cases}$ As we see, the CNOT operation is implemented by a coupling of the two electrons and a π rotation.

SUPECONDUCTING QUBIT : INTRO

Let's assume to have a LC resonator: the capacitance accumulates charge $\to$ determines a voltage $\to$ determines a flux Φ of the magnetic field through the inductance.



The behaviour resembles an electromagnetic wave that oscillates back and forth between the capacitance and the inductor.

$$U_C = \int \underbrace{V \cdot I}\, dt = \int C \cdot \frac{dV}{\cancel{dt}} \cdot V \cdot \cancel{dt} = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$\to$ we already found this formula for defining the electrostatic energy of the quantum dot and the corresponding Coulomb Blockade.

energy stored in the capacitance

Power integrated in time

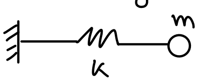
energy of the inductance

$$U_L = \int VI\,dt = \int L\frac{dI}{\cancel{dt}}I\,\cancel{dt} = \frac{LI^2}{2} = \frac{\Phi^2}{2L} \longrightarrow U = U_C + U_L = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$\Phi = LI$

$\to$ Total energy of the oscillating circuit. The energy is continuously exchanged in the form of charge or electric field (into the C) to the form of flux or current (into the L) and viceversa.

The LC acts as a pendulum or a mass oscillating WRT a point since it is affected by an elastic force we know that this system has an energy that is composed by a potential one given by



$\hookrightarrow$ elastic constant

the elastic force and the other by the kinetic energy of the mass. Also in this case we have a continuous exchange between the two types of energy and we have the possibility to find a parallelism.

$$U = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

$\frac{p^2}{2m}$ → POTENTIAL; $\frac{1}{2}kx^2$ → KINETIC

The charge energy can be seen as a potential energy and the flux energy can be seen as a kinetic energy. We want then to move to a quantum visualization and individuate the operators for Φ and Q.

$x \longleftrightarrow \Phi$

$p \longleftrightarrow Q$

$L \longleftrightarrow \frac{1}{k}$

$C \longleftrightarrow m$

The momentum operator is $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ → we can do the same for Q ⟶ $\hat{Q} = -i\hbar\frac{\partial}{\partial \Phi}$

$\hat{x}$ and $\hat{p}$ are conjugate variables → $[\hat{x},\hat{p}] = i\hbar$ it means that the commutator is different from zero: $\hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$ → In quantum physics being conjugate variables means to respect such rule and also to the uncertainity principle $\Delta x \Delta p \geq \frac{\hbar}{2}$

$\hat{Q}$ and $\hat{\Phi}$ are characterized by the same commutator and the same uncertainty principle applies.

At this point we can write the Hamiltonian $\hat{H} = \frac{\Phi^2}{2L} - \frac{\hbar^2}{2C}\frac{\partial^2}{\partial \Phi^2}$ → QUANTUM HARMONIC OSCILLATOR

$i^2 = -1$

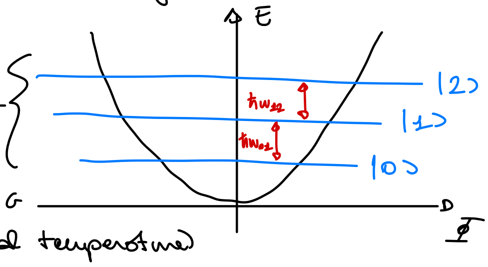
Solutions are given by solving the Schrödinger's equation $\hat{H}\Psi = E\Psi$ (time independent Schrödinger's equation)

By solving analitically we find

QUANTIZED ENERGY LEVELS

where $E_n = \left(n + \frac{1}{2}\right)\hbar\omega_0$

ω_0 in the case of mechanic oscillator is $\omega_0 = \sqrt{\frac{k}{m}}$ → $\omega_0 = \frac{1}{\sqrt{LC}}$



$\hbar\omega_{01} = \hbar\omega_{12}$

PROBLEM FOR THE USE AS QUANTUM BITS

In a REAL CIRCUIT we always have a resistance BUT thanks to the SUPERCONDUCTING STATE we can consider $R=0$. (For $T < T_c$ critical temperature)

Resistance introduces SCATTERING → DEPHASING and ENERGY LOSSES.

This system allows to create a quantum circuit but it actually presents a critical point: we have to limit the energy levels to only $|0\rangle$ and $|1\rangle$ → If I'm in the $|1\rangle$ state and I apply a Rabi oscillation, I'm not sure to switch my state toward $|0\rangle$ or $|2\rangle$. The superconductive regime will let us achieve such result.

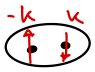
An alternative picture for the description of the Hamiltonian is achieved by looking at the capacitive charge expressed by means of Cooper pairs.

$$U_C = \frac{Q^2}{2C} = \frac{(2Ne)^2}{2C}$$

$Q = 2Ne$

e ↳ charge of the single electron $= 1{,}6 \cdot 10^{-19}$ C

N ↳ number of pairs of electrons



→ In a Cooper pair we have two electrons with antiparallel SPIN → so the overall SPIN of the pair is 0 → it is not a Fermion anymore but it is a BOSON : obeys to Bose-Einstein statistic.

We have to rewrite the Hamiltonian but this time in terms of $\hat{N}$ operator:

since $\hat{Q} = -i\hbar\frac{\partial}{\partial \Phi}$, I can introduce the flux quantization that puts phase and flux in relation

$\phi = 2\pi\frac{\Phi}{\Phi_0}$

ϕ → PHASE IN [rad]

Φ_0 → QUANTUM FLUX → the magnetic field flux is quantized in nature. We have always an integer number $\Phi_0 = \frac{h}{2e}$

$$\hat{Q} = -i\hbar\frac{2\pi}{\Phi_0}\frac{\partial}{\partial\phi} \xrightarrow{\text{SWITCH TO } \hat{N}} \hat{N} = \frac{\hat{Q}}{2e} = -i\frac{\partial}{\partial\phi}$$

As a result the new Hamiltonian will be

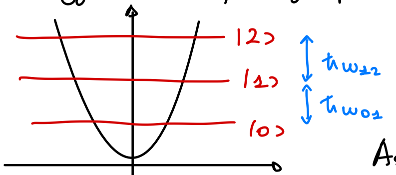
$$\hat{H} = -\frac{4e^2}{2C}\frac{\partial^2}{\partial\phi^2} + \frac{\Phi_0^2}{2L(2\pi)^2}\phi^2 = -4E_C\frac{\partial^2}{\partial\phi^2} + \frac{E_L}{2}\phi^2$$

$\frac{e^2}{2C}$ → E_C (charging energy)

$\frac{\Phi_0^2}{L(2\pi)^2}$ → inductive energy

By plotting the new Hamiltonian we have pretty much the same problem of before. We must introduce the superconducting principle.

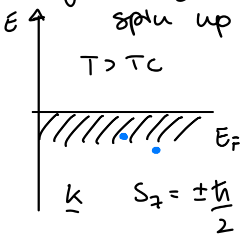
We've seen that due to the harmonic behaviour of the LC circuit, it is impossible to use that system as a qubit. In fact we've seen the QHO (Quantum Harmonic Oscillator) Hamiltonian that presents more energy levels, equally spaced in terms of energy difference between consecutive levels.



We now introduce the principles of:

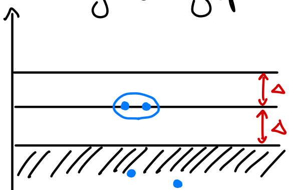
- SUPERCONDUCTIVITY
- JOSEPHSON JUNCTION (NON HARMONIC OSCILLATOR)

As we already pointed out, superconductivity is achieved under a critical temperature T_c, at which the observed material shows a NON RESISTIVE BEHAVIOUR. For mercury (Hg) $T_c = 4.2K \rightarrow @\ T < 4.2K \rightarrow R = 0$. This means to have a current even in absence of Voltage.
(For $Al \rightarrow T_c = 1.2K$ and $Nb \rightarrow T_c = 9K$) Below such temperature there's a phase transition inside the materials that involves the carriers.
In a typical metal we have a Fermi level → below such Fermi level there are states filled by electrons (tipically two electrons) and each state presents two electrons with spin up and spin down. This is the situation at environment temperature, where there's an abrupt transition from void level to under Fermi level.



↓
In this region it is very difficult to find any electron.

