Course notes by Lorenzo Perlo. These are free, so if you've paid for them, you've been scammed.

I've studied this subject by reading the book provided by the professor on WeBeep before attending the lessons. Many topics are just summarized here because the book explains them well, while others are covered in more detail. Rare topics that are not in the book can be found here (though not fully covered, as the professor's slides were clearer). Some parts are not covered, as they are in the slides (where images are more helpful than words).

My advice: use these notes alongside the slides and the book, you'll get more out of the class (be sure to attend, the professor is really good at what he does).

Please excuse any possible grammatical mistakes.

SLAB WAVEGUIDE

$$\vec{E}(x,y,z,t) = E(x,y)\, e^{-j\beta z} e^{j\omega t}$$

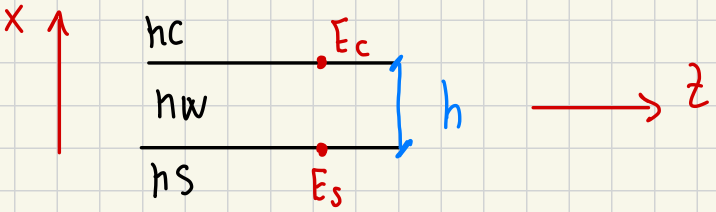
Families of solution :

$$\text{TE} \rightarrow H_y = 0 = E_x = E_z$$

$$\text{TM} \rightarrow H_x = 0 = E_y = H_z$$

A waveguide usually carries both, maybe one better than the other.

$$\frac{d^2 E_y}{d x^2} = \left(\beta^2 - n^2(x) \underbrace{k_0^2}_{\frac{2\pi}{\lambda}} \right) E_y$$



The possible solutions for TE :

$$\left\{\begin{array}{lll} \textcircled{1} & E_{y_s} = E_s\, e^{\gamma_s x} & x<0 \\ \textcircled{2} & E_{y_c} = E_c\, e^{\gamma_c (x-h)} & x>h \\ \textcircled{3} & E_{y_g} = E_g \cos(K_g x - \underbrace{\phi_s}) & 0<x<h \end{array}\right.$$

If the peak is not at the center of the core ($\frac{h}{2}$), ϕ_s bring it there and it's $\neq 0$.

Now if you substitute the solutions in the wave equation, you find β for each. So put ① in eq., Find β or γ_s or γ_c, repeat for ② and ③.

$$\begin{cases} \gamma_s^2 = \beta^2 - k_0^2 n_s^2 \\ \gamma_c^2 = \beta^2 - k_0^2 n_c^2 \\ k_q^2 = k_0^2 n_w^2 - \beta^2 \end{cases}$$

$\Rightarrow$ Related to the solutions

The boundary condition:

$$\begin{cases} x = 0 & E_{y_s} = E_{yq} \\ x = h & E_{y_c} = E_{yq} \end{cases}$$

$\rightarrow$ From here find

$$H_x = -\frac{\beta}{\omega\mu} E_y$$

$$\Downarrow$$

$$\mathrm{tg}\left(k_q h + 2\underbrace{N}\pi\right) = \frac{k_q(\gamma_c + \gamma_s)}{k_q^2 - \gamma_c \gamma_s}$$

$\hookrightarrow$ Integer number of solutions, solved numerically

E_q, E_c and E_s share the same β! They move at the same speed.

Cannot Find β = something, because it's the eigen solution of the N equation.

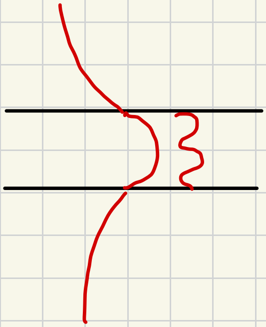
↳ After find n_w, n_c, n_s, h and ω | find β, change ω or the other, change β.

FINALLY

$$\beta = \frac{2\pi}{\lambda} \underbrace{n_{eff}}$$

Vary with ω and material and shape

Change β, change shape of $\vec{E}$ (eigenvector).



The slab is the only one to have both TE and TM modes, the other are hybrid (so also quasi TE/TM).

↳ 6 components ($E_x, E_y, E_z, H_x, H_y, H_z$ not null)

Special cases for $n_c = n_s$ (symmetric)

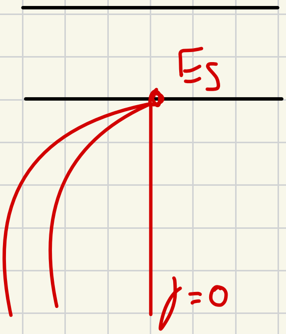
- $$\underset{k_0 \to 0}{\overset{\shortparallel}{\omega \to 0}} \quad : \quad \beta = \frac{2\pi}{\lambda} n_s$$

$$\omega \to \infty \quad : \quad \beta = \frac{2\pi}{\lambda} n_\omega$$

$$k_0 n_s < \beta < k_0 n_w$$

MODE WITH THIS β ARE GUIDED

$\gamma_s \to 0$: cut-off, the mode is NOT confined in the wave guide, but it extend everywhere



$$tg(k_q h + 2N\pi) = 0$$

$$k_q h + 2N\pi = N\pi$$

$$\frac{h}{\lambda_{cut_{off}}} = \frac{N}{2\sqrt{n_w^2 - n_s^2}}$$

I have a cut-off for every mode.

- $\gamma \rightarrow \text{Im} \rightarrow$ The field is radiate away

  ↙

  RADIATIVE MODE (plane wave)

- IF WAVEGUIDE SYMMETRIC

  ↳ NO CUT-OFF AT $\omega = 0$ for 1^{st} mode

- IF ASYMMETRIC

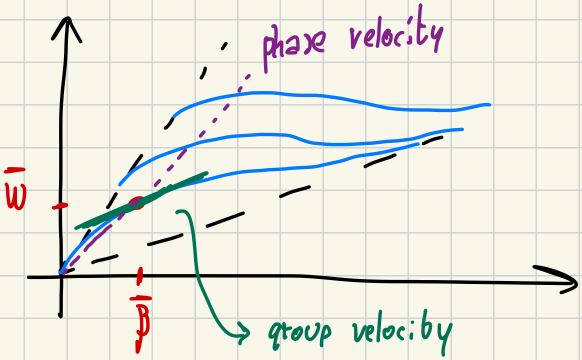
  ↳ CUT-OFF also for low frequency and 1^{st} mode

IF YOU DON'T EXCITE A MODE (RADIATIVE OR GUIDED) AT $\overline{\omega}$, THAT MODE DOESN'T PROPAGATE EVEN IF AT THAT ω IS A SOLUTION FOR THAT GUIDE

↳ MISMATCH AND ROUGHNESS CAN BRING β to RADIATIVE (NOT SEEN IN PROJECT, BUT AFTER MANUFACTURING)

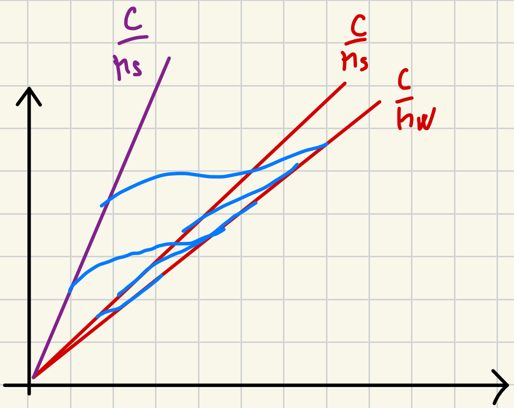
- $$V_p = \frac{\omega}{\beta} = \frac{c}{n_{eff}}$$

- $$V_g = \frac{\partial \omega}{\partial \beta} = \frac{c}{n_g} = \frac{c}{n_{eff} + \omega \frac{\partial n_{eff}}{\partial \omega}}$$



For real wg with low Δn and hig Δn

↳ Silicon



low $\Delta n \rightarrow n_{eff} \simeq n_g$

high $\Delta n \rightarrow n_g \gg n_{eff}$

In free space $\Delta n = 0$, $n_{eff} = n_g$, because you have only TE or TM, not other. If you have also E_z or H_z (z direction of prop.), you have high energy (with high Δn) or low energy (low Δn) that is reactive in the z direction, so slow down the wave

↳ So in wg (where E_z and H_z can be $\neq 0$) the field is slower.

For silicon $n_g = 4.2$

Special case

Launch TE_0 in a dual mode waveguide of fixed w, but shifted along the X position, cause the excitation of two modes (TE_0 and TE_1) and they have different neff (so $\beta_0 \neq \beta_1$).

↳ Both are guided, but there is a periodicity in the total field due to the interference.

↳ If the x-offset is low, I excite more TE_0 than TE_1. Depending on the size, the x-offset is more or less degrading.

Modes don't change shape and don't exchang power, but the total fiel has L_B

↳ $\downarrow\downarrow d$, the x-offset count less and the guide is more mono-modal and after some z μm the total field is like the one of only 1^{st} mode.

THE ALIGNEMENT IS IMPORTANT, BUT IF THERE IS A X-OFFSET, THE PHOTONIC DEVICE MUST NOT BE PUT TOO NEAR THE ENTRANCE!

↳ More mode propagating is possible, but for most application only mono mode is used and separate wrongly excited mode is difficult.

$$L_B = \frac{\lambda}{\Delta n_{eff} \text{ of modes}} = \frac{\lambda}{n_{eff_1} - n_{eff_2}}$$

$n_{fondamentale} > n_{modo\ superiore}$

LEAKY MODE => LOOSE POWER WHILE PROPAGATION AND AFTER SOME μm in z DIRECTION THEY ARE RADIATED AWAY CHANGING THE SHAPE.

↳ No plane wave like radiative mode, but packet of wave.

Birefringence

Different n_{eff} between different polarization (TE and TM)

↳ If wg sym, TE and TM have same $\lambda_{cut\text{-}off}$, but $n_{eff_{TE}} \neq n_{eff_{TM}}$

SO THE BEHAVIOUR OF THE DEVICE VARY WITH THE POLARITAZION (EXCITE WHAT YOU NEED!).

↳ The fiber never conserve the the polarization, so most of the time I don't know what arrive.

I want polarization independent device. Two solution

① ↳ $B = 0 \Rightarrow n_{eff_{TE}} = n_{eff_{TM}}$

$$\underbrace{\Delta\phi_{\substack{TE\\TM}}}_{10^{-2}/10^{-3}} = \frac{2\pi}{\underbrace{\lambda}_{10^{-6}}} B \underbrace{L}$$

→ Typically $\sim 10^{-2}$

↳ So $B < 10^{-6}$, so neff must be controlled to the sixth digit

↓

IMPOSSIBLE FOR SILICON PHOTONICS

ok for other material

② POLARIZATION DIVERSITY SCHEME

↳ Polarization beam splitter

rotator

Device

identical

↓ ↓

Device / rotator

↓

Recombine

Difficult but possible, V silicon photonics.

Losses

Caused by δw, but with same δw, they are greater on material with high Δn ($n_g \neq \neq n_{eff}$) w.r.t. low Δn ($n_g \simeq n_{eff}$).

Depend on roughness leaved by etching ($w_{real} = w_{ideal} \pm \delta w$)

↳ I can excite radiative modes

↳ Some of fundamental mode is coupled with the counter propagating mode fundamental.

⇓

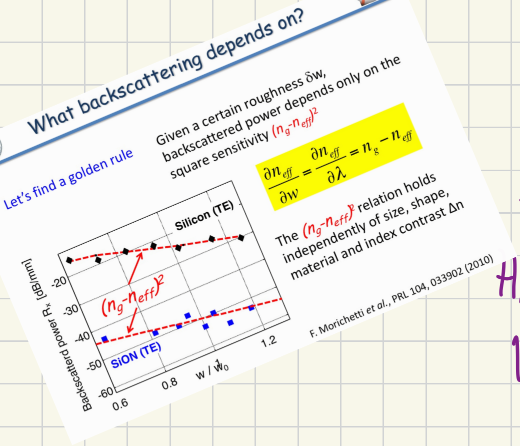
Important for laser, they are sensible to light that comes back.

For the same wg, TE and TM experience different losses and backscatter

↳ IMPORTANT EVERY TIME YOU ARE SENSIBLE TO RADIATION THAT COMES BACK

LIDAR
(I measur what comes back, backscatter creates errors)

Laser (light comes back, laser works differently)



What backscattering depends on?

Let's find a golden rule

Given a certain roughness δw, backscattered power depends only on the square sensitivity $(n_g-n_{eff})^2$

$$\frac{\partial n_{eff}}{\partial w} = \frac{\partial n_{eff}}{\partial \lambda} = n_g - n_{eff}$$

The $(n_g-n_{eff})^2$ relation holds independently of size, shape, material and index contrast Δn

F. Morichetti *et al.*, PRL 104, 033902 (2010)

EVERY LASER IN WAVEGUIDE HAS ISOLATOR BEFORE COUPLING WITH WAVEGUIDE.

BENT WAVEGUIDE

Conformal transform → transform the problem to one system of coordinate to one other, so I can see a bend wg like a straight one and every value must be transformed like $\underbrace{n}_{\text{refractive index in bend guide}}(U,V) \rightarrow \underbrace{n}_{\text{equivalent as if it was straight}}(x,y)$

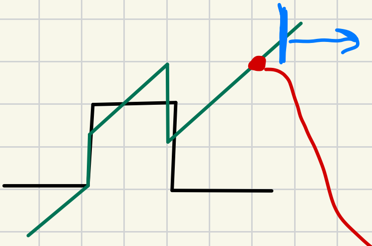
$$\boxed{U,V} \xrightarrow[\Gamma]{} \boxed{X,Y}$$

$$\Gamma = R \ln\left(\frac{\overbrace{W}^{U+jV}}{R}\right)$$

$$n(U,V)\, e^{\frac{2X}{R}} = n\left(1+\frac{2X}{\underbrace{R}_{}}\right)$$

If straight $R \to \infty$, otherwise this term play a role

The index profile is tilted a linear term that increase going away from the waveguide due to bend.