Below the T_c, we have the formation of a gap around E_F:

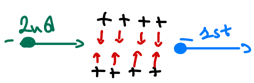


At E_F we find the Cooper pairs (pair of electrons within a boundary).
Such pair present opposite $k \rightarrow \pm k$ and spin $S_z = \pm \frac{\hbar}{2}$. Although the single electrons are Fermions, the Cooper pair is a Boson with overall SPIN equal to zero. ($S = 0$)
↓
Obeys therefore to Bose-Einstein statistic. Under a certain temperature such Bosons can go toward Bose Condensation.

Bose condensation:
Bosons are characterized by electrons with opposite spins, therefore the overall spin of the Cooper pair is zero and they condensate right at Fermi level.
The Cooper pairs show a coherent behaviour and don't SCATTER. Therefore we have no RESISTIVE EFFECT.

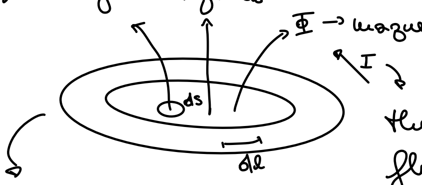
NOTE: The electrons of the Cooper pair are not one close to the other, they are coupled by the opposite spins but they can be very far from each other inside the material.

BARDEEN-COOPER-SCHRIFFER THEORY (BCS) → We can assume that when an electron is moving, all the positive ions in the surroundings will experience a Coulomb attraction → they react to the presence of the electron and I experience an elastic effect. Such effect creates a local increase of potential because there's an accumulation of positive charges. Such positive charges will induce the second electron pair to move coherently WRT the first electron. The attraction of positive ions towards the electron constitutes a PHONON and, the fact that the second electron moves on the same trajectory of the first one, makes the Cooper pair COHERENT.



Because of this COOPER PAIRS in SUPERCONDUCTING MATERIAL we have some particular properties:

1) The first regards the relation between FLUX and PHASE.



The current can flow even without any voltage applied.

Within this loop I have electrons and Cooper pairs flowing → I have to figure out the wave function with which they are moving

density $[cm^{-3}]$

$$\psi = \sqrt{n_s} \cdot e^{i\phi} \longrightarrow J = |\psi|^2 \cdot v\ [s^{-1}\ cm^{-2}]$$

current flux

Now, within the loop, I can compute the phase around it: $\phi = \frac{2\pi}{\Phi_0} \oint \underline{A}\, d\underline{l}$ line integral

↳ A is the VECTOR POTENTIAL * related to the B field

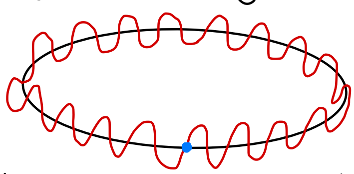
↳ FLUX QUANTUM $= \frac{h}{2e} = \Phi_0$

* $\underline{A}$ is defined in order to have $\underline{B} = \nabla \wedge \underline{A}$

$\underline{B}$ is the ROTOR of the potential vector A (or the "CURL")

Since $\underline{B}$ is the ROTOR of A we can shift to a surface integral: $\phi = \frac{2\pi}{\Phi_0} \int_S \underbrace{\underline{B} \cdot d\underline{S}}_{\text{B flux across the loop}} = 2\pi \cdot \frac{\Phi}{\Phi_0}$

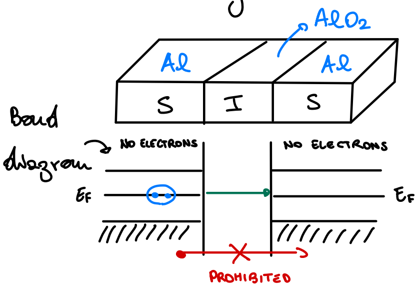
since we are in a loop, the phase must be coherent (like a Bohr atom electron that travels along an orbit and reaches the same places with the same phase). This means that such phase must cover an integer number of 2π $\longrightarrow$ $\phi = 2\pi \frac{\Phi}{\Phi_0} = 2\pi n$ $\rightarrow$ From this we understand that we created a sort of artificial atom with



electrons orbiting and keeping their phase, but also that the flux is a QUANTIZED value since it can be expressed as an integer number of the quantum flux: $\Phi = n\Phi_0$ The B field is not quantized, it is continuous.

JOSEPHSON JUNCTION

It consists of a SUPERCONDUCTOR - INSULATOR - SUPERCONDUCTOR Junction. It resembles a capacitance (Metal - Ins. - Metal). Below the Tc we must reason in terms of Cooper pairs. The band diagram shows the parity between the two Fermi levels and we have the gaps around with the accumulation of Cooper pairs at E_F. We have that these pairs can tunnel. Electrons instead remain confined due to the absence of free states in the other region and the absence of an applied voltage.

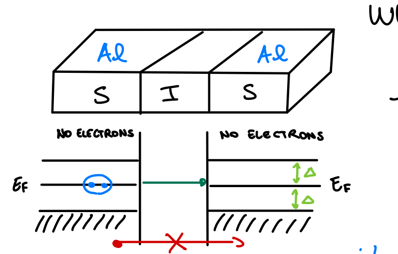


The tunneling looks more difficult since we have to let 2 electrons to tunnel in a coherent way. In reality it appears to be easier than what appears and such tunneling happens at $V = \emptyset$. Such current expression comes from first Josephson relation:

$I = I_0 \sin\phi$ $\rightarrow$ where this ϕ is the phase difference across the Josephson Junction.

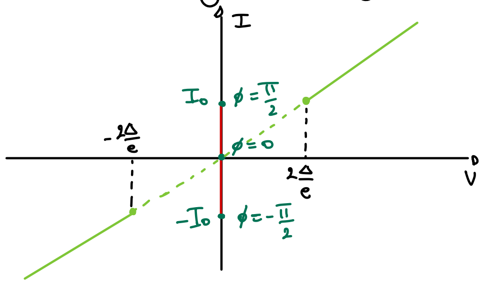
What is driving the current is the PHASE instead of a voltage

$\phi = \phi_2 - \phi_1$



$\psi_1 = \sqrt{n_s}\, e^{i\phi_1}$ $\qquad$ $\psi_2 = \sqrt{n_s} \cdot e^{i\phi_2}$

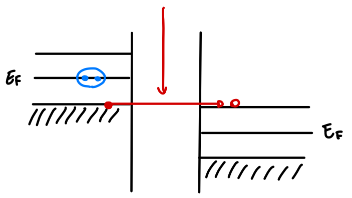
If we have to write the I-V curve we see:



in green we have the Ohmic regime that verifies for a voltage above $\frac{2\Delta}{e}$

In Ohmic regime we shift our left band up of 2Δ and in this way the electrons are able to reach the free states region on the other side.

ALLOWED

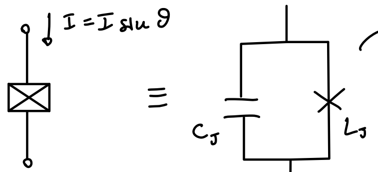


Since the movement of such electrons is not coherent then they introduce scattering $\rightarrow$ that's why we have the Ohmic regime $\rightarrow$ finite resistance. In such regime we don't use the material as a superconductor. We will use this regime in order to apply reading operations.

The second Josephson relationship regards the voltage: $V = \frac{d\Phi}{dt} = \frac{\Phi_0}{2\pi} \frac{d\phi}{dt} = \frac{\hbar}{2e} \frac{d\phi}{dt}$ $\rightarrow$ this ϕ is related to the phase difference

this relation tells us that if the phase changes with time, we experience a (small) voltage.

USE OF JOSEPHSON JUNCTION IN A CIRCUIT



$\rightarrow$ with such representation we split the capacitive component from the inductive one. The capacitive effect is not so strong due to the small value of $C_J = \varepsilon_0 \varepsilon_r \frac{A}{t_{ox}}$ $\rightarrow$ for $1\,\mu m^2$ and $t_{ox} = 10\,nm$ $C_J = 80\,fF$ (very small)

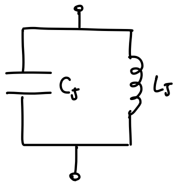
Usually J-Junction is much smaller

Where does the inductance come from? If we rewrite the second Josephson relation: $V = \frac{\hbar}{2e} \frac{d\phi}{dt} = \frac{\hbar}{2e \cdot I_0 \cos\phi} \frac{dI}{dt}$ $\rightarrow$ we obtain an expression CONSTANT $\cdot$ DERIVATIVE OF CURRENT and such CONSTANT VALUE is the INDUCTANCE.

$\frac{dI}{dt} = I_0 \cos\phi \cdot \frac{d\phi}{dt}$ $\qquad = L_J \frac{dI}{dt}$ $\rightarrow$ JOSEPHSON INDUCTANCE $L_J = \frac{L_{J0}}{\cos\phi}$ $\rightarrow$ it is not a real constant, it is controlled by the phase.

Therefore the Josephson Junction can be seen as a RESONATOR where the inductance depends on the phase (we are still not matching the requirement of a NON HARMONIC OSCILLATOR)

We now want to write the Hamiltonian for the Josephson Junction:



first we compute the potential energy

$$U_J = \int_0^t \underbrace{V \cdot I}_{} \cdot dt = \int_0^t \frac{\Phi_0}{2\pi} \frac{d\phi}{\cancel{dt}} I_0 \sin\phi \, \cancel{dt} = \int_0^{\phi} \frac{\Phi_0 I_0}{2\pi} \sin\phi \, d\phi =$$

I can replace with J. relations

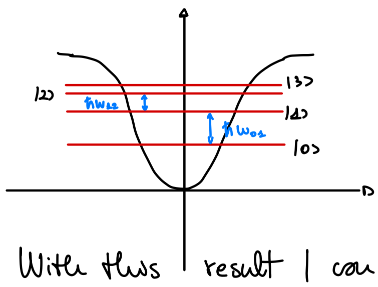
$$= \underbrace{\frac{\Phi I_0}{2\pi}}_{E_J}(1-\cos\phi)$$

$\hat{H} = U_C + U_L = 4E_C \hat{N}^2 + \frac{1}{2} E_L \phi^2 \rightarrow$ we are used to write our Hamiltonian in this way.

$\hat{N} = -i \frac{\partial}{\partial\phi}$ $\quad E_C = \frac{e^2}{2C} \rightarrow$ charging energy

What I do now is replacing the U_L with U_J since we deal with a Josephson Junction.

$\hat{H} = U_C + U_J = -4E_C \frac{\partial^2}{\partial\phi^2} - E_J \cos\phi$



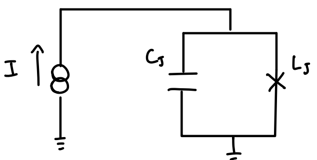
↳ $\cos\phi$ plot resembles a parabola but it is not the same thing: in fact what we get from the eigenvalues calculation is that the different states are not equally spaced and there's a distinction among the energy differences $\hbar\omega_{01}$, $\hbar\omega_{12}$ ...

So I get a NON HARMONIC OSCILLATOR and this can be used as a qubit.

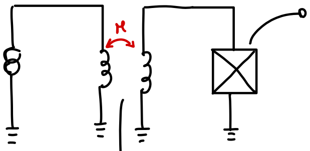
With this result I can use the Josephson Junction as an element of my quantum circuit.

We have a variety of Josephson Junctions: (when we see "x" QUBIT it means that the quantity "x" is a good quantum number for the application)

PHASE QUBIT



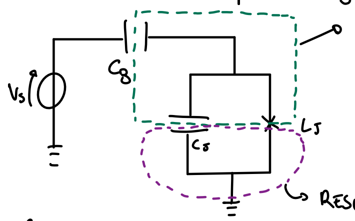
FLUX QUBIT: I can bias my Junction with a flux



Magnetic coupling

Also here I have the inductance that constitutes the NON HARMONIC OSCILLATOR.

CHARGE QUBIT: capacitively driven Junction



COOPER PAIR BOX: it is a sort of island, isolated piece of my circuit capacitively coupled to the voltage source on one side and with the RESERVOIR on the other.

The exchange between island and reservoir allows TUNNELING → I insert cooper pair in my island. This means that we bias the Junction by inserting charges (charge qubit)

↳ RESERVOIR

The industry is actually focusing on this kind of qubit which can be further differentiated in:

1) COOPER PAIR BOX (the one already shown)
2) TRANSMON (most used, actually)

} They are almost the same but we have some differences in terms of energies: $E_J \leq E_C \rightarrow$ we have a Cooper pair box
$E_J \gg E_C \rightarrow$ we have a TRANSMON

COOPER PAIR BOX VS TRANSMON

I have to recompute the Hamiltonian since before we did it only for the Junction, but now we have more elements:

$\hat{H} = 4E_C(\hat{N} - N_g)^2 - E_J \cos\phi =$

Net charge

↳ Inductive potential due to Junction

↓ Can be achieved with a large capacitance

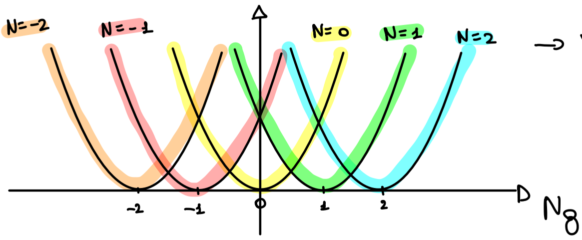
↳ $E_C = \frac{e^2}{2C}$ we cannot do much with E_J, but we can operate on E_C.

$N_g = \frac{C_g V_g}{2e}$ NON INTEGER NUMBER → we can't count the number of electrons on a capacitance plate since it changes depending on the continuously varying voltage. It is possible to have an average value of N_g by keeping the voltage constant.

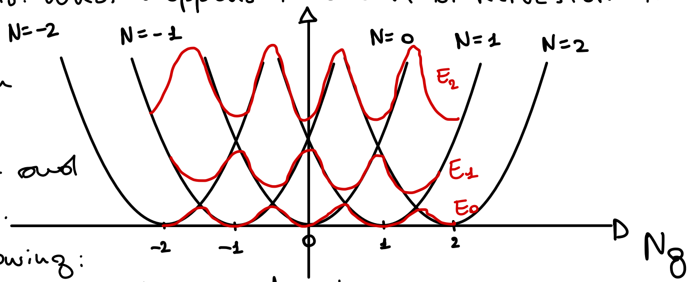
Such value acts as a knob that we can manipulate by tuning V_g. I can use it to bias my Cooper box to a certain point in order to achieve the needed properties.

$= 4E_C\left(-i\frac{\partial}{\partial\phi} - N_g\right)^2 - E_g \cos\phi$

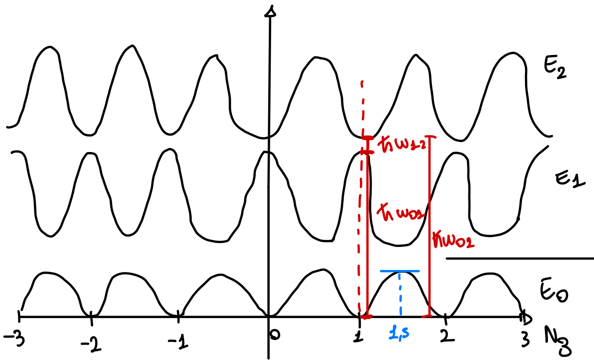
Depending on N_g I'll have different eigenvalues. Moreover, depending on E_J or E_C we have a different impact of the capacitive VS inductive component. Assuming $E_J = 0$, the energies will look like independent parabolas (the different parabolas are defined on the basis of $N \rightarrow$ number of COOPER PAIRS INSIDE THE BOX.



→ The role of E_J in these parabolas is to promote the coupling within the bands. What happens is a sort of REPULSION in correspondance of the interaction points of the parabolas. The larger the E_J, the larger the repulsion and the GAP in in correspondance of the interaction points.



In terms of energy bands, what I get is therefore the following:



We get that the eigenvalues can be tuned with N_g and therefore also the splitting and the qubit frequency can be changed.