$\eta = \sqrt{\frac{\mu}{\varepsilon}}$ is less and less

From here light is NO more confined

So if there is enough evanescent field it starts to radiate laterally

The light is pushed away from the center as R↓↓

↪ Different modes with more bend, leaky and not guided

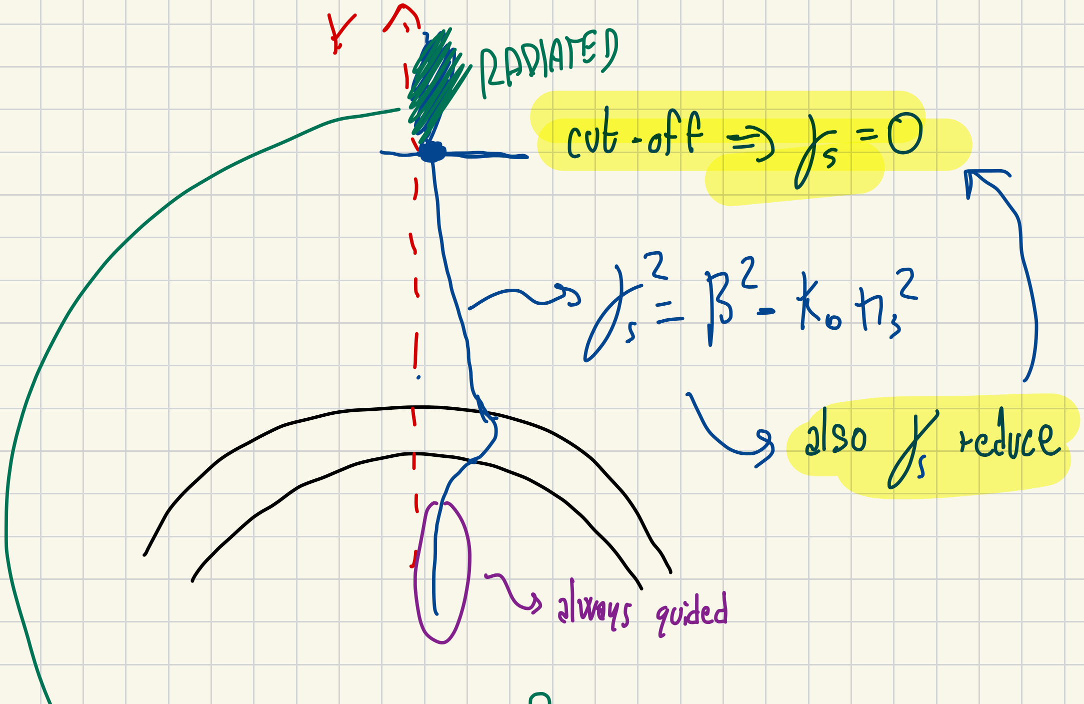
With the same β, in a straight guide every point of the mode is at the same speed, NOT in a bent one, the phase font move slower as $r \uparrow\uparrow$, so β vary with r:

$$r \frac{\partial \theta}{\partial t} = \frac{\omega}{\beta(r)}$$

$$r = R \Rightarrow \beta_0 = \beta(R)$$

$$\beta(r) = \beta_0 \cdot \frac{R}{r}$$

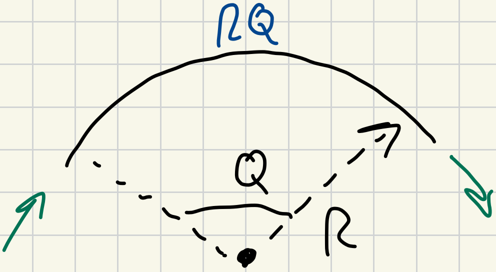
With $r \uparrow\uparrow$, $\beta \downarrow\downarrow$ reaching the cut off at some point were the tale of the modes is radiated.



$$\alpha_r \propto e^{-cR}$$

radiated loss

So the insertion loss (from input to output):



$$IL = e^{-\alpha \overbrace{L}^{QR}} = e^{-c^{-CR}RQ}$$

↳ SE INIZIO A VEDERE UNA PERDITA, LA RELAZIONE È COSÍ FORTE CHE PASSANDO DA DRITTO A CURVO TUTTO È LEAKY.

Con β variabile con r, piú mi allontano e piú il campo è lento, cioé perdo il campo.

Bending → the fondamental mode is more and more asymmetrical and distorted, so as cut-off approach the cut-off, all is leaky

EVERY TIME I START TO BEND
I LOOSE SOMETHING! ~ RADIATED OR EXCITE HIGHER ORDER MODES

↳ Shape of the mode ≠

↳ neff dif of a bend waveguide (little bit)

The efficiency of coupling between straight and bent is lower than 1

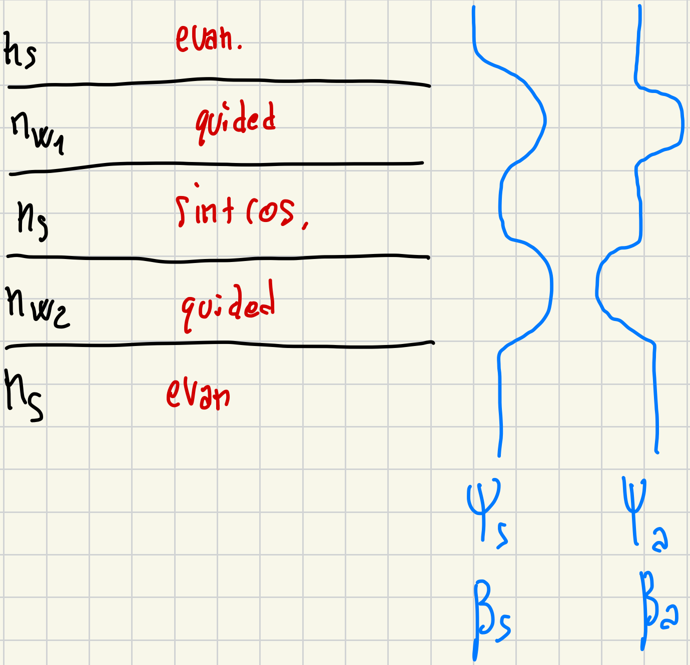
COUPLED MODE THEORY

Use the info of one wg to understand how the field change with two wg next to each other. It's a guiding structure and we want a simple method to study it.

The structure is at least bimodal :

$$\underbrace{\psi_1}_{\text{mode of single wg Without coupling}} \simeq \frac{\psi_a + \psi_s}{2} \qquad \psi_2 \simeq \frac{\psi_a - \psi_s}{2}$$

ψ_a and ψ_s real mod in coupled guide from the Maxwell equations.



Coupled mode theory = combine real solution as linear combination of simple modes

$$\left.\begin{matrix}\psi_1\\ \psi_2\end{matrix}\right\}$$ coupled mode

$$\left.\begin{matrix}\psi_a\\ \psi_s\end{matrix}\right\}$$ NOT coupled, real mode of the structure

$$\psi_a \simeq \psi_1 - \psi_2$$

$$\psi_s \simeq \psi_1 + \psi_2$$

$$\beta_{s,a} \propto \beta_1, \beta_2$$

I know how to obtain ψ_1 and ψ_2, from here I find ψ_a and ψ_s.

$$\nabla_t^2 \psi + \frac{\partial^2}{\partial z^2} \psi + k_0^2 n^2(x,y) \psi = 0$$

$$\psi \cong A(z)\, \psi_1(x,y)\, c^{-j\beta_1 z} + B(z)\, \psi_2(x,y)\, e^{-j\beta_2 z}$$

ψ_1 and ψ_2 are not modes because their amplitude depends on z, but they are solution of the single wg.

Real solution (field seen in simulation and propagation)

$$\psi = \underbrace{A}_{} \psi_a e^{-j\beta_a z} + \underbrace{B}_{} \psi_s e^{-j\beta_s z}$$

const.

$$\begin{cases} \Delta n_1^2 = n^2(x,y) - n_1^2(x,y) \\ \Delta n_2^2 = n^2(x,y) - n_2(x,y) \end{cases}$$

Δn_1 is the perturbation that the second wg gives to the first and viceversa.

From wave equation substitute ψ :

$$K_0^2 \Delta n_1^2 A \psi_1 + K_0^2 \Delta n_2 B \psi_2 e^{j\Delta\beta z} - 2j\beta_1 \psi_1 \frac{\partial A}{\partial z} - 2j\beta_2 \psi_2 \frac{\partial B}{\partial z} e^{j\Delta\beta z}$$

Multiply for ψ_1^* or ψ_2^* and integrate along x, y:

$$\frac{dA}{dz} = -jK_{11} A(z) - jK_{12} B(z) e^{j\Delta\beta z}$$

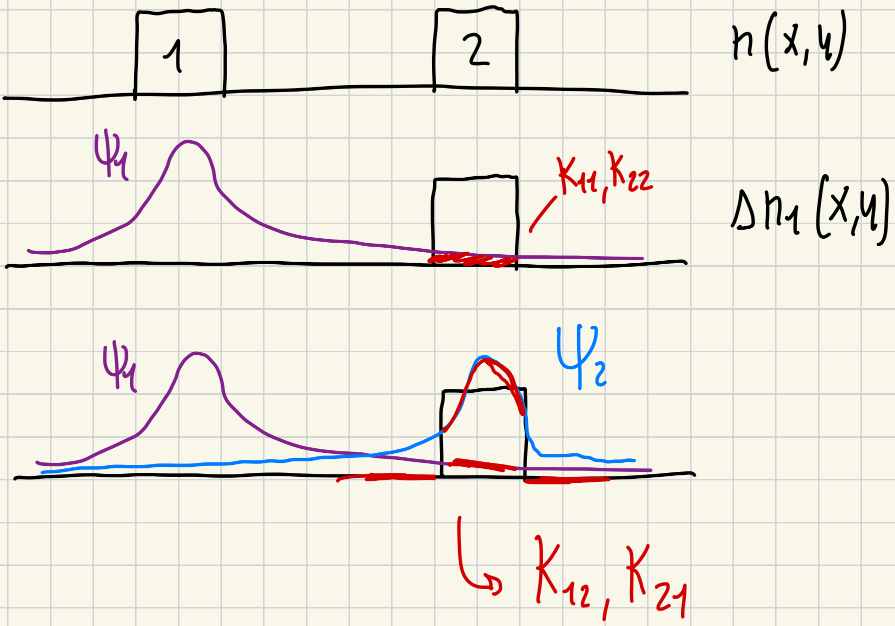
$$\frac{dB}{dz} = -jK_{22} B(z) - jK_{21} A(z) e^{j\Delta\beta z}$$

Remember ψ_1 and ψ_2 are orthogonal [they do not interact:

$$\int \psi_1 \psi_2 = 0$$

$$K_{11} = \frac{k_0^2}{2\beta_1} \iint \psi_1 \Delta n_1^2 \psi_1^* \quad , K_{22}$$

$$K_{21} = \frac{k_0}{2\beta_2} \iint \psi_1 \Delta n_2^{\,2} \psi_2 \quad , \; K_{12}$$



$K_{11}, K_{22} \to 0$, $\underbrace{K_{12}, K_{21}}_{\downarrow}$ much larger.

FIELD COUPLING COEFFICIENTS

What if the guide are far away?

- $K_{12}, K_{11} \to 0$

For coupling the guide must be close

- $\dfrac{dA}{dz} \to 0 = \text{const.}$, ψ_1 is a mode (appunto della guida singola)

Redefine variables:

$$\left.\begin{aligned} a(z) &= A(z)\, e^{-j\beta_1 z} \\ b(z) &= B(z)\, e^{-j\beta_2 z} \end{aligned}\right]$$

Propagating amplitudes of ψ_1 and ψ_2

Substitute:

$$\begin{cases} \dfrac{da}{dz} = -j\left(\beta_1 + \cancelto{0}{K_{11}}\right) a(z) - jK_{12}\, b(z) \\[2ex] \dfrac{db}{dz} = -j\left(\beta_2 + \cancelto{0}{K_{22}}\right) b(z) - jK_{21}\, a(z) \end{cases}$$

$\circledast$ ↓

Now I know the exact solution ($a_a e^{-j\beta_a z}$, $a_s e^{-j\beta_s z}$), I substitute and find :

$$\begin{cases} a_s\left(\beta-\beta_1\right) - K_{12}a_a = 0 \\ -a_s K_{21} + a_a\left(\beta-\beta_2\right) = 0 \end{cases}$$

The determinant must be 0 to have solutions :

$$\beta_{s,a} = \frac{\beta_1+\beta_2}{2} \pm \boxed{\sqrt{\frac{(\beta_1-\beta_2)^2}{4}+K_{21}K_{12}}}\ \delta$$

$$\Big\Downarrow \quad \beta_1=\beta_2=\beta_0$$

$$\beta_{s,a} = \beta_0 \pm \boxed{\sqrt{K_{21}K_{12}}}$$

C = coupling coefficient

So ψ_a and ψ_s have different β, so different speed. They are the real modes.

↳ So at different z, ψ_1 and ψ_2 vary

$$\psi_s = \frac{\psi_1 + \psi_2}{2} \quad \text{con } \beta_s$$

$$\psi_a = \frac{\psi_1 - \psi_2}{2} \quad \text{con } \beta_a$$

After $\beta_s z - \beta_a z = \pi$ all the field from one wg is gone in the second. It seems there is a power exchange between the two, but it's not true, it's just the field that has a max in one and then in another as travelling along z.

$$\text{Periodicity} \rightarrow (\beta_s - \beta_a) L = \pi$$

↳ if $\beta_1 = \beta_2$ (identical wg)

$$(\beta_0 + C - \beta_0 + C) L_c = \pi$$

Se different

$$L_c = \frac{\pi}{2\delta} = \frac{\pi}{\beta_s - \beta_a}$$

$$\boxed{L_c = \frac{\pi}{2C}}$$

So if "coupled" wg is long one L_c, all the power given in one of the two wg "pass" on the other.

Real modes have different n_{eff}.

The coupled mode theory is an approx, for $d \to 0$ is NO more correct and remember that

$$C \propto e^{-bx}$$

So if $x \to 0$, $C \uparrow\uparrow$ and viceversa, also L_c.

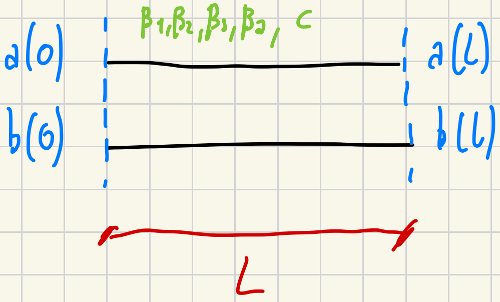
If $\beta_1 \neq \beta_2$ (different wg), it's not possible to couple all the power between the two. So power remain more in a wg.

↳ $\Delta\beta = 0$ ⇒ control n_{eff}
to the sixth digit
to make sure
is $\Delta\beta = \frac{2\pi}{\lambda}(n_{eff_1} - n_{eff_2})$

Directional coupler → solve the system ⊛ ↑

Solution :

$$\begin{cases} a(z) = a_s e^{-j\beta_s z} + a_a e^{-j\beta_a z} \\ b(z) = \text{book} \end{cases}$$



In $z=0$:

$$a(0) = a_s + a_a$$

$$b(0) = \dots$$

"Put this in a TC" and you find $b(L)$ and $a(L)$:

$$T_c = \begin{bmatrix} \cos(\delta z) - jR\,\mathrm{sen}(\delta z) & -jS\,\mathrm{sen}(\delta z) \\ -jS\,\mathrm{sen}(\delta z) & \cos(\delta z) - jR\,\mathrm{sen}(\delta z) \end{bmatrix}$$

$$R = \frac{\Delta\beta}{2\delta} \qquad S = \frac{C}{\delta}$$

$$\delta = \sqrt{\frac{\Delta\beta^2}{4} + C^2}$$

$$\det(T_c) = 1$$

$$\Downarrow$$

$$\underbrace{R^2 + S^2 = 1}_{}$$

If identical wg $\Rightarrow \beta_1 = \beta_2 = \beta_0$

$$R = 0 \, , \quad S = 1 \, , \quad \delta = c$$

$$T_c = \overbrace{\begin{bmatrix} \cos cz & -j\,\text{sen}\, cz \\ -j\,\text{sen}\, cz & \cos cz \end{bmatrix}}^{\text{NO LOSS here}} \underbrace{\sqrt{e^{-\alpha L}}}_{\text{yes loss (in power)}}$$

Special case $(a(0) = 1 \, , \; b(0) = 0)$:

$$|a(z)|^2 = 1 - \frac{c^2}{\delta^2} \sin^2 \delta z$$

$$|b(z)|^2 = \frac{c^2}{\delta^2} \sin^2 \delta z$$

$$\boxed{\delta \geq c}$$

If $\Delta\beta = 0 \Rightarrow$

$$|a(z)|^2 = \cos^2 cz$$

$$|b(z)|^2 = \sin^2 cz$$

It seems like a change of power, but it's not, it's just a phase mismatch between the two real modes that sum with different result in the total field.

The max power exchangeable:

$$P_{max} = \frac{c^2}{\delta^2} = K$$

The relative phase of the two device

The two field in output are in quadrature, for synchronous coupler.

For asynchronous couplers the mismatch depends on the length of the coupler.

↳ NON CERCARE L TALE CHE

$b(L) = 1$ e $a(L) = 1$

↓

NON POSSIBILE SE ASINCRONO,
SOLO SE SINCRONO

↓

Mentre se asincrono posso ancora fare un -3dB coupler.

↳ Extreme case $\Delta\beta = 2C$

$$\delta = \sqrt{\frac{4C^2}{4} + C^2} = C\sqrt{2}$$

$$L = \frac{\pi}{2\sqrt{2}C}$$ to have a splitter async

Se $\Delta\beta > 2K$, two wg too different so I cannot couple all the power from one to another, but it stays more in one of the two (the one that I excite).

Coupling efficecy :

$$K = \frac{P_2}{\underbrace{P_1+P_2}_{\text{on output}}} \underset{\text{IF NO LOSS}}{\boxed{=}} \frac{P_2}{P_0} = \overbrace{\sin^2(cL)}^{\text{SYNC}}$$

$K = 1$ ONLY IN SYNC ONES

If I don't want coupling is useful to have $\Delta\beta \neq 0$

$$\boxed{T_C = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix}}$$

Splitter

$\Delta B = 0$

$cL = \frac{\pi}{4}$

$$TC = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

ASYNC

$\delta L = \frac{\pi}{4}$

$\Delta\beta = 2K$

The difference is the phase → SYNC : quadrature

→ ASYNC : both

So it's preferable SYNC because I know the outputs will be in quadrature.

Dependance of δ to λ in couplers

$$K_{ij} = \frac{K_0^2}{\underbrace{2\beta}_{\propto \lambda^{-1}}} \underbrace{\int \varphi_i \, \varphi_j \, \Delta n^2}$$

→ The shape of the modes $\propto \lambda$

Change λ, change shape modes

$\downarrow$

More confined or less

$\downarrow$

Change overlap → so K_{ji}

$\lambda \downarrow\downarrow \Rightarrow$ MODES CONFINED $\uparrow\uparrow$

$\Downarrow$

$K_{ji} \downarrow\downarrow$

$C = C_0 \lambda$

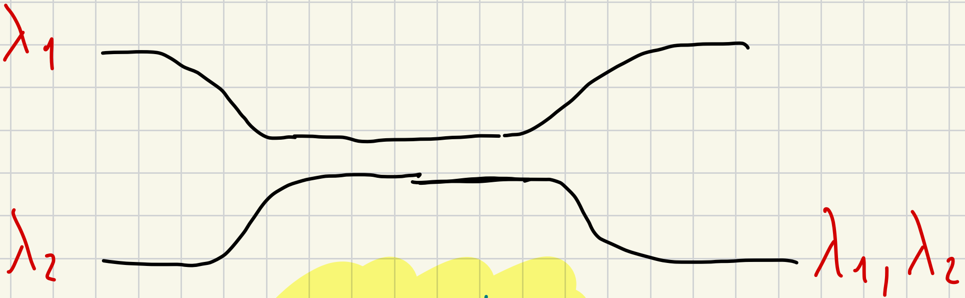
$\lambda \uparrow\uparrow \Rightarrow$ MODES CONFINED $\downarrow\downarrow$

$\Downarrow$

$K_{ji} \uparrow\uparrow$

So low Δh not so variation for different λ ($C_0 \downarrow\downarrow$), but with $\Delta h \uparrow\uparrow$, $C_0 \uparrow\uparrow$, great variation of c with λ.

So use it to split λ in input to do MUX or DEMUX



WDM coupler

$$\begin{cases} P_{CROSS}(\lambda_1) = \sin^2\left(\overbrace{c_0 \lambda_1 L}^{\frac{\pi}{2}+2N\pi}\right) = 1 \\ P_{BAR}(\lambda_2) = \cos^2\left(c_0 \lambda_2 L\right) = 1 \end{cases}$$

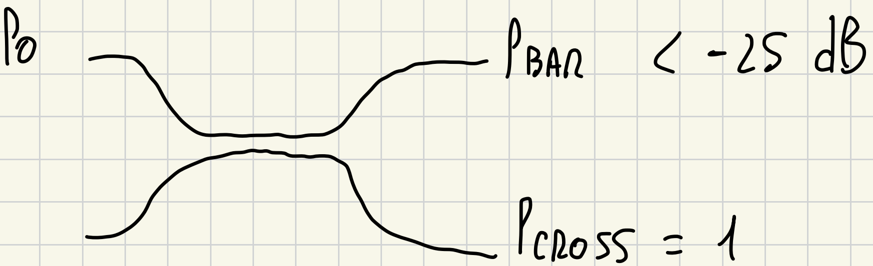
$K = \sin^2(cL)$

$$c_0 L (\lambda_1 - \lambda_2) = \frac{\pi}{2}$$

$$N = \frac{1}{2} \frac{\lambda_1}{|\lambda_1 - \lambda_2|}$$

N must be an integer, if it's NOT I cannot have solutions, so it works only for λ_1 far away from λ_2, like pump + signal in laser and amplifier.

Precision required



$$L_c = \frac{\pi}{2C}$$

$$P_{BAR} = \cos^2 c' L_c = 3{,}16 \cdot 10^{-3}$$

$$c' L_c = \cos^{-1}\left(\sqrt{3{,}16 \cdot 10^{-3}}\right)$$

1,5145 $\quad \frac{\pi}{2} \quad$ 1,6270

1,57 nominal value ($P_{BAR} = 0$)

Accuracy required on the process : $\varepsilon = \pm 3,7\%$ w.r.t. nominal one

between 1,5145 and 1,6270 $P_{bar} < -25$ dB.

$$c \propto e^{-\gamma g}$$

$$c' \propto e^{-\gamma (g + \delta g)}$$

So error on L are linear, while error on the gap cause great exp variation on c' so error greater on directional coupler.

If I want $cL = \frac{\pi}{2} + \pi$ I need $\varepsilon = \pm 1,2\%$

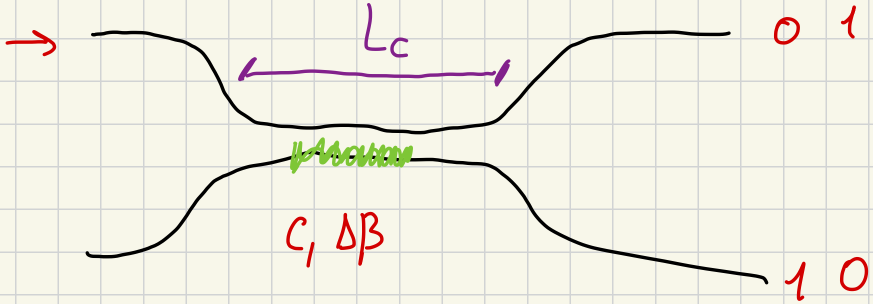
$$\Downarrow$$

$cL \uparrow\uparrow \Rightarrow$ more stress on the accuracy of the process

small error on gap and $\varepsilon\%$ is not achievable

SO CHOOSE ALWAY THE FIRST CL

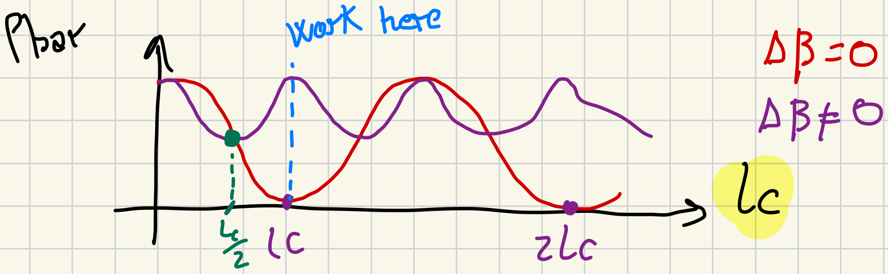
Switch



$\Delta\beta = 0$ $\quad L_c = \frac{\pi}{2C} \rightarrow P_{bar} = 0$, but with a particular $\Delta\beta$:

$$P_{bar} = \cos^2\left(\sqrt{\frac{\Delta\beta^2}{4} + C^2}\; L_c\right) = 1$$

Induce a change on $\Delta\beta$ (with electro-optic or temperature or UV) to have $P_{bar} = 1$ or 0, but changing the characteristic:



Mi muovo da $\cos^2(c L_c)$ a $\cos^2\left(\sqrt{\frac{\Delta\beta^2}{4}+c^2}\, L_c'\right)$

$$\sqrt{\frac{\Delta\beta^2}{4}+c^2}\,L_c' = \frac{\pi}{2} \rightarrow L_c' = \frac{\pi}{2}\,\frac{1}{\sqrt{\frac{4\Delta\beta^2}{4}+c^2}} = \frac{L_c}{2}$$

$$\frac{\cancel{\pi}}{\cancel{2}}\,\frac{1}{\sqrt{\frac{\Delta\beta^2}{4}+c^2}} = \frac{\cancel{\pi}}{2c}\cdot\frac{1}{\cancel{2}}$$

$$\frac{\Delta\beta^2}{4}+c^2 = 4c^2$$

$$\Delta\beta^2 = 4\cdot(3c^2)$$

$$\Delta\beta = 2\cdot\sqrt{3}\,c$$

$$\Delta\beta = \frac{\sqrt{3}\,\pi}{L_c}$$

Difficult to achieve, better other structure !

Dependence on gap

g large → low loss

→ large BW

→ no excitation of higher order modes, low distortion

→ large coupler

→ "easy" fabrication

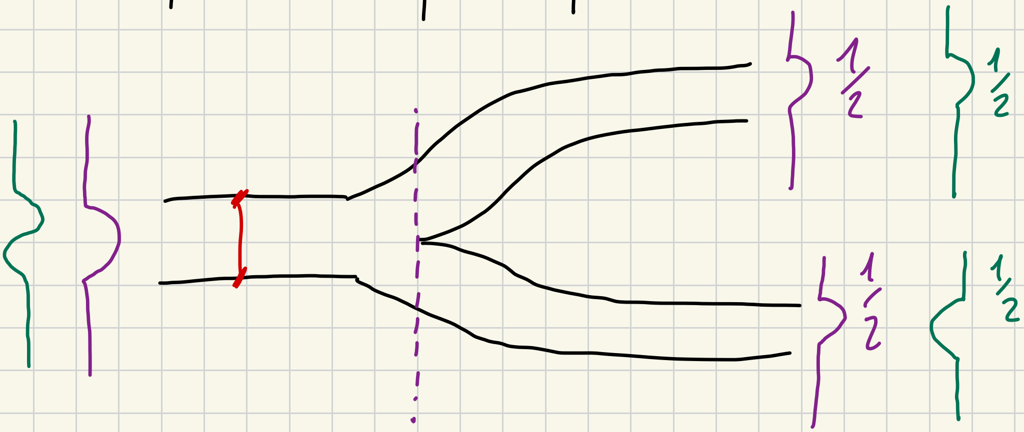
I cannot go to low, I'm limited by the process

$$c \propto \int \psi_1 \psi_2 \Delta n \propto e^{-g}$$

Y - BRANCH

Always two input and two output, for more MMI or Star coupler.