$E_1 - E_0 = \hbar\omega_{01}$

$E_2 - E_1 = \hbar\omega_{12}$ → I have to make sure that this value is different from the one before, this states the operating point of the Cooper pair box. We find out that the best operating points are for N_g = integer number

→ As we see from the plot $\hbar\omega_{01} >> \hbar\omega_{12}$ BUT we have a problem since $\hbar\omega_{01} \simeq \hbar\omega_{02}$ → this might introduce wrong transitions from state $|0\rangle$ to $|2\rangle$, resulting in a LEAKAGE effect

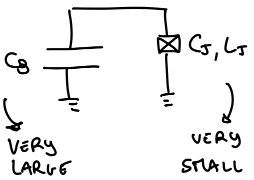
Due to the problem we explained, it might be better to use the HALF-INTEGER option for N_G.

Instead of seating at 1, 2 ... we seat at 1.5 → At this point I have a FLAT EIGENVALUE, resulting in $\frac{\partial \omega_{01}}{\partial N_g} = 0$ → I need this property since N_g is not only defined by the V_g, but also from the charge noise → In my coupling capacitance C_g there could be interference (mainly 1/f noise) that makes my N_g to move → If the eigenvalues change over time it is a mess → we couldn't calibrate correctly our system and also the decoherence would be affected.

The solution to such problem comes with the necessity of having completely flat eigen values: the solution is to have $E_J >> E_C$ → TRANSMON

↳ The N_g contribution can be made negligible. The coupling between the bands becomes so large that the eigenvalues are practically flat.

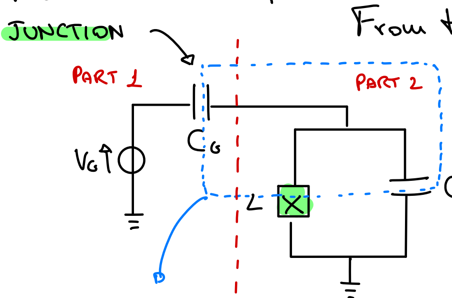
My TRANSMON will be:



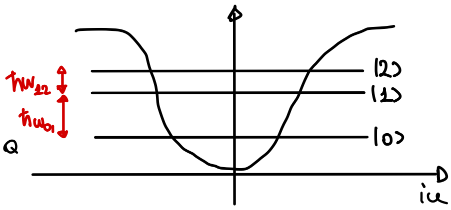
$$\hat{H} = -4E_C \frac{\partial^2}{\partial\phi^2} - E_J \cos\phi$$

We were dealing with SUPERCONDUCTING QUBITS and we saw that our purpose is to build an ARTIFICIAL ATOM by means of a METAL-INSULATOR-METAL plate, where the metal is kept under its critical temperature. We have the non linear device: **JOSEPHSON JUNCTION**

Lecture 15 - 21/12/23



From this circuit we have the HAMILTONIAN as the one that follows:



in Josephson Junction case we get an Hamiltonian whose different levels are NOT EVENLY SPACED → this allows to properly tune the frequency in order to modulate the qubit state.

We identify this as the Cooper pair box. Here we can isolate a certain number of Cooper pairs. Depending on the amount of charge we can have different 0s and 1s (the eigenvalues are different). We have two cases depending on E_J and E_C

The Hamiltonian can be written as: $\hat{H} = -4E_C \frac{\partial^2}{\partial\phi^2} - E_J\cos\phi$, with $E_C = \frac{e^2}{2C_{tot}}$ and $E_J = \frac{I_0 \Phi_0}{2\pi}$

} Being sensitive to charge means to be sensitive also to charge noise → SHORT COHERENCE TIME → we must be immune to charge noise → C_Q MUST BE LARGE.

C_Q large means E_C small and this brings to $E_J \gg E_C$ → TRANSMON

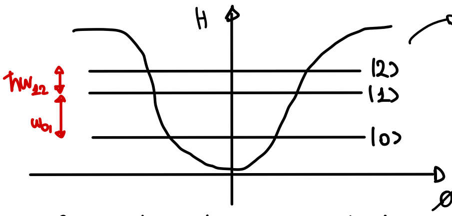
In this limit we are not sensitive to N_G (number of Cooper pairs) and we can get rid of the PART 1 of the circuit.

The **advantage of TRANSMON is the LONG COHERENCE TIME**: T_1, T_2 or T_ϕ (PARAMETERS) ($\approx 100\,\mu s$)

↳ Very high coherence time. One of the highest among the different qubits.

↳ most critical time; it is the dephasing time (jitter). Although we are not loosing energy we are dephasing (it happens much more frequently)

A **drawback is the SMALL UNHARMONICITY**:



when we introduced the Josephson junction, we did it in order to gain an Hamiltonian such that the different eigenvalues would have been sufficiently distant and to get a behaviour as far as possible from the PARABOLA, which highlights an HARMONIC BEHAVIOUR. Due to the dominating parameter $E_J\cos\phi$ related to the Josephson Junction (in the case of TRANSMON with $E_J \gg E_C$), we see that $\cos\phi$ is not an UNHARMONIC function and what will happen is that all the eigenvalues will collapse toward the bottom.

To illustrate such unharmonicity we introduce the formula for the TRANSMON EIGENVALUES:

$E_n \simeq \hbar\omega_0\left(n+\frac{1}{2}\right) - \frac{E_C}{12}(6n^2+6n-3)$ → analytically derived unharmonic function due to E_C contribution

$n = 0,1,2$ quantum number

↳ formula for the harmonic oscillator (see second lecture)

$\omega_{01} = \frac{E_1 - E_0}{\hbar} = \omega_0 - \frac{E_C}{12\hbar}(6+6\cancel{-3}\cancel{+3}) = \omega_0 - \frac{E_C}{\hbar}$ → frequency at which I have to tune in order to manipulate $|0\rangle$ and $|1\rangle$ states

E_1 and E_0 are calculated with the expression above

$E_0 = \frac{\hbar\omega_0}{2} + \frac{3E_C}{12}$

$E_1 = \hbar\omega_0\frac{3}{2} - \frac{9E_C}{12}$

$\frac{E_1 - E_0}{\hbar} = \frac{\hbar\omega_0\frac{3}{2} - \frac{\hbar\omega_0}{2} - \frac{9E_C}{12} - \frac{3E_C}{12}}{\hbar} = \omega_0 - \frac{E_C}{\hbar}$

$\eta = -\frac{E_C}{\hbar}$

We want an UNHARMONIC FUNCTION, we compute also ω_{12} in order to see how differs from ω_{01}: $\omega_{12} = \frac{E_2 - E_1}{\hbar} = \omega_0 - \frac{E_C}{12\hbar}(24+\cancel{12}\cancel{-3}\cancel{-6}\cancel{-6}\cancel{+3}) = \omega_0 - \frac{2E_C}{\hbar}$

we can go even further and compute $\omega_{23} = \omega_0 - \frac{3E_C}{\hbar}$

As we see the difference is VERY SMALL since E_C is SMALL. This represents an issue since when I tune my RABI OSCILLATION in order to induce a transition between $|1\rangle$ and $|0\rangle$, I'll be inducing also a transition toward $|2\rangle$, due to the similar frequencies. This problem is known as LEAKAGE.

To evaluate the unharmonicity we introduce the parameter $\alpha = \frac{\omega_{01} - \omega_{12}}{\omega_{01}} \sim \frac{E_C/\hbar}{\sqrt{E_C E_J}}$ — ω_{01} is PROPORTIONAL TO $\sqrt{E_C E_J}$

L and C are inversely prop. to E_C and E_J ← remember $\omega_0 = \frac{1}{\sqrt{LC}}$

Therefore we have that $\alpha \sim \sqrt{\frac{E_C}{E_J}}$ → As we keep lowering E_C, we also decrease the UNHARMONICITY. The good thing is that the coherence time increases exponentially.

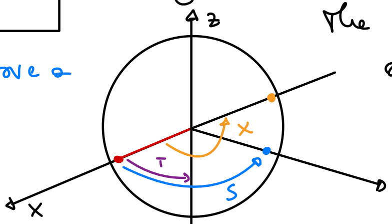
$\omega_0 \sim 5-30$ GHz

$\Delta\omega \sim 200$ MHz

$\frac{\Delta\omega}{\omega_{01}} \sim 9\%$

HOW DO I OPERATE A TRANSMON?

First of all I have to implement a Z gate (rotation).

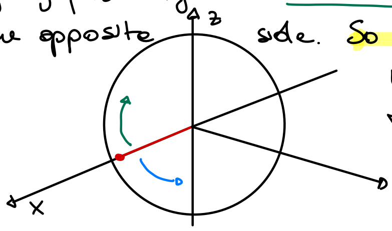


In blue we have a $\frac{\pi}{2}$ rotation.