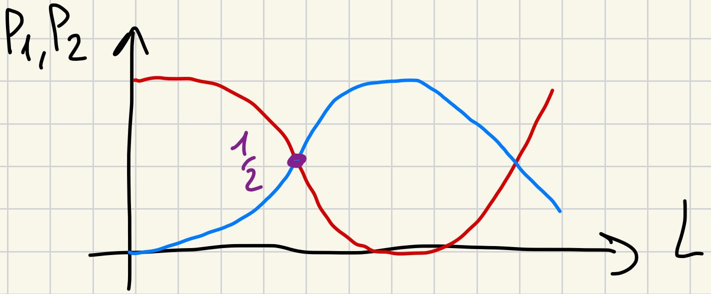
Bimodal wg that split apart :



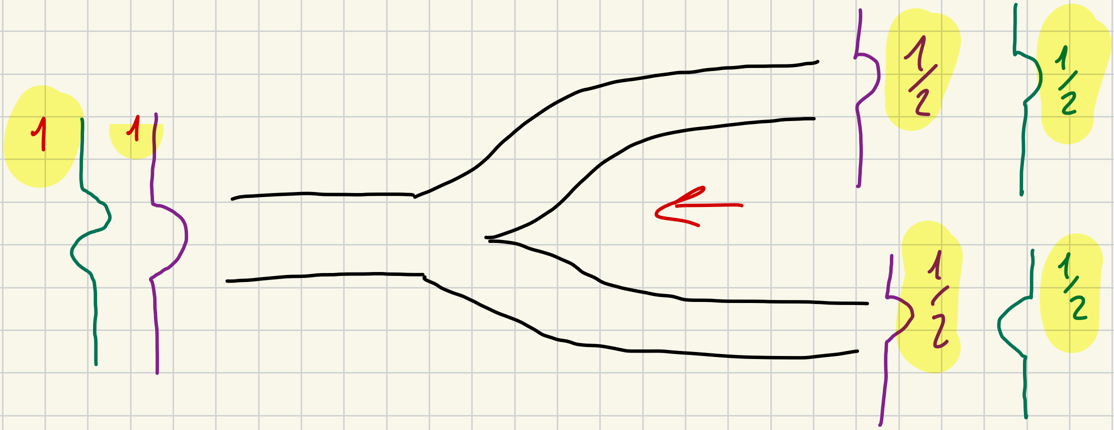
It' a power splitter and with zero phase mismatch for the fundamental mode and with 180° phase for the second mode of the single wg :

$$T_c = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

If adiabatic the bifurcation, T_c does NOT depend on λ, extremly robust.



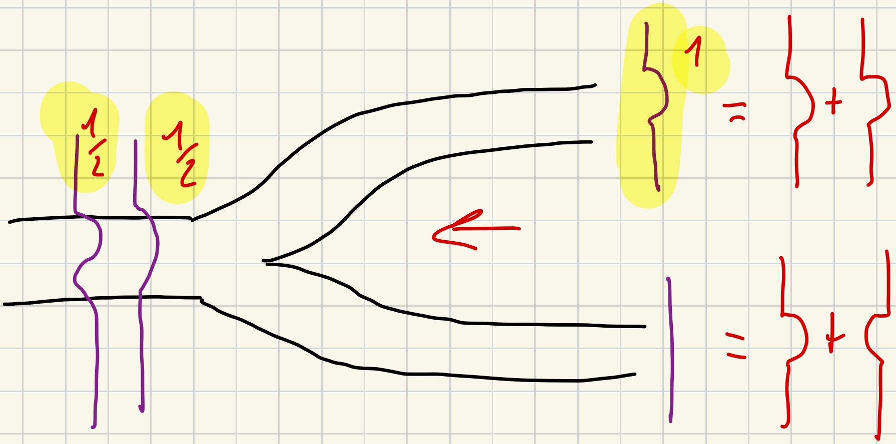
On the reverse direction:



It' a power combiner

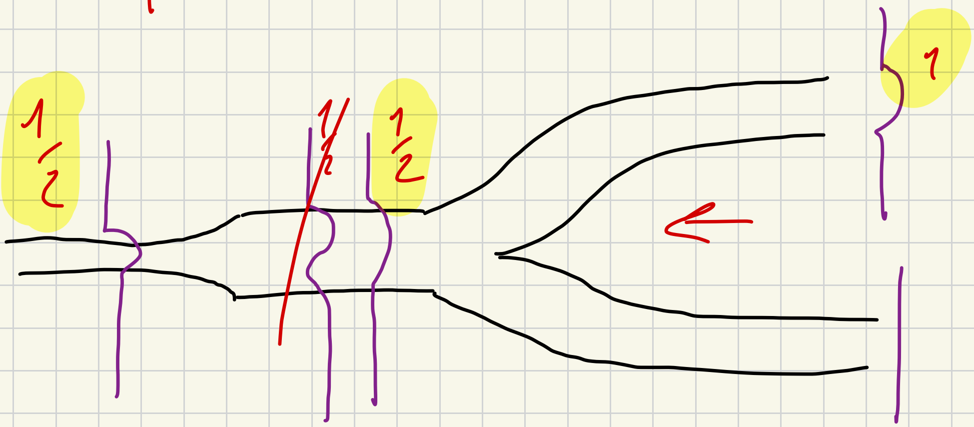
Special case

Field only on one



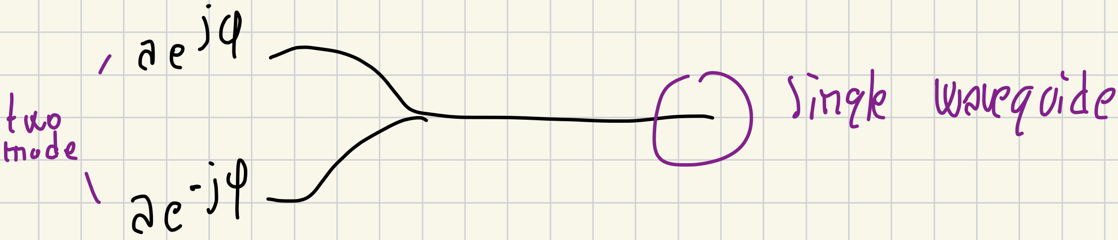
I excite the two modes.

With taper



Only the fondamental there is, but I lost half of the power because I lost the higher order mode.

General case with two wave



$$\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a\,e^{j\varphi} \\ b\,e^{-j\varphi} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} a\left(e^{j\varphi} + e^{-j\varphi}\right) \\ a\left(e^{j\varphi} - e^{-j\varphi}\right) \end{bmatrix}$$

$$= \begin{bmatrix} a \cos\varphi \\ j a \operatorname{sen}\varphi \end{bmatrix}$$

1° mode

2° mode (irradiato x monomodal)

So the output amplitude depend on the phase difference between the two input

$\downarrow$

↗ impossible with wave

IT'S NOT A SUM OF POWER, BUT AN INTERFERENCE BETWEEN THE FIELD OF THE TWO.

So with two field with $\varphi = \frac{\pi}{2}$ I can have NO power on the fundamental mode, while with $\varphi = 0$ only the fundamental has power.

$$\begin{cases} b_0^2 = a^2 \cos^2 \varphi \\ b_1^2 = a^2 \operatorname{sen}^2 \varphi \end{cases}$$

SO I CAN CONTROL THE PHASE OF THE INPUT TO CONTROL THE AMPLITUDE OF THE OUTPUT

↳ making waves interfere

FILTERS

Characteristic and periodicity (depends on phase so are periodic)

I can also have destructive/constructive interference, so same wave in phase or not.

General TDF :

$$\underbrace{H(f)} = \sum_{\underbrace{m}_{\text{interference}}}^{N} c_m e^{j\phi_m} e^{-j(2m\pi f T)} \qquad \text{FIR}$$

Fourier expansion of the function that I want

$$H(f) = \left[H_{FIR}(f) \right]^{-1} \qquad \text{IIR}$$

with $T = \underbrace{\Delta L}_{L_1 - L_2} n_g / c$

$L_1 - L_2$ → due path diversi causano uno sfasamento tra due λ

The phase delay that I accumulate depends on the λ, so I see the phase shift between two λ:

$$\Delta\phi = \frac{2\pi}{\lambda}\left(n_{eff_1}(\lambda)\, L_1 - n_{eff_2}(\lambda)\, L_2\right)$$

$$\Delta\phi(\underbrace{f_m}) = \frac{2\pi f_m}{c} \underbrace{\Delta n_{eff}(f_m)\, L}_{\Delta L_{eff}(f_m)} = 2\pi m$$

f with same $\Delta\phi$

$$n_{eff}(f_m) = n_{eff}(f_0) \pm \frac{FSR}{2} \frac{\partial n_{eff}}{\partial f}$$

$$FSR = \frac{c}{n_g \Delta L}$$

The phase shift depends on n_{eff}, while the periodicity on n_g.

↳ Always true for interference filter!

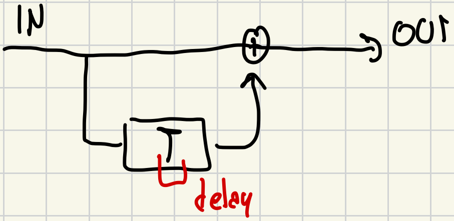
TDF

$N = 1$

$$H(\omega) = 1 + \underbrace{C_1}_{\text{if } 1} e^{-j\omega T}$$

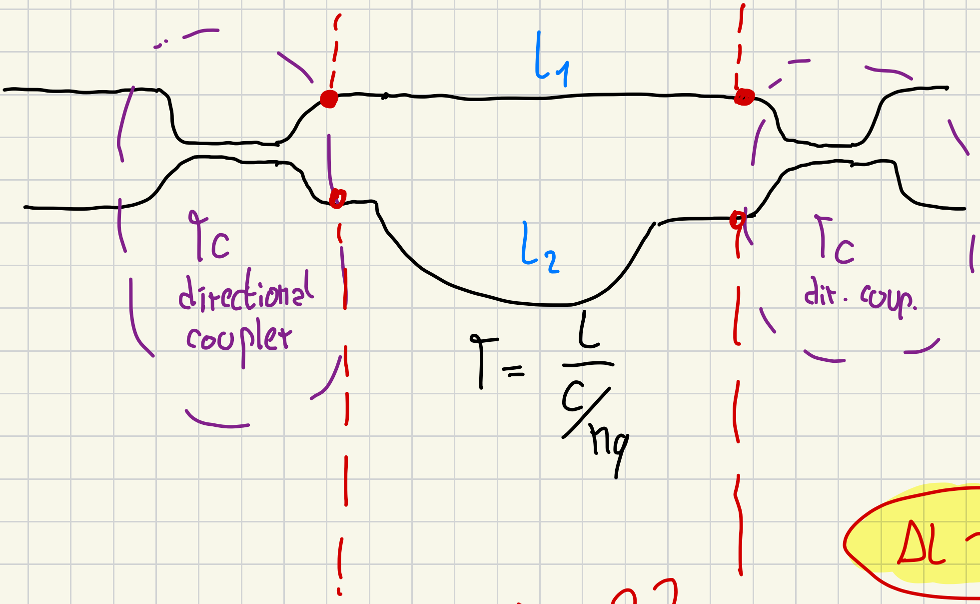
$$↳ \quad e^{-j\frac{\omega T}{2}} \underbrace{\left(e^{j\frac{\omega T}{2}} + e^{-j\frac{\omega T}{2}} \right)}_{\cos\left(\frac{\omega T}{2}\right)}$$

It' a cosine, so a periodic filter, NOT Fantastic, but it's the building block for FIR filter of higher order (N > 1).



GENERAL

⇓ In photonic



$$T = \frac{L}{c/n_g}$$

$\Delta L \sim$ mm

Γ_L ??

$$T_2 = \begin{bmatrix} \overbrace{e^{-j\frac{2\pi}{\lambda} n_{eff_1} l_1}}^{\varphi_1} e^{-\alpha L_1} & 0 \\ 0 & e^{\underbrace{-j\frac{2\pi}{\lambda} n_{eff_2} l_2}_{\varphi_2}} e^{-\alpha L_2} \end{bmatrix}$$

$$= e^{-j\varphi_1} \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Delta\varphi} \end{bmatrix}$$

$$\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \underbrace{T_{c_2} T_2 T_{c_1}}_{\hookrightarrow \text{Opposite direction}} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Different MZ, but with the same $\Delta\varphi$ in the end behave equally.

$$P_{bar} = |T_{11}|^2 = \cos^2\frac{\Delta\varphi}{2}\cos^2 2c\underbrace{L}_{\text{same length for couplers}} + \operatorname{sen}^2\frac{\Delta\varphi}{2}$$

$$P_{CROSS} = |T_{21}|^2 = \cos^2\frac{\Delta\varphi}{2}\,\mathrm{sen}^2 2cl$$

If $K_1 = K_2 = \frac{1}{2} \Rightarrow cl = \frac{\pi}{4}$ (SYNC)

$$\begin{cases} P_{bar} = \mathrm{sen}^2\frac{\Delta\varphi}{2} \\ P_{cross} = \cos^2\frac{\Delta\varphi}{2} \end{cases}$$

No loss considered

$\Delta\varphi$ is periodical in ω, NOT in λ

↓

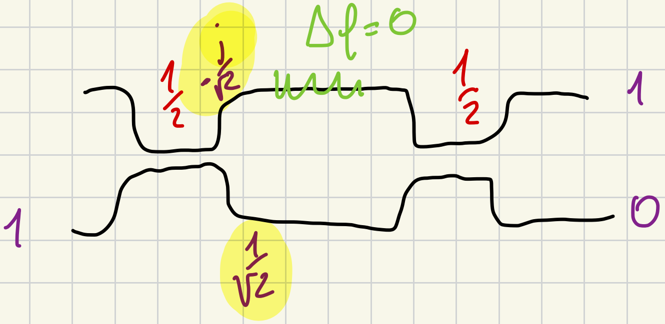
So also $H(\omega)$

↳ Channel of WDM are equally spaced in frequency not in λ.

↙ FSR [Hz]

↳ $$\Delta\varphi \propto \underbrace{\omega}_{\omega\uparrow,\ \Delta\varphi \text{ is periodic}} \text{ or } \frac{2\pi}{\underbrace{\lambda}_{\lambda\uparrow,\ \Delta\varphi\to 0}}$$

Esercizi visivi



$$\Gamma_{c_{\frac{1}{2}}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix}$$

- $b_0 = \frac{1}{2}\left[a_0 - j a_1\right]$

  $= \frac{1}{2}\left[-j - j\right] = -j$

- $b_1 = \frac{1}{2}\left[-j a_0 + a_1\right]$

  $= \frac{1}{2}\left[-1 + 1\right] = 0$

If $\Delta\varphi = \pi \Rightarrow |b_0|^2 = 0$

$|b_1|^2 = 1$

Max and min of P_{bar}/P_{cross}

$$\hookrightarrow \frac{\Delta\varphi}{2} = \pi + 2N\pi + \frac{\pi}{2}$$

↳ if I want $P_{bar} = 1$

Periodicity → $FSR = \dfrac{c}{\underbrace{n_g}_{} \Delta L} = \dfrac{1}{\tau_g}$

always because $n_{eff} \propto \omega$

$$n_g = n_{eff_0} - \omega \frac{\partial n_{eff}}{\partial \omega}$$

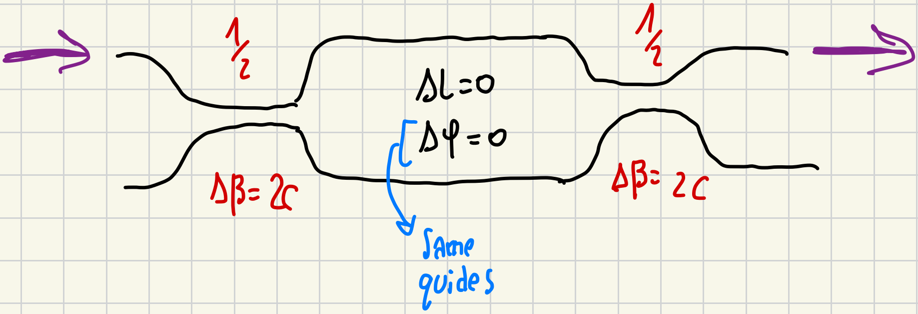
$$\frac{2\pi}{\lambda} \underbrace{n_{eff}}_{} \Delta L = 2N\pi$$

↳ Interested in phase = n_{eff}

What if -3 dB coupler is $\Delta\beta = 2c$?

$$\Delta\beta = 0 \rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix}$$

$$\Delta\beta = 2K \rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$



What if two wg different with $\Delta L = 0$?

$$\Delta\varphi = \frac{2\pi}{\lambda}\left(n_{eff_1} l_1 - n_{eff_2} l_2\right)$$

But $n \propto$ Temperature :

$$n = n_0 + K_T \Delta T$$

$K_T = 10^{-5}$ SiO_2, SiN

$K_T = 10^{-4}$ Si, InP, polimers, TiO_2

$K_T > 0$ (SiO_2, SiN, Si, InP)

$K_T < 0$ (polimers, TiO_2)

In MZ I want to switch from cross to bar ($\Delta L = 0$), with an heater :

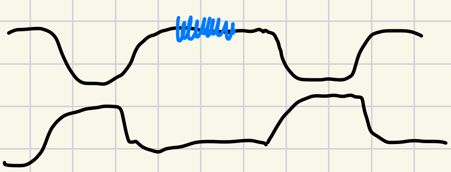
$$\frac{\Delta\varphi}{\cancel{2}} = \frac{\pi}{\cancel{2}}$$

$$\frac{2\pi}{\lambda} \Delta n_{eff} L = \frac{2\cancel{\pi}}{\lambda} K_T \Delta T L = \cancel{\pi}$$

$$\Delta T = \frac{\lambda}{2 K_T L}$$

For SiO_2 with $L = 1mm$ $\Delta T = 100°C$

If Si $K_T = 10^{-4}$ $\Rightarrow$ $\Delta T = 10°C$.



$\Delta\varphi \propto \Delta T$

Recap MZ with $\Delta\beta = 0$ splitter:

$$T_{MZ} = \begin{bmatrix} \text{sen}\left(\frac{\Delta\varphi}{2}\right) & \cos\left(\frac{\Delta\varphi}{2}\right) \\ \cos\left(\frac{\Delta\varphi}{2}\right) & \text{sen}\left(\frac{\Delta\varphi}{2}\right) \end{bmatrix}$$

Useful for switch, modulatore and variable optical attenuator.

Filter MZ

$$H(\omega) = 1 - C e^{-j\omega T}$$

At max/min
$@\ \omega T = \pi + 2N\pi$

$$\tau_g = -\frac{\partial \varphi(\omega)}{\partial \omega} \boxed{=} \frac{2T\overline{C}}{1+\underbrace{C}}$$

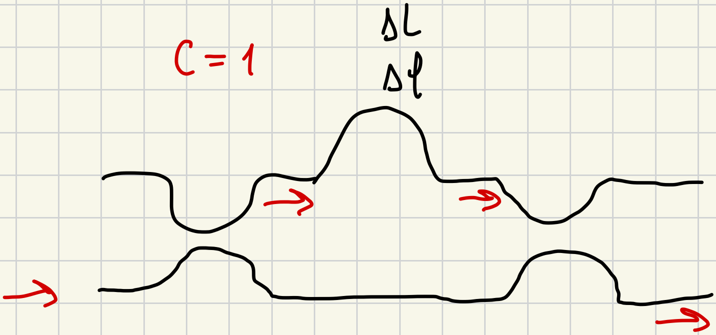
speed of light

↳ coupling of the couplers

C=0

$\tau_g = 0$

C = 1



$\tau_g = T$

For $C = \frac{1}{2} \rightarrow \tau_g = \frac{T}{2}$

So MZI can be used as tunable delay line.

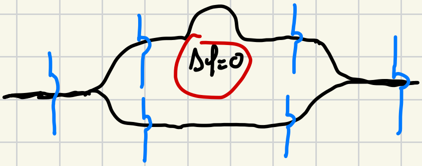
↳ So in the max/min

$$\tau_g = c\,T$$

And with bifurcations?

In this case a_2 is the second mode, if monomodal guides are used is radiated (power loss if excited).

Particular case



Excited only 1^{st} mode?
At exit only the 1^{st} will exit

General

$$\frac{1}{2}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & e^{-j\Delta\phi}\end{bmatrix}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\ e^{-j\Delta\varphi} & -e^{-j\Delta\varphi}\end{bmatrix}$$

monomodal wg

$$\boxed{\begin{matrix}a_1 = 1\\ a_2 = 0\end{matrix}}$$

$$\frac{1}{2}\begin{bmatrix}1+e^{-j\Delta\varphi} & 1-e^{-j\Delta\varphi}\\ 1-e^{-j\Delta\varphi} & 1+e^{-j\Delta\varphi}\end{bmatrix} \rightarrow b_1 = 1+e^{-j\Delta\varphi} \cdot \frac{1}{2}$$

$$= e^{-j\frac{\Delta\varphi}{2}}\left(e^{j\frac{\Delta\varphi}{2}} + e^{-j\frac{\Delta\varphi}{2}}\right)\cdot\frac{1}{2}$$

$$|b_1|^2 = \cos^2\left(\frac{\Delta\varphi}{2}\right)$$

So – $\Delta\varphi = 0 \rightarrow |b_1|^2 = 1$

└ $\Delta\varphi = \pi \rightarrow |b_1|^2 = 0$

} opposite of -3dB coupler

How to design ?

1) $FSR = 2\Delta f = \frac{c}{n_g \Delta L} \rightarrow \Delta L$

2) $P_{cross}(\lambda_1) = 1 \rightarrow \underbrace{\frac{2\pi}{\lambda} n_{eff} \Delta L = N\pi}$

3) Find cL

$$N \rightarrow \text{int}(N)$$

$$\boxed{FSR'} \leftarrow \boxed{\Delta L'}$$

Right ones

So I optimize P_{cross} for λ_1, but accepting xtalk $P_{CROSS}(\lambda_2) \neq 0$ (low, depend on the application what is acceptable).

$$\Delta L \, n_{eff} = \lambda \cdot \overset{\sim 1000}{N}$$

Now the order of magnitude :

$$\Delta\varphi = \underbrace{\frac{2\pi}{\lambda}}_{\sim 10^{-6}} \; \underbrace{n_{eff}}_{n_{eff_0} + \underbrace{\delta n_{eff}}_{\text{due to aging, process error, change of temperature, stress . . .}}} \; \overbrace{\Delta L}^{\sim 10^{-3}}$$

$$\lambda_0' = \frac{n_{eff}\,\Delta L}{N} + \frac{\partial n_{eff}\,\Delta L}{N} \qquad \frac{\lambda_0}{n_{eff}}$$

$$\delta\lambda_0 = \lambda_0' - \lambda_0 = \delta n_{eff}\,\frac{\lambda_0}{n_{eff}}$$

$$\boxed{\frac{\partial\lambda}{\lambda} = \frac{\partial n_{eff}}{n_{eff}}} = \frac{df}{f}$$

WHATEVER IS THE DEVICE, IF THERE IS A SHIFT OF THE Neff DUE TO SOMETHING CAUSE A SHIFT OF THE TDF OF $\delta\lambda$

For example with FSR = 200 GHz :

$$\frac{200\cdot 10^{9}}{200\cdot 10^{12}}\cdot 1,46 = \delta n_{eff} = \underbrace{1,46\cdot 10^{-3}}$$

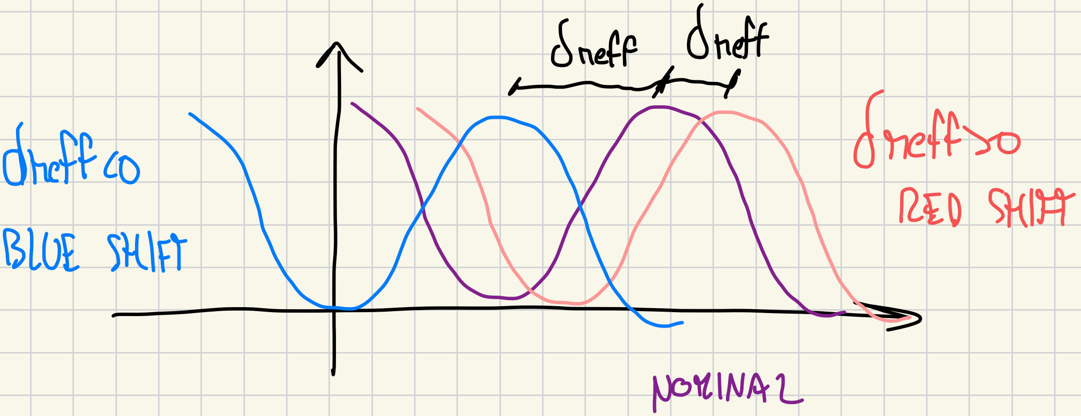
A variation of neff bigger cause a shift the FSR

Suppose that

$$\text{Acceptable } \delta f \simeq \frac{FSR}{100} \longrightarrow \delta n_{eff} = 1,46\cdot 10^{-6}$$

Then $\delta n_{eff} = \underbrace{k_T}_{10^{-5}} \underbrace{\Delta T}$

↳ Control the temperature to max change of 1,46° otherwise I shift the FSR of 2 GHz in the example above



$$P_{bar} = \sin^2\left(\frac{\Delta\phi}{2}\right) = 0$$

$$\simeq \frac{\Delta\phi^2}{4} = 0 = \left(\frac{\pi\,\delta n_{eff}\,\Delta L}{\lambda_0}\right)$$

$$\delta n_{eff} = \frac{\lambda_0 \sqrt{P_{BAR}}}{\pi \Delta L}$$

FOR $P_{BAR} \leq -30$ dB

↳ $\delta n_{eff} = 1,5 \cdot 10^{-5}$

Now if $\delta n_{eff} = 0$:

$$\underbrace{\delta \Delta L}_{} = \frac{\lambda_0 \sqrt{P_{BAR}}}{\pi n_{eff}} = 23 \text{ nm} = 16 \text{ atoms}$$

↳ Also this shift the transfer function

⇓

NEED FOR PRECISE PROCESSES

Is better to control $\delta n_{eff} / \delta \Delta L$ of MZ or the $2cl_c = \frac{\pi}{2}$ of the -3dB coupler?

$\Delta L \sim 10^{-3}$ m so $\delta \Delta L$ so small is very difficult to control

Better spend money to control better cl_c.

↳ MORE CRITICAL TO CONTROL C, SO THE COUPLER

MZ is extreme sensible to parameter

↳ It's good also for sensor!

CASCADED MZ !