The S rotation is part of the Clifford set of quantum gates and together with T ($\frac{\pi}{4}$ rotation) it belongs to Clifford + T set. The Z gate consists of a rotation along the z-axis: to carry it out we assume to have our superposition state (see the red vector) that is doing a precession along z at frequency ω_{01}. I can write my Hamiltonian $\hat{H} = -\frac{\hbar}{2}\begin{pmatrix} \omega_{01} & 0 \\ 0 & -\omega_{01} \end{pmatrix}$ → UNPERTURBED HAMILTONIAN → no poles are applied and the TRANSMON is doing a precession at ω_{01}. We have only $|0\rangle$ and $|1\rangle$ states.

$\hat{\sigma}_z$ operator → responsible for the precession.

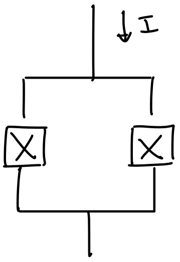
I prefer to set a rotating reference axis that is rotating together with the state at ω_{01} → in this way the precession is almost canceled → In such reference system the qubit is fixed and we want to apply a phase change → this means to detune WRT the characteristic frequency my qubit so that, for a given time, I change the angular velocity of my qubit if it is decelerating it moves in clockwise direction. If it is accelerating it moves toward the opposite side. So change of phase or z-gate consists in a change in the angular velocity.



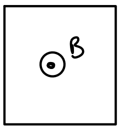
ω_{01} depends on E_J and E_C and it is an intrinsic value of my system that appears difficult to manipulate. It has been developed a method to achieve such modification: THE SQUID

The squid is a SUPERCONDUCTING QUANTUM INTERFERENCE DEVICE

it consists of two Josephson Junctions in parallel:



Let's assume to have a superconducting loop → such loop will see a B field pointing out from the spring. A property of the superconducting material tells me that



$$\frac{2\pi}{\Phi_0}\oint A\, d\ell = \frac{2\pi}{\Phi_0}\int_S B\, dS = 2\pi\frac{\Phi}{\Phi_0} = 2\pi n$$

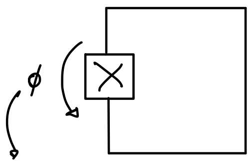
curl of B (potential vector)

n must be an integer number, otherwise the interference would be destructive and the wave would stop to exist.

I assume that this is a sort of Bohr atom whose electrons move according to a stationary wave.

What happens if I now introduce a Josephson Junction inside my superconducting loop?

It happens that the Junction introduces a discontinuity in the phase:

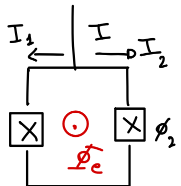


phase of Josephson Junction.

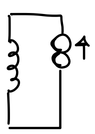
there should be a minus (-) but we generally disregard it for the phase

$$\phi + 2\pi\frac{\Phi}{\Phi_0} = 2\pi n \iff \phi = 2\pi\frac{\Phi}{\Phi_0}$$

In the case of the SQUID I use such superconducting wire is used to drive a current. The situation is the following: the two Josephson Junctions are equal (geometrically speaking - we deal with SYMMETRICAL SQUID)



The magnetic flux is generated by an external circuit like this:



$I = I_1 + I_2 = I_0 \operatorname{sen}\phi_1 + I_0 \operatorname{sen}\phi_2$ since I_0 (critical current) are the same, we can write $= I_0(\operatorname{sen}\phi_1 + \operatorname{sen}\phi_2) = I_0 \cos\frac{\phi_1 - \phi_2}{2} \cdot \operatorname{sen}\frac{\phi_1 + \phi_2}{2}$

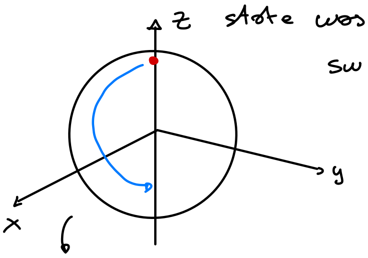
FROM PROSTAFERESI FORMULAS

Along the loop I have two discontinuities, but since I meet them in opposite ways along the loop, I get that the overall phase discontinuity will be given by $\phi_1 - \phi_2 = \phi$ → we can replace in the found expression: $\underbrace{I_0 \cos\pi\frac{\Phi_e}{\Phi_0}}_{I_{0,eq}}\ \underbrace{\operatorname{sen}\frac{\phi_1 + \phi_2}{2}}_{\phi_{eq}}$ → I can view this squid as an equivalent Josephson Junction where

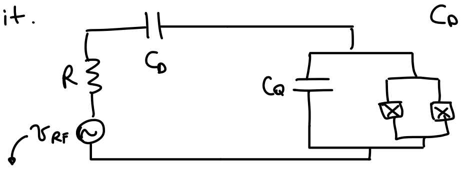
Therefore $I = I_{0,eq} \cdot \operatorname{sen}\phi_{eq}$ → this ϕ_{eq} is used in the Hamiltonian

The most important aspect regarding the previous expression is that $I_{0,eq}$ impacts $E_J = \frac{I_0 \Phi_0}{2\pi}$ →
→ such I_0 in this case is tunable by means of the magnetic flux we apply and allows me to tune the characteristic frequency. This kind of system is known as TUNABLE QUBIT.
A drawback is the phase noise coming from magnetic noise → it introduces JITTER and therefore we have a decrease of decoherence time (TRADE OFF).
So I get my Z-gate which consists of a squid that detunes the qubit.

X-Y GATE – in the case of Z-gate we had a situation in which there was no energy exchange, the qubit state was always at the same energy during the rotation. In the case of XY rotation, we switch between $|0\rangle$ and $|1\rangle$ state and therefore we have an exchange of energy: where does this come from? We must have an external circuit that is providing it.



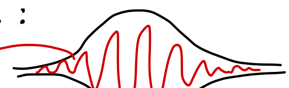
Moving from $|0\rangle$ to $|1\rangle$ implies the use of energy



Radio frequency voltage

C_D is the driving capacitance that couples to C_Q of the qubit.
NOTE: For the first part of the circuit we can also avoid to work under the critical temperature. (also at room temp.)
↳ At room temperature. All the voltage generators are put away from the cryogenic state for dissipation purposes. $v_{RF} = e(t) \sin(\omega_{0} t + \delta)$
↳ tuned to the qubit frequency
$e(t)$ can be a Gaussian:

sinewave at ω_0 multiplied by an envelope function



← envelope voltage

In terms of Hamiltonian we have a similar situation to the one of the SPIN QUBIT → in that case we applied an oscillating magnetic field that induces an x or y rotation. In this case instead of a magnetic field we apply an RF voltage. (or electric field).

$$\hat{H} = \underbrace{-\frac{\hbar}{2}\hat{\sigma}_z}_{} - \frac{e(t)}{2}\frac{C_D}{C_D + C_Q}\boxed{\sqrt{\frac{\hbar}{2 Z_Q}}}\left(\hat{\sigma}_x \cos\delta + \hat{\sigma}_y \sin\delta\right)$$

→ Z_Q is the qubit impedance

same as ESR where I need to stay in phase with the precession in order to perform x rotation and in quadrature for a y rotation.

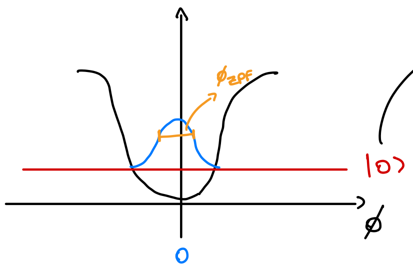
Unperturbed Hamiltonian responsible for precession

Fraction of voltage the effectively drops across the qubit

→ Q_{ZPF} → equivalent charge
Zero Point Fluctuation Charge

→ I will have an oscillating charge between the qubit and C_Q, the average charge will be zero but the deviation will not. Such Q_{ZPF} corresponds to such std. deviation. $Q_{ZPF} \sim \sigma_Q$

To better understand the Q_{ZPF} → let's plot the Hamiltonian and observe the wave function at state $|0\rangle$.



corresponds to the ground energy point
in blue I have my wave function WRT the phase as I can see the average phase is zero BUT I have some deviation → this deviation is the ϕ_{ZPF}. Q_{ZPF} is the conjugate of ϕ_{ZPF} and together they express $(Q_{ZPF} \cdot \phi_{ZPF})$ an Heisenberg uncertainity principle (like $\underset{\text{position}}{\Delta x} \cdot \underset{\text{momentum}}{\Delta p} \geq \frac{\hbar}{2}$)
↳ $Q_{ZPF} \cdot \phi_{ZPF} \sim \frac{e}{2}$ → The larger the fluctuation of the phase the smaller will be the fluctuation of the charge →
→ this is the case of the Cooper pair box where the charge was a nice quantum number.