↳ SIMPLE

↳ I NEED PERFECTLY ALIGNEMENT OF THE STAGES

↳ DIFFICULT

LATTICE MZI

↳ LESS SIMPLE

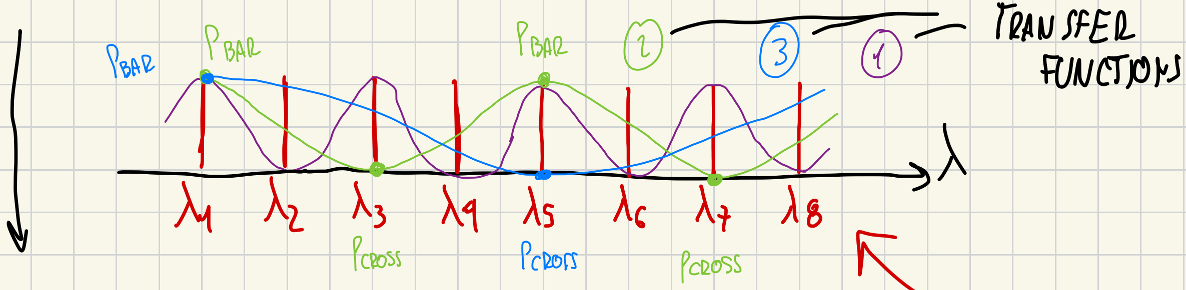
↳ SHAPE TDF MORE CONTROLLABLE

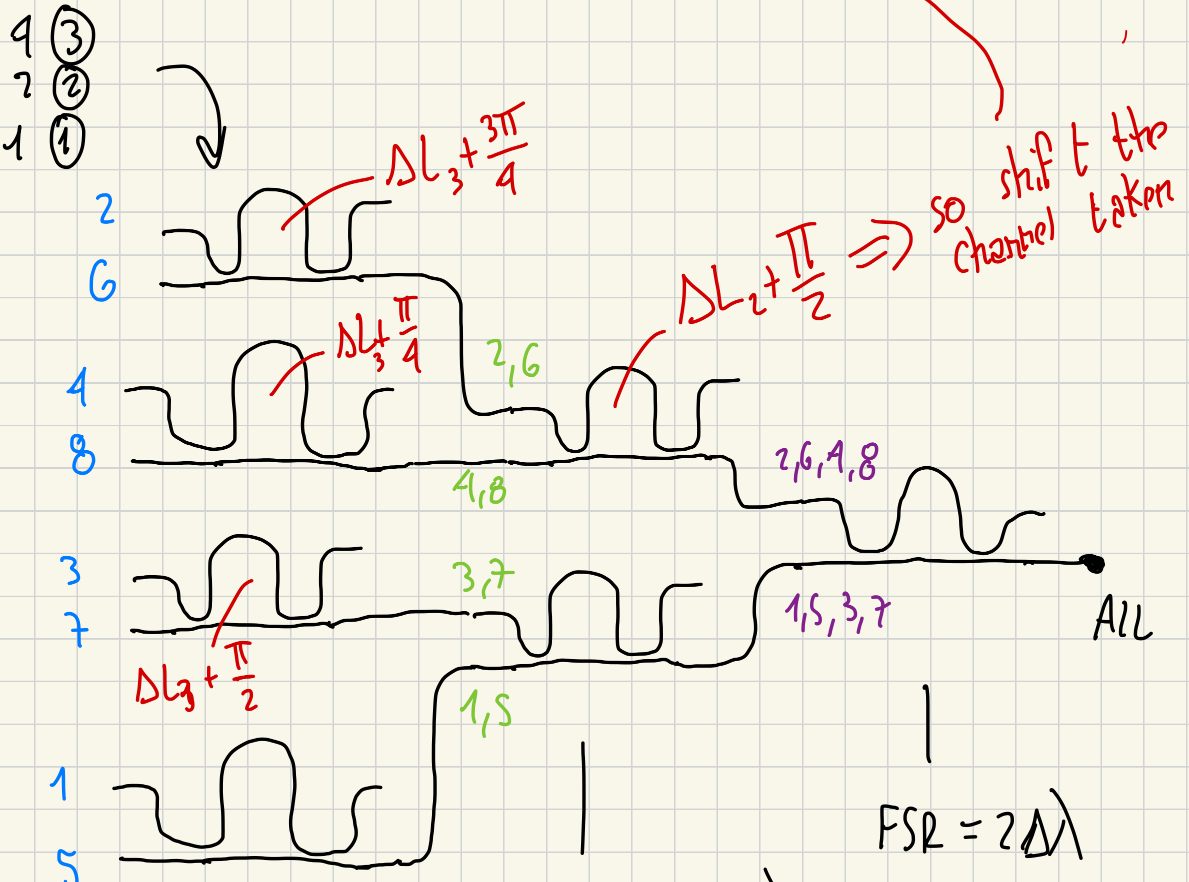
↳ LESS SENSITIVITY ON MISALIGN

MUX

↳ Combine 8 λ in 1 output

I need 7 MZ





$FSR = 2\Delta\lambda$

ΔL_1

$\underbrace{}$ Largest one with FSR lowest

$FSR = 4\Delta\lambda$

$\Delta L_2 = \frac{\Delta L_1}{2}$

$FSR = 8\Delta\lambda$

$\Delta L_3 = \frac{\Delta L_1}{4}$

How to design every MZI

↳ FSR → ΔL

↳ $P_{cross}(\lambda_i) = 1$

↳ $N \rightarrow \Delta L'$

There are losses (exits from unused output) but the overall function is preserved.

DEMUX

It's the reversed of the MUX

↳ Now, consider losses, they are crosstalk (the two output are used both, so leakage is not radiated)

↙

NO WAY TO SEPARATE THE CHANNELS AFTER XTALK

⇓

MORE DIFFICULT TO DESIGN A DEMUX

For more channel ?

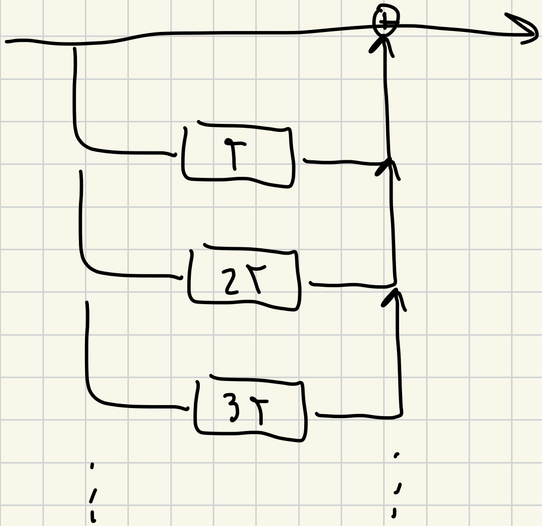
No, it becomes huge and difficult to design

⇓

AWG

Arrayed waveguide grating

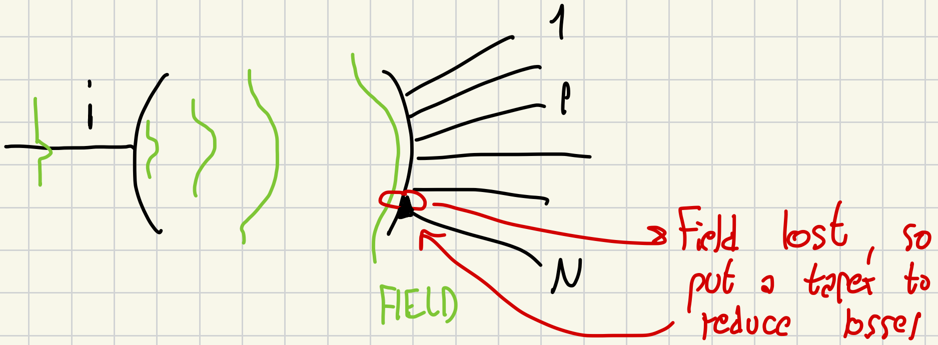
How to implement higher order FIR?



MZ is difficult

STAR COUPLER

In the FPR the field is confined vertically but NOT laterally.

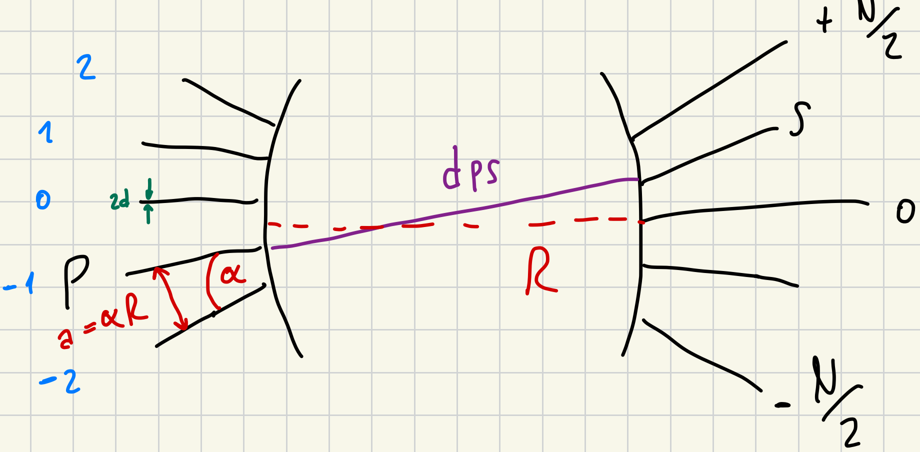


Field lost, so put a taper to reduce losses

The field exiting the input has a Gaussian shape and the TDF:

$$T_{ip} = \frac{\overset{<1}{\boxed{\eta}}}{\sqrt{N}} e^{j\phi_{ip}}$$

I can have M input:



With this convention:

$$d_{ps} = R\left(1 - ps\,\alpha^2\right)$$

Note if $\rho=0 \rightarrow$ dps equal for all output

The width of the (enlargin while travelling) beam is :

$$W(R) = \frac{d}{2}\sqrt{1+\left(\frac{4\lambda R}{\pi h_{eff} d^2}\right)^2}$$

The beam is spreaded reaching the outputs , so choose :

$$R = \frac{N\pi h_{eff} a^2}{2\lambda} \quad \text{to have } \eta = 0{,}5$$

$\downarrow$

FEW OUTPUT (MOLTO VICINI)
THE FIEL IS UNIFORM (THEY SEE THE SAME POWER APPROXIMALLY) BUT I LOSE A LOT

|

↓

OR VICEVERSA

$\alpha \uparrow\uparrow$, Uniform ↑↑

$\alpha \downarrow\downarrow$, uniform ↓↓

↓

Depend on the application

How to choose a ?

$a \to 0$: coupling → AVOID IT !

$a \to \infty$: losses ↑↑ → AVOID IT !

LEZIONE 16 1:36:00

↳ DI CIÓ CHE ESCE E ARRIVA NELLA ZONA DI FAR FIELD, SUGLI OUTPUT NE ARRIVA LA TRASFORMATA DI FOURIER

↓

SE VUOI CAMPO UNIFORME SU TUTTI GLI OUT, QUINDI UN RECT DEVI PARTIRE DA UN SINC E SI PUÓ ACCOPPIANDO PIÚ GUIDE D'ONDA IN INPUT.

↳ Start gaussian → end gaussian (transform to itself)

$$\text{AWG} \;\text{o}\; \underbrace{\text{W}\underbrace{\text{G}}_{\text{grating}}\underbrace{\text{R}}_{\text{router}}}_{\text{Less common name}}$$

It's like phase array. It's better to have $a = (2 \div 3)\, d$ and the wg on the extern more next to each other.

1° star coupler :

$$T_{ps} = \frac{1}{\sqrt{M}} e^{j\varphi_{ps}}$$

$$\varphi_{ps} = \frac{2\pi}{\lambda} n_{eff} \underbrace{d_{ps}}_{\simeq \underbrace{R}\left(1-ps\alpha^2\right)}$$

R must be a certain value to have an exact phase front on the outputs

$$\frac{M\pi n_{eff}\, a^2}{2\lambda}$$

2° Star coupler

↳ It's a phase array, depending on the phase of the input focalize the beam on an output wg

↳ This depends on the λ (on wich depends also $\Delta\phi$)

↳ So it's a phase array on a circle that focus the beam on a circle

Change the phase → change the phase front

Change the point of focus

SLIDE 12-13-14

③ Gratings

$$T_s = e^{j\varphi_s}$$

Total :

$$E_{qsp} = \frac{1}{\sqrt{M}} e^{j\varphi_{ps}} \; e^{j\underbrace{\varphi_s}} \; \frac{1}{\sqrt{M}} e^{j\varphi_{sq}}$$

$$\downarrow$$

$$\frac{2\pi}{\lambda} \underbrace{n_{eff}} (L_0 + s\,\Delta L)$$

$$\downarrow$$

Assume that neff wg is equal to star coupler

not true, but not so wrong, make simple the IDF

$$\Delta\varphi_{pq} = \varphi_{psq} - \varphi_{p,s-1,q} = \frac{2\pi}{\lambda} n_{eff} \left(\Delta L - R(p+q)\alpha^2\right)$$

Now sum the contribution of all the waveguide in the grating :

$$E_{qp} = \sum_{s=0}^{M-1} E_{psq} = \frac{1}{M} \sum_{s=0}^{M-1} e^{j\varphi_{psq}}$$

$$|\Gamma_{qp}|^2 = \frac{1}{M^2} \frac{\text{sen}^2\left(M \frac{\Delta\varphi_{pq}}{2}\right)}{\text{sen}^2\left(\frac{\Delta\varphi_{pq}}{2}\right)}$$

oscillate M times faster than

If $\Delta\varphi_{pq} \to Q\,2\pi$: $\text{sen}^2(\cdot) \to 0$

$$|\Gamma_{pq}|^2 = \frac{1}{M^2} \frac{M^2 \frac{\Delta\varphi_{ps}^2}{4}}{\frac{\Delta\varphi_{ps}}{4}} \to 1$$

Special case $M = 2$

↳ $|\Gamma_{qp}|^2$ is the TDF of a MZ.

How λ, ΔL, R are related to the TDF?

The TDF is periodical, I'm interested in the position of the max, the band and the out of band rejection.

The max are when:

$$\Delta \phi_{pq} = \frac{2\pi}{\lambda} n_{eff} \left(\Delta L - R(p+q)\alpha^2\right) = 2Q\pi$$

$$\lambda_{pq} = \frac{n_{eff}\left(\Delta L - R(p+q)\alpha^2\right)}{Q}$$

$$= \overbrace{\frac{n_{eff}\Delta L}{Q}}^{\lambda_{00}} - \overbrace{\frac{n_{eff} R (p+q) \alpha^2}{Q}}^{\Delta\lambda}$$

$$= \underbrace{\lambda_{00}} - (p+q)\underbrace{\Delta\lambda}$$

Doesn't depend on the star coupler (from porto 0 to 0)

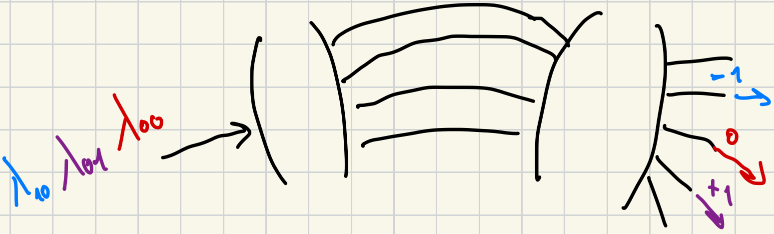
Channel spacing

What is the TDF of p and q-1 ?

It's the same but shifted of $\Delta\lambda$. The other ports have max where the others have zeros.

$$\begin{cases} M-1 \text{ zeros} \\ \text{Each TDF is shifted of } \Delta\lambda \\ FSR = \dfrac{c}{n_g \Delta L} \\ B = \dfrac{FSR}{M} \end{cases}$$

↳ MUX/DEMUX AT THE SAME TIME



Design ?

① FSR → ΔL

② $\Delta L \rightarrow Q <$ λ_0, $\Delta L'$

③ Q "+" $\underbrace{\Delta\lambda}_{\text{given}} \rightarrow R \cdot \alpha^2 \rightarrow$ Then play with the star coupler

④ Find M depending on B

It's an approx! ↳ $n_{eff_{SC}} \neq n_{eff_{gratings}}$

Because in the SC I can have reflected waves that bounce back and forth, changing phase

↓

Do not change too much the TDF, but this is the reason why star couplers have strange shape (to radiate away the reflected power)

↓

So in the star coupler I have different λ

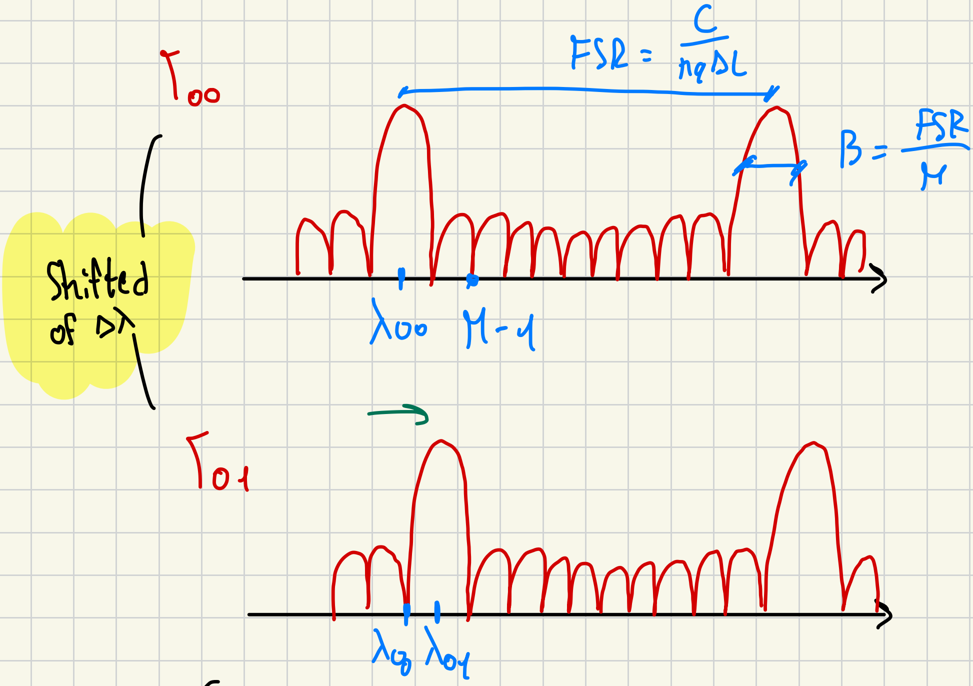
↓

$$\lambda_{pq} = \underbrace{\lambda_0}_{\frac{n_{eff}\Delta L}{Q}} - (p+q)\,\underbrace{\Delta\lambda}_{\frac{n_q R \alpha}{Q}}$$

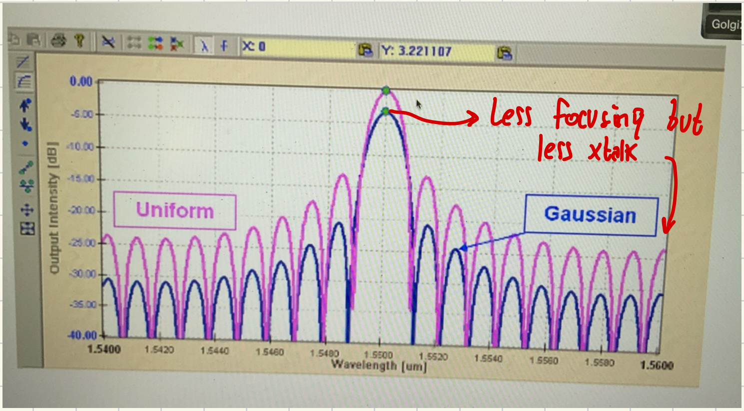
The right one

! ↗

(FIR) → Every time the number of channel increase, increase the number of zero and B ↓↓.



Then there are AWG with different shape, like gaussian where you pay a little attenuation in band while rejectin more



REMEMBER TO USE USE TAPER BETWEEN STAR COUPLER AND INPUT/OUTPUT WAVEGUIDES

↳ Catch the max possible of field and avoid reflections reducing losses.

$\Downarrow$

MORE SIMPLE DESIGN AWG
BUT REALIZING AND OPTIMIZING
IT IS MORE DIFFICULT

Example of design

"$\Delta\lambda$" = 100 GHz
8 channels
25 Gbit/s NRZ $\simeq$ 20 GHz BW of signal

$n_g = 1{,}5$
$n_{eff} = 1{,}45$

↳ PHOTO PC ON WHY

$$FSR = \frac{c}{n_g \Delta L} = N \cdot \text{"}\Delta\lambda\text{"} = 800 \text{ GHz}$$

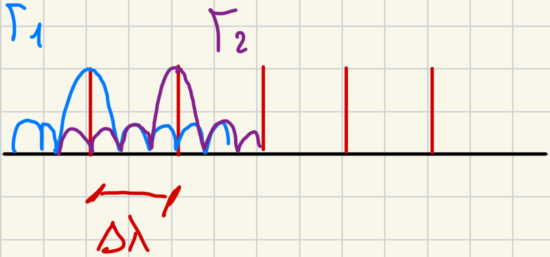
↓

$\Delta L = 250\ \mu m$

↙

$$\lambda_0 = \frac{n_{eff} \Delta L}{Q}$$

$= 1550 \text{ nm}$



$Q' = 233,8$ ↙

↳ $Q = 234 \rightarrow \Delta L' = 250,14\ \mu m$

For SC :

$$\Delta\lambda = \frac{n_g R \alpha^2}{Q} = \text{"}100 \text{ GHz"} = 0,75 \text{ nm}$$

↙

From here S-C $\leftarrow R\alpha^2 = \frac{\Delta\lambda Q}{n_g} = 124,8 \cdot 10^{-9}$

Glass on silicon → $W \approx 5\ \mu m$

$$R\alpha^2 = \frac{a^2}{R} \rightarrow \text{scelgo io, } 15\ \mu m$$

$$\rightarrow \boxed{R = 1{,}8\ mm}$$

And then how many wg in the array?

$$M = \frac{FSR}{B} = \frac{800\ GHz}{20\ GHz} = 40$$

If $M > 40$ I filter the signal, so M alway less or equal to $\frac{FSR}{B}$.

↳ But $M \downarrow\downarrow$, number of zero $\downarrow\downarrow$, $\uparrow\uparrow$ xtalk (out of band lobes)

Now I need to keep aligned the "filter" with the signal

↳ HOW WELL I HAVE TO CONTROL TEMPERATURE, δn_{eff}, $\delta \Delta L$?

$\Delta T = 0{,}1°C$ acceptable for Glass on Si

$$\frac{\delta\lambda}{\lambda} = \frac{\delta n_{eff}}{n_g} = \frac{\overset{10^{-5}}{K}\Delta T}{n_g}$$

$(10^{-4}$ on Si$)$

With this ΔT depending on the material I have FSR of:

$\Delta T = 1°C$ {

$SiO_2 \rightarrow \overbrace{1\ GHz}^{\text{shift}}$] $B = 20\ GHz$, small impact

[$Si, InP \rightarrow 10\ GHz$

↳ Impact big → $B = 20\ GHz$

↳ For these materials M ↓↓ most, like $M=38$, so $B=21$ GHz and I can attenuate the effect of 0.1°C in Si. For ΔT ↑↑, greater complexity!

$\Delta T=0.1°C$

↳ Si → 1 GHz

shift

Acceptable shift are $\frac{1}{10}\frac{FSR}{M} = "\delta\lambda"$

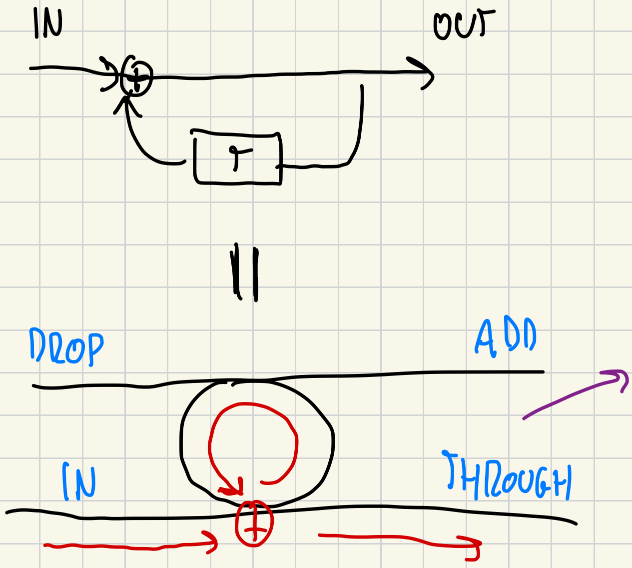
But δn_{eff} can be also given by other fenomena.

↳ In a datacenter every 6 months change something.

AWG usually are on Glass on Silicon

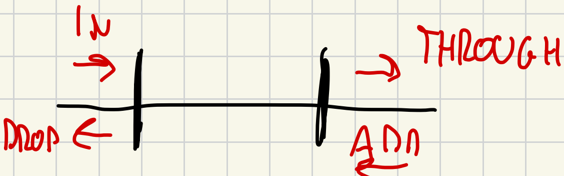
RING RESONATOR

↳ IIR → opposite of FIR

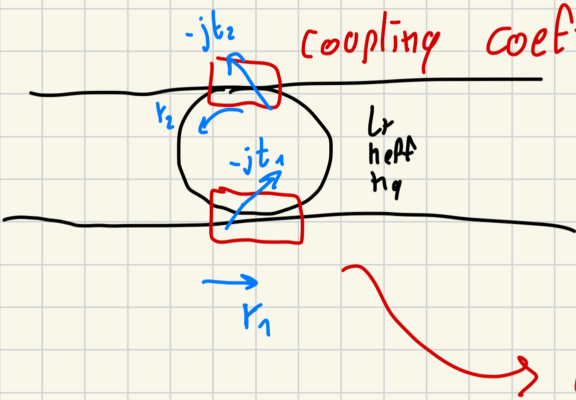


Another ring another T, higher order IIR

The ring is a resonatore and it's a Fabry Perot cavity, in fact another mode to implement a resonator is to use partial reflector mirror:



coupling coefficient t



like a coupler, but different matrix:

$$\gamma = e^{-\alpha L_r} \leq 1 \qquad \text{(LOSSES)}$$

For couplers:

$$\Gamma_c = \begin{bmatrix} r_1 & -jt_1 \\ -jt_1 & r_1 \end{bmatrix} \qquad r_1^2 + t_1^2 = 1$$

For ring, on the through port:

$$H_t(\omega) = r_1 + \underbrace{\left(-jt_1\gamma r_2\right)e^{-j\beta l_r}\left(-jt_1\right) +}_{\text{Light with one round}}$$

$$+ \underbrace{\cdots\cdots}_{\text{second roun}} +$$

$$+ \cdots$$

→ INFINITE RESPONSE FILTER

$$H_t = \frac{r_1 - \gamma r_2 e^{-j\beta l_r}}{1 - \gamma r_1 r_2 e^{-j\underbrace{\beta}_{} l_r}}$$

$$\rightarrow \beta = \frac{2\pi}{\lambda} n_{eff}$$

Depending on λ H_t can be null or different from 0.

$$H_d = 1 - H_t = \frac{-t_1 t_2 \sqrt{\gamma}\, e^{-j\beta \frac{L_r}{2}}}{1 - \gamma r_1 r_2 e^{-j\beta L_r}}$$

H_d has not zero but only a pole (can go to ∞) while H_t has also a zero

$H_d \neq 0$ always

When $H_t = 0$?

together $\begin{cases} e^{-j\beta l_r} = 1 \\ r_1 = \gamma r_2 \end{cases}$ → $\beta l_r = N 2\pi$

$$\frac{2\pi}{\lambda} n_{eff} l_r = N 2\pi$$

$$\lambda_0 = \frac{n_{eff} l_r}{N}$$

RESONANT λ OF THE RING

< in $\gamma < 1$

$H_d = 1$ AT λ_0

Add$\overset{d}{}$ in phase with the light arriving from the ring

$\kappa_1 = \kappa_2 \gamma$

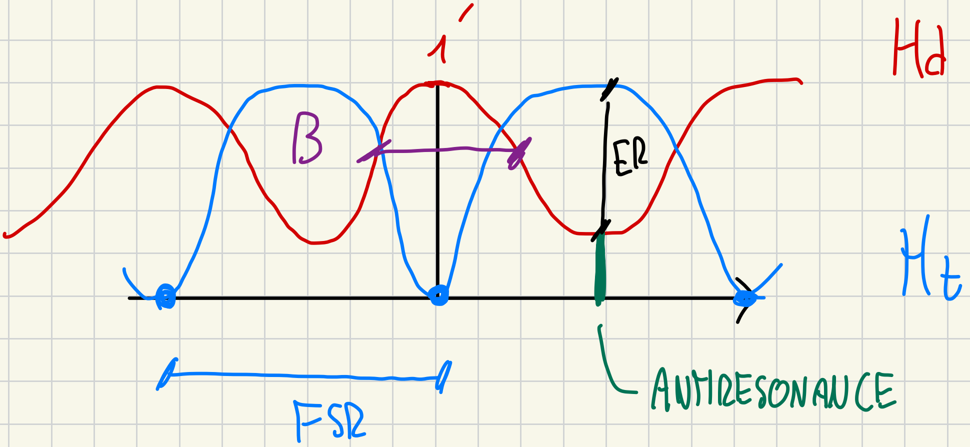
If the two directional coupler are identical ($r_1 = \gamma r_2$) apart for the losses, I always have $H_t = 0$ at resonance.

Light enter → couple in the ring → some goes in the H_t, some in the ring

Keeps recirculating in the ring and at λ_0 all exit from drop

If no reflection nothing exit from ADD port.