In the case of transmon the fluctuation of the phase is smaller (ϕ is very well defined) but the charge has higher fluctuation.

HOW TO SOLVE THE DIFFERENTIAL EQUATION

$\hat{H}|\psi\rangle = i\hbar \frac{\partial}{\partial t}|\psi\rangle$

wave function at time t, when I finished my computation and I'm in the final state

$$\frac{\partial}{\partial t}|\psi\rangle = -i\frac{\hat{H}}{\hbar}|\psi\rangle \longrightarrow |\psi(t)\rangle = e^{-\frac{it}{\hbar}\hat{H}}|\psi(0)\rangle$$

evolution operator $\hat{U}$ → this consists of the gate we apply to our qubit and therefore it comprises all the rotations that we perform on it.

Before we had that $\hat{H}$ was constant WRT time, but in this case it depends from the envelope $e(t)$ and therefore we have to write an integral.

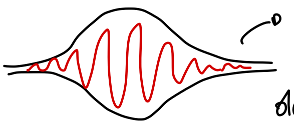
$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar}\int_0^t \hat{H}\,dt\right)|\psi(0)\rangle$$

Assuming to be in phase with the qubit frequency ($\delta = 0$ → then we are doing $\hat{x}$ rotation) we get the following evolution operator:

$$\hat{U} = \exp\left(-\frac{i}{2\hbar}\frac{C_0}{C_0+C_a}\sqrt{\frac{\hbar}{2Z_q}}\,\hat{\sigma}_x\int_0^t e(t')\,dt'\right) = \exp\left(-i\frac{\vartheta}{2}\hat{\sigma}_x\right) = R_x(\vartheta)$$ → Rotation around x of angle $\vartheta = \frac{C_0}{C_0+C_a}\sqrt{\frac{\hbar}{2Z_a}}\cdot\frac{1}{\hbar}\int_0^t e(t')\,dt'$

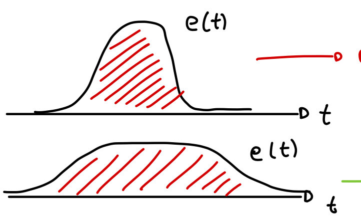
→ See first lectures where it is explained that the operator can be written also in this modality.

The $Z_q = \sqrt{\frac{L}{C}}$ characteristic qubit impedance ($\simeq 300\,\Omega$ usually)



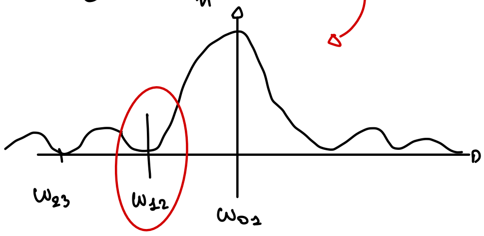
→ Envelope function with sinewave in phase with the precession. Depending on my envelope function, I define the angle of rotation → the area of the envelope determines the angle. To calculate the area we can consider e(t) as a Gaussian with high amplitude and lower width (or the contrary). Why do I use a Gaussian? Because in frequency domain it will still be a Gaussian. It is important to have a limitation in frequency due to the UNHARMONICITY NEED. If I tune my poles to a certain frequency (let's assume to be at ω_{01}) I have to ensure that my Gaussian doesn't overlap with the $\eta = -\frac{E_C}{\hbar}$ value that states the point in which the energy can excite the transfer from $|1\rangle$ to $|2\rangle$ (instead of $|0\rangle$) → LEAKAGE EFFECT → To avoid it I need my Gaussian, in f domain, to be narrower, in order to make ω_{12} negligible. This can be done by changing my poles / envelope shape. We have to use a SLOWER ENVELOPE → drawback is that we must wait more time for gate → TRADE OFF with GATE TIME MINIMIZATION → the overall computation must be kept as low as possible since we have to deal with DECOHERENCE TIME.

POSSIBLE e(t) REPRESENTATIONS



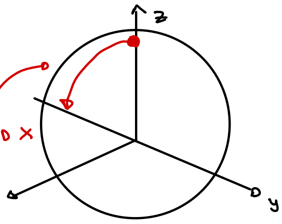
ENVELOPE IN FREQUENCY DOMAIN

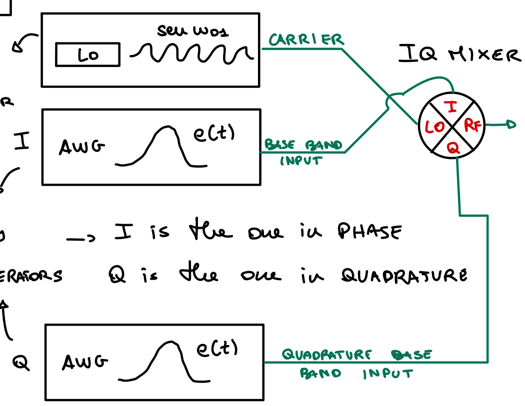
A trick to solve this issue can be the one to redesign my poles in order to introduce a NOTCH on top of the $-\frac{E_C}{\hbar}$ point:



What we have found until now is a X gate → in order to apply a $\hat{Y}$ rotation I just need to change my ϑ_{RF} phase: the setup in my transmon will be something like this

ANTICLOCKWISE ROTATION AROUND X





produces the signal of frequency ω_{01} ← LOCAL OSCILLATOR

I

ARBITRARY WAVE GENERATORS → generator of arbitrary wave function. This is the one that supplies the Gaussian envelope

→ I is the one in PHASE
Q is the one in QUADRATURE

Q

IQ MIXER → Performs the MIXING of the input and gives back the RF signal

RESULTING RF SIGNAL ⇒ $e(t)\,(I\sin\omega_{01}t + Q\cos\omega_{01}t)$

I can have also $I = Q = \frac{1}{2}$ → 50% in phase and 50% in quadrature → corresponds to a rotation around a bisector $\hat{n} = I\hat{x} + Q\hat{y}$

By playing with I and Q I can perform a rotation along any arbitrary axis.

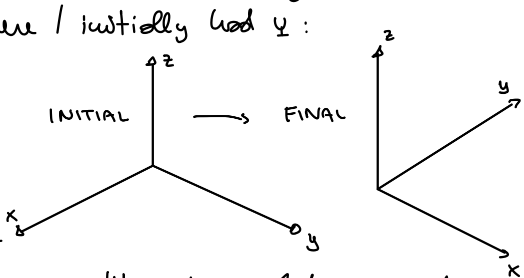
Most of the times I don't care about the rotation around Y → WHY? Because it is MAPPED IN THE PHASE

We mainly rely on X rotations.

What we do is to rely on the SOFTWARE VIRTUAL ROTATION, in the sense that I make my initial x axis "move" and apply it as a reference to the previous position where I initially had y:

so, the system I have is — $\boxed{X_\vartheta}$ — $\boxed{X_\vartheta^\phi}$ —

INITIAL → FINAL



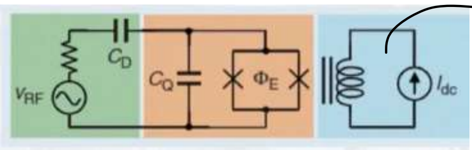
We apply a rotation around Z axis of ϕ

So instead of applying physically the rotation, I can just MAP the z rotation inside the I and Q parameters and run the z rotation with software ⇒ LESS QUANTUM GATES → GOOD FOR DECOHERENCE TIME.

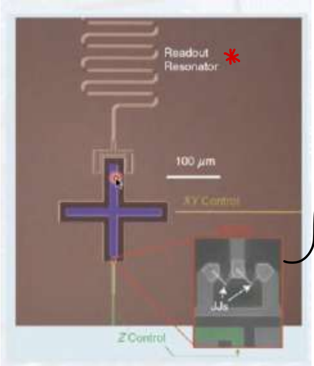
$$\hat{X}_\vartheta^\phi\,\hat{X}_\vartheta = \hat{Z}_{-\phi}\,\hat{X}_\vartheta\,\hat{Z}_\phi\,\hat{X}_\vartheta$$

I MAP THE Z-AXIS ROTATION INTO THE X GATE

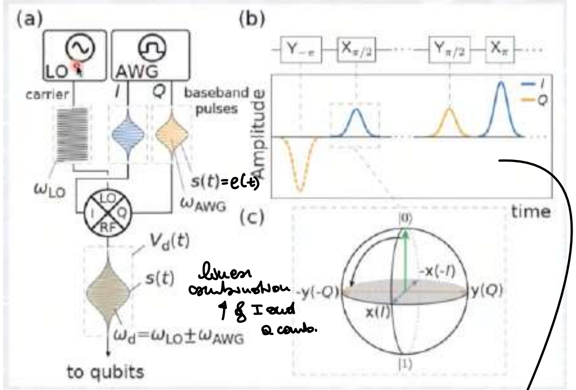
Lecture 16 - 22/12/23



As already anticipated this part of the circuit consists of the z line that is responsible for the generation of a magnetic field that is coupled to the SQUID (composed by the two J.J.) and tunes the qubit frequency.



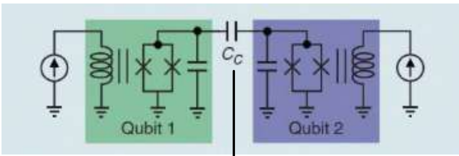
The cross on the left represents the Cooper pair box. The cross shape minimizes the inductance and maximizes the capacitance.
The SQUID is also connected to the bottom plate capacitance and the z-line.
The XY control is the one that applies a driving voltage that can be either in phase or in quadrature with the carrier and is used to induce RABI oscillations.
As we've seen in the previous lesson, the resulting envelope $s(t)$ (or $e(t)$ as we used to call it) will be at a frequency ω_d → note that $\omega_d = \omega_{LO} \pm \omega_{AWG}$ where ω_{AWG} is obtained by linear combination of I and Q contributions, must be $\omega_d = \omega_{01}$ → → so ω_{LO} is kept slightly different from ω_d and the fine tuning is applied by means of ω_{AWG} contribution



VIRTUAL Z-GATE → Not carried out in terms of qubit detuning but with SOFTWARE → the idea is the one to change the reference and apply the rotation.

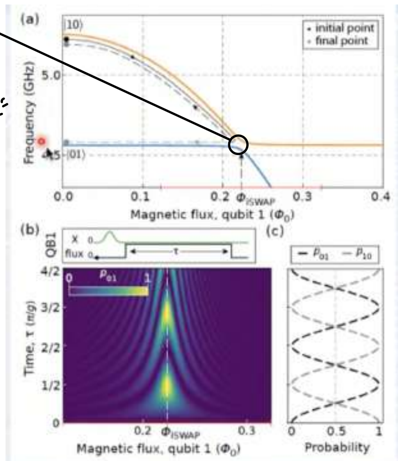
Remember that the angle of rotation depends on the area of the envelope

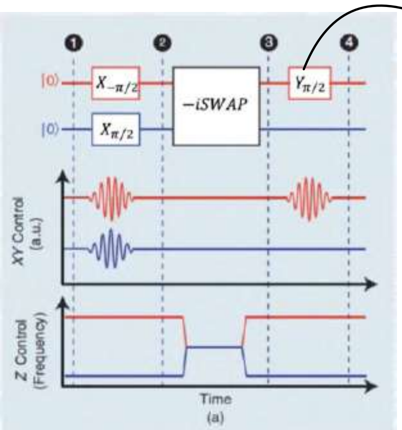
2 QUBIT GATES → essential for entanglement. How do we operate them in the case of superconducting qubit? We have two oscillators capacitively coupled: the energy exchange is possible only in case of strong coupling and the characteristic frequencies are close one to each other.
The qubit frequencies of the two qubits are made different enough in order to be able to operate on the single qubits. When we need to enable the coupling of the qubits, we have the z-line that detunes the frequency of the qubits → great advantage of Transmons.



integrated capacitance responsible for the coupling.

At this point I have basically changed the frequency of QUBIT 1 in order to make it "equal" to the QUBIT 2 one. At that point (or slightly before reaching it) we experience the "AVOIDED CROSSING" which consists in the bending of the two energies in order to avoid the crossing. Such behaviour resembles the one of a bandgap presence. At this point we have the oscillation starting, the first one is excited while the second is at ground state, so the energy moves from the first qubit to the second one → SWAP OPERATION
The longer the time, the longer the energy exchanged
The SWAP by itself doesn't introduce the entanglement. What determines it is the iSWAP (swap with additional $\frac{\pi}{2}$ shift).

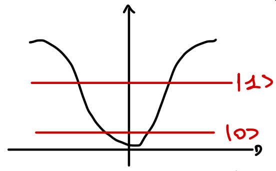




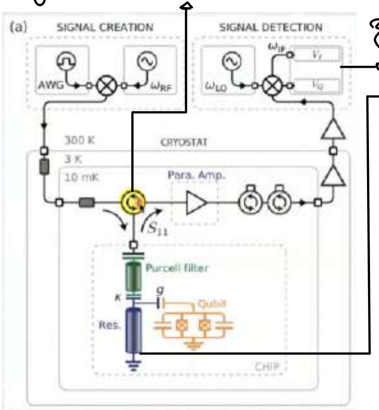
applied in order to create INTERFERENCE: it means that we make a ROTATION in parallel to different states and they cancel each other. In this case we get that $(|0\rangle - |1\rangle)$ turns into $|0\rangle$ and $(|0\rangle + |1\rangle)$ into $|1\rangle$ and at the end we obtain the Bell state (SEE SLIDE 31)

READOUT OF THE TRANSMON → In the case of transmon is like having an oscillator that can be either excited or at ground state. The excited oscillator presents a photon → this photon is a quantum of energy and it is NOT A LIGHT PHOTON → therefore it is very difficult to read. We deal with RF photons (μeV energy range): they are difficult to detect. What is used to measure our qubit is the impedance. The idea is that the transmon has a state dependent impedance. If we take into account a LC oscillator we would have the same impedance, no matter what is the energy → this is because of the HARMONICITY of the LC oscillator. But the TRANSMON

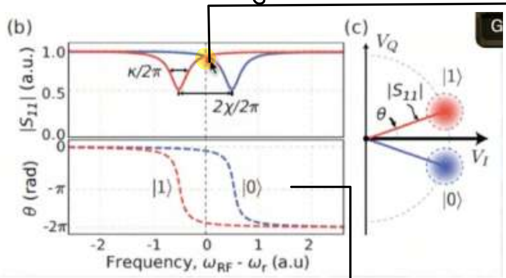
is unharmonic and therefore the impedance changes based on the energy stored. Remembering the parallelism made with a mechanical spring, the elastic constant k corresponds to $\frac{1}{L}$.
By looking at the Hamiltonian, in the case of Transmon we don't have an effective parabola.



The curvature depends on the k elastic constant and the smaller the curvature the larger the k → this means that in correspondance of the state |0⟩ we have large k → this implies a SMALL INDUCTANCE $\left(k = \frac{1}{L}\right)$
In state |1⟩ the "parabola" gets wider → the INDUCTANCE IS INCREASING.
In principle, in order to read such inductance I need to create a signal and read what gets reflected. CIRCULATOR → input and output depend on the entry point. It is responsible for the ROUTING of the signals.



SPECTRUM ANALYZER → Reads the amplitude and phase of my signal.
RESONATOR* → remember that in the cross structure of the transmon, there was also such resonator. The transmission line doesn't go directly toward the qubit but goes first to the RESONATOR which is capacitively coupled to the transmon. We don't want this signal probing to be injected into the transmon or, better to say, we don't want the transmon to be continuously coupled to a readout line where LEAKAGE can occur. Since the resonator and the transmon are coupled, the overall impedance will be dependent from the qubit state. The signal we produce will be at the resonant frequency of the RESONATOR: such characteristic frequency will be dependent on the state → I see max or min. reflectance depending on the state. L for state |0⟩ is lower → characteristic frequency is larger.



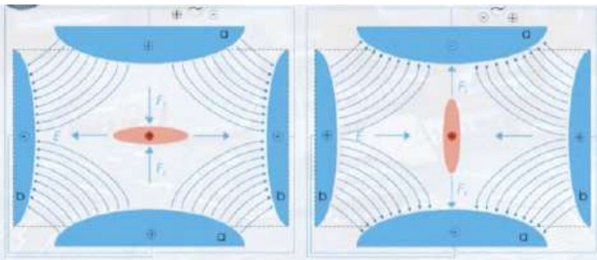
N.B. The difference between the two frequencies (for |0⟩ and |1⟩) are not so different. This means that due to noise it may be difficult to succesfully readout the actual state basing the measurement on this parameter. What we do to avoid noise related issues is to produce a frequency right in the middle of the two characteristic freq. → as we see the amplitude is the same both for |0⟩ and |1⟩. Instead of looking to the amp. we look to the PHASE which is instead very different. From the phase-quadrature plot, we can see that the probe costellation returns states with the same amplitude BUT DIFFERENT PHASE.
PURCELL FILTER → we have such bandpass that acts as a notch for the qubit frequencies. We must avoid the excitation of the qubit otherwise the continuous coupling to the readout circuit would determine the LEAKAGE effect.

END OF SUPERCONDUCTING QUBIT

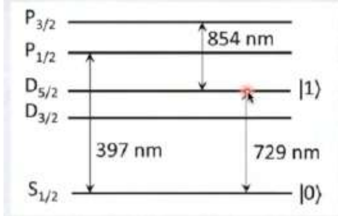
WRAP-UP (LAST GROUP OF SLIDES)

Among the different qubit topologies, there are 3 main parameters that we have to take in consideration: COHERENCE TIME (quite comparable for electron spin and superconducting) GATE TIME → how much time we employ to carry out a quantum operation. These two times give us the parameter $N = \frac{t_{coh}}{t_{gate}}$ that tell us the number of operations we are able to complete before the decoherence takes place. Both for electron spin and superconducting we are in the order of 2000 − 2500 (not so much). Solutions like FAULT TOLERANT COMPUTERS are under development in order to increase such number. The last important parameter is the FIDELITY.
Nuclear spin is not included among the different topologies since it results too variable (strictly depending on the material) and therefore it results in a difficult to SCALE technology.
TRAPPED ION → we use NATURAL QUBIT (like atoms) that offer high stability and therefore long coherence time (eventhough it results difficult to manipulate).
NV center → Nitrogen Vacancy in diamond is used as qubit → relies on the defects of the diamond. Mostly interesting for QUANTUM SENSING more than COMPUTING.

TRAPPED ION QUANTUM COMPUTING → based on ION TRAPPED in VACUUM → we align the ions in a HIGH VACUUM SPACE thanks to an electric field imposed by electrodes with opposite charges → this condition creates a SADDLE where the ions will seat in a unique direction

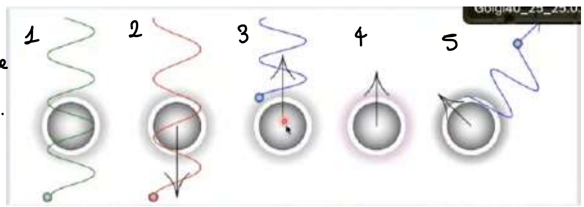


The alignment relies on the fact that the ions are positively charged and therefore the space between them is determined by the natural Coulombic repulsion. Tipically based on alcalyne ions like Ca.
Calcium ions will present different levels → the electron will generally stay in the $S_{1/2}$ level ($|0\rangle$ state) - $S_{1/2}$ means orbital s, where L=0, and 1/2 is the SPIN of the electron.



The level for the $|1\rangle$ state is the $D_{5/2}$ → the excitation from one state to the other is done by means of laser (729 nm - red light). We have basically an optical qubit.
I can use also RF qubits → energy difference between $|1\rangle$ and $|0\rangle$ state in this case is much smaller. I can obtain much smaller energy difference by applying a static magnetic field → such field determines a Zeeman splitting in the order of μeV at the $S_{1/2}$ level. μeV correspond to the energy of RF signal.

HOW DO WE FREEZE THE ION? We must cool the qubit down in order to trap the ions. In the case of trapped ion technology, I don't cool down to mK range but I employ the DOPPLER COOLING.
This technique is based on lasers. Assume that I'm applying a 397 nm laser (actually not exactly tuned at this energy but slightly less). If the ION has v=0 (CASE 1) in the direction of the light, the laser doesn't get absorbed because the energy won't be enough to make the transition from $S_{1/2}$ to $P_{1/2}$.



If the ion is moving in the same direction of light (CASE 2) the apparent frequency due to DOPPLER EFFECT will be lower and therefore also in this case no transition occurs. In CASE 3 we see that the ION moves in the opposite direction and it experiences a DOPPLER SHIFTING → the effective frequency is larger $\nu' = \nu(1 + \frac{v}{c})$ where ν is the laser frequency. In this case the energy will be enough to induce the transition from $|0\rangle$ to $|1\rangle$. The photon gets absorbed. When the ion receives a photon it will also absorb a RECOIL → the ion absorbs also the momentum of the photon. This determines a sort of kick back effect since the energy/velocity of the ion will decrease because of such "kick". After this, the final step (CASE 5) consists in the RANDOM RE-EMISSION of the photon. By repeating absorbtions and emissions, the velocity of the ions will reduce more and more, determining, in average, v=0 → the ions will FREEZE.

INITIALIZATION

Excitation to $P_{3/2}$ state which is a metastable level → we use a 854 nm PUMP LASER → if we use it on a $|1\rangle$ state qubit it will jump to $P_{3/2}$ and then, thanks to the fast deexcitation, it goes to GND state $|0\rangle$. If we use it on $|0\rangle$ nothing happens because the energy is not enough to promote such transfer.

QUBIT CONTROL

Each ion is separated from the other by around 1 μm space → with lasers we are able to focus on the single ions. • Two-qubit control is possible even among non-adjacent qubits. Every ion will have its own frequency: I apply a gradient magnetic field that induces different Zeeman splitting and thus, all the trapped ions will have a different frequency.

DECOHERENCE

Extremely long since the ions are sufficiently spaced and there's "no" interaction/scattering.

ADIABATIC COMPUTING

It consists of a FLUX-QUBIT approach. We have a situation similar to a TRANSMON with Josephson Junction where the distinction of the state is based on the current in the loop that circulates in clockwise or counterclockwise direction. On the basys of the current direction we have the electrons moving in the opposite direction and generating an "ARTIFICIAL SPIN". In adiabatic computer I start with an initial and simple Hamiltonian and we slowly change it in order to arrive to a final Hamiltonian that represents the problem to solve. The following theorem is applied: ADIABATIC THEOREM

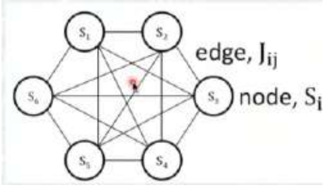
Adiabatic theorem: if a Hamiltonian $\hat{H}$, changes slowly in time, then the state remains in the ground state → the ground state ever exists → in case it is the solution to the problem, the result is immediately found. (SEE SLIDES)

Such approach is more useful for OPTIMIZATION PROBLEM (larger ones).

ISING MODEL → The Ising problem is one of the most difficult to solve and it is based on a model where each node is linked to the other N-1 nodes. In our case the interest is the one to compute the Hamiltonian for a system with N qubits.

$$\hat{H} = \sum_{j=1}^{N} h_j \hat{S}_j + \sum_{i \neq j} J_{ij} \hat{S}_i \hat{S}_j$$

By increasing the number of nodes, we also increase the complexity. The role of the adiabatic computation is the one to build from scratch the Hamiltonian in order to solve the problem.



QUANTUM INSPIRED CLASSICAL COMPUTING → We use physical computing / physics law in order to run "quantum" computations. In QUICC I have a physical bit at which I apply pulses, B field or electric field (from the external side of the place where the qubit is stored) in order to manipulate my qubit. This is done at room temperature and there is NO ENTANGLEMENT.

BOLTZMANN MACHINES → similar to adiabatic computing.

LAST PART OF THE LECTURE HAS BEEN JUST "READING OF SLIDES"