$\gamma = 1$ supposed ($\gamma \leq 1$)



Hd cannot go to zero!

$$\boxed{FSR = \frac{c}{n_g L_r}}$$

$$\boxed{B = \frac{FSR}{\pi} \frac{K}{\sqrt{1-K}}}$$

assuming identical couplers and $\gamma = 1$

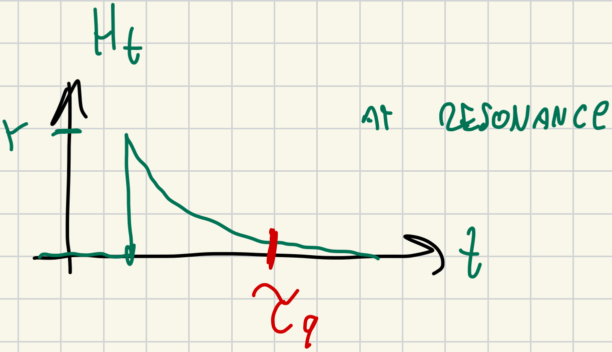
Antiresonance (between the two resonance), when the light that enters the ring arrive π shifted to the first coupler, adding in antiphase.

↳ Here $H_t - H_d$ = Extintion ratio

$$\boxed{ER = \frac{(K-2)^2}{K^2}}$$

What happens during transient?

Not seen here, all off this is at steady state.



Design?

Given L_r, n_{eff}, n_g, r and K find B, FSR λ_0 and ESR

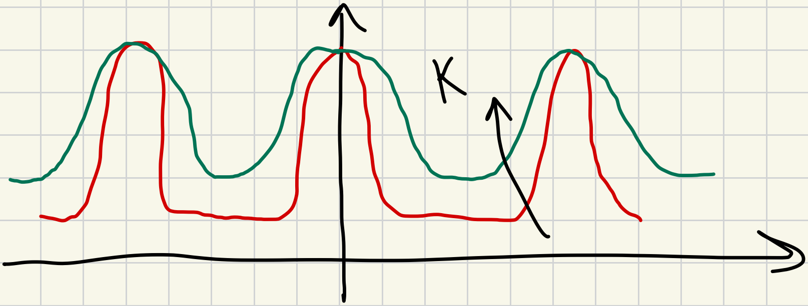
Difference with MZ

MZ is a sine the TDF and BW cannot be changed

↳ With ring, increasing K I can change the BW and ER

↓

Difficulty: have large BW with low ER



Other important parameter to see how selective is the ring with respect to FSR:

$$\boxed{F = \frac{FSR}{B}}$$ FINESSE

And the quality factor :

$$\boxed{Q = \frac{\overbrace{f_0}^{\text{From } \lambda_0}}{B} = N \cdot F \frac{n_{eff}}{n_g}}$$

$$Q = \omega \cdot \frac{\text{energy}}{\text{power loss}}$$

↳ For any resonant structure

Power loss {
- BAD — power loss due to imperfection
- WANTED — power loss by loading the cavity (drop port extract power from the resonant ring to work).

$$\frac{1}{Q} = \underbrace{\frac{1}{Q_L}}_{\text{Due to the coupler}} + \underbrace{\frac{1}{Q_\gamma}}_{\text{attenuation (INTRINSIC Q)}} \simeq \frac{1}{Q_L} \quad \text{if } \underbrace{Q_\gamma \gg Q_L}$$

$\hookrightarrow \frac{1}{Q_\gamma} = 0$ if $\gamma = 1$

Given Q I can design the ring

$\hookrightarrow$ MORE SELECTIVE? $K \uparrow\uparrow$ UNTIL ATTENUATION $\left(\frac{1}{Q_L}\right)$ BECOMES COMPARABLE WITH $\frac{1}{Q_\gamma}$.

But in photonics we don't work with first order resonance $(N=1)$, but higher order, so the most important parameter is F, not Q.

What does F tell me?

How much at resonance I enhancing all the other phenomena.

↳ Time to one round trip:

$$T = \frac{L_r \boxed{n_g}}{c}$$

$$FSR = \frac{1}{T}$$

$$\Downarrow$$

The finesse is the number of round trip, related to the photon life time

in average

in reality infinite with decreasing Amplitud

So the group delay:

$$\boxed{\tau_q = F \cdot T}$$

Don't see chromatic dispersion here

I stay in the ring for that much before going to drop port, so the insertion losses approx is (at resonance):

$$\boxed{IL \simeq F \cdot \gamma}$$

How well do I have to control ΔT?

$$\frac{\delta\lambda}{\lambda} = \frac{\delta n_{eff}}{n_g} = \frac{K \Delta T}{n_g}$$

F times better than MZ, because as AWG and MZ with a shift of 2π in the phase I shift of one FSR the spectrum.

$$B = \frac{FSR}{F}$$

If I shif of $\frac{2\pi}{F}$, I shift of one B (vor acceptable):

$$F = \pi \frac{\sqrt{1-K}}{K}$$

Without loss

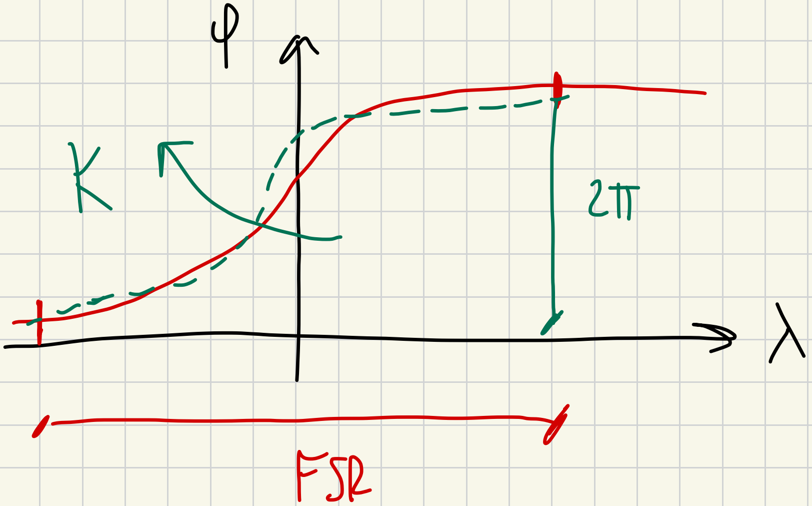
Change K to change F, valid until $Q_\gamma \gg Q_L$, so $\gamma \to 1$

How much time takes the transient?

$$\tau_g = F T = \frac{1}{B}$$

The phase response

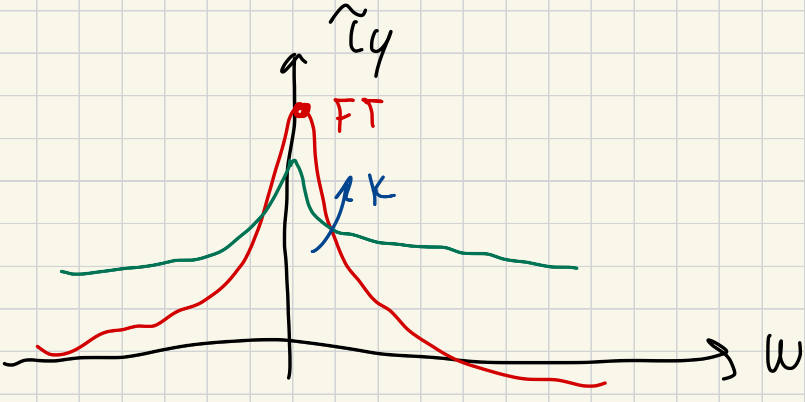
In MZ the phase is linear, here NO!



Jump of 2π between the two resonance over one FSR

↳ Increase B increase the phase jump around resonance

$$\tilde{\tau}_g = \frac{\partial \varphi}{\partial \omega} = \text{slope of } \varphi$$



Selective filter → K ↓↓ , B ↓↓ , $\tilde{\tau}_g$ ↑↑
at resonance

CHROMATIC DISPERSION = $\frac{\delta \tau_g}{\delta \omega}$

→ DETRIMENTIVE IMPACT ON THE SHAPE OF THE PULSE

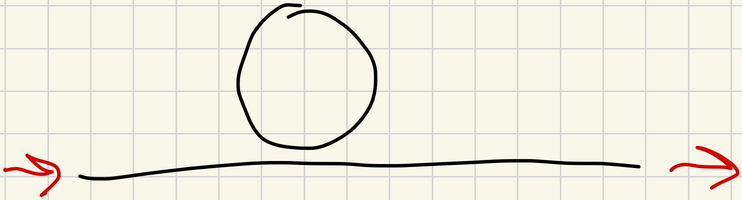
Phase shifter

$r_2 = 1$, $t_2 = 0$, ALL PASS FILTER

↓

Only phase shift

$$T(\omega) = \frac{r - \gamma e^{-j\beta L_r}}{1 - \gamma r e^{-j\beta L_r}}$$



All the light going IN go in OUT without losses so $T = 1$ for every λ. But if I have losses I can have NOTCH in the TDF, because I enhance losses during round trip at λ_0.

At λ_0, $e^{-j\beta Lr} = 1$ so if $\underbrace{\gamma = 1-t^2}$, $\Gamma \to 0$

↙ Critical coupling (Notch at λ_0)

The notch goes to zero only when $\gamma = 1-t^2$, otherwise with $\gamma \neq 1-t^2$ there are notch at λ_0 but not zero.

↳ Only with critical coupling I can switch off the light at λ_0 on the output

↳ Change phase of ring / shift Γ → modulator

No γ, No notch!

Easier to design but more sensitive to tolerances.

⇓

Field intensity can be big in the ring causing greater losses depending on material used and change in neff

↘ So the filter shift

⇓

We try to work with $y \to 1$

But can go out of resonance and field intensity ⇓

↙

So the signal can cause self modulation with itself

↓

Only in material where neff $\propto$ field intensity (most dielectric). I need high power to see these effects.

$$n = n_0 + \underbrace{n_1}_{\text{Kerr effect}} I$$

But also losses depends on I, so also IL.

When happen resonance?

When $L_r = N\lambda_0$, without phase added on the ring by something. In this case I have to add this phase shift when finding resonance:

$$\frac{2\pi}{\lambda} n_{eff} L_r + \varphi_{TH} + \varphi_R \ldots = 2N\pi$$

RING CAN SLOW DOWN THE LIGHT

↳ $V = \frac{c}{n_g L}$ ($n_g \uparrow\uparrow$ and L const, $V \Downarrow$)

$\tau = \frac{L}{c} n_g$

↳ but $\tau \uparrow\uparrow$, also losses increase

$IL = e^{-\alpha L n_g}$

NO MEMORY WITH PHOTONICS

Backscatter → $r_b = \underbrace{F}_{\text{enhanchement of losses}}^{2} \cdot r_b'$

MZ - RING FILTERS

The enhance of loss are on the transition of the TDF, NOT in the flat BAND.

TEST COST A LOT

⇓

BETTER SEND DEVICE TO TEST ONCE THE DEVICE WORK

⇓

ELECTRONIC FEEDBACK
(heater, UV trimmic, electro-optic ...)

Why slow light?

To make shorter device.

↳ But for memory insted?

Not possible in photonic

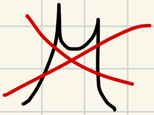
[– Must be small

[– loss must not corrupt data

Not possible in potonic

PHOTONIC IS GOOD TO MOVE THINGS AROUND

Oscillation in group delay are bad, keep it as smooth as possible!



✓

FOR ANY STRUCTURE

MAGNETO - OPTIC

99% of lasers has an isolator to protect it from reflections coming from illuminated wg.

$$\Gamma_c = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \rightarrow \text{NOT SYMMETRIC}$$

Until now all devices were symmetric, what break symmetry here?

↳ Only if device is NOT reciprocal, NOT asymmetric

NON reciprocity

Some material in prefence of a magnetic field change the ε tensor that is NO more symmetric.

↳ But the material must sensitive to $\vec{H}$ or also in time varing or NON linear material.

I'm interested in SYMM and RECIPR material with $\vec{H}=0$, and ASYM and NON RECIPR. with $\vec{H} \neq 0$

↳ So there's optical activity

$$\varepsilon = \begin{bmatrix} \varepsilon_\perp & j\delta_\varepsilon & x \\ j\delta_\varepsilon & \varepsilon_\perp & x \\ x & x & \varepsilon_{\parallel} \end{bmatrix}$$

$\mu = 1$

Transparent

$\delta \propto \alpha H$

What happens to a plane wave inside this material?

Wave equation in form of matrix:

$$\begin{bmatrix} -\beta^2 + \omega^2 \mu_0 \varepsilon_0 \varepsilon_\perp & -j\omega^2 \mu_0 \varepsilon_0 \delta\varepsilon \\ -j\omega^2 \mu_0 \varepsilon_0 \delta\varepsilon & -\beta^2 + \omega^2 \mu \varepsilon_0 \varepsilon_\perp \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = 0$$

Now E_x and E_y are coupled. With $\delta\varepsilon = 0$, the solution is $e^{-j\beta z}$. Now in order to have a solution the determinant must be null:

$$\beta_{\underbrace{R,L}} = \frac{2\pi}{\lambda}\sqrt{\varepsilon_\perp \pm \delta\varepsilon}$$

EIGENVALUE

Right and left because ↓

$$E_x = \pm j E_y$$

EIGENVECTOR

$E_z = 0$ (plane wave) and E_x in that way is a circular polarization wave.

↳ Linear polarized wave cannot be a solution in a material like that.

So the circular right pol. wave and left wave have:

$$n_R = \sqrt{\varepsilon_\perp + \delta\varepsilon}$$

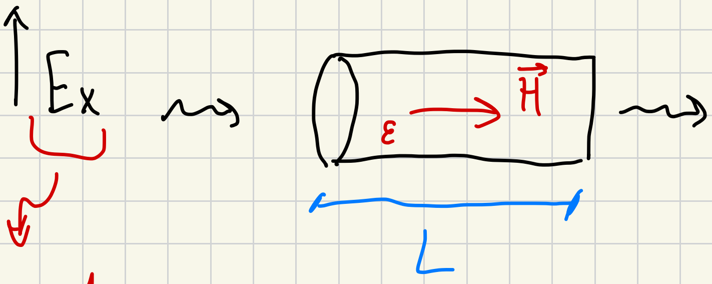
$$n_L = \sqrt{\varepsilon_\perp - \delta\varepsilon}$$

$$\boxed{B_c = n_R - n_L \simeq \frac{\delta\varepsilon}{\sqrt{\varepsilon_\perp}}}$$

Circular Birefringence

Excercise

Linear vertical polarized wave in input (NOT a solution, but it's a combination of two circular polarized):



$$\frac{1}{2}E_R + \frac{1}{2}E_L$$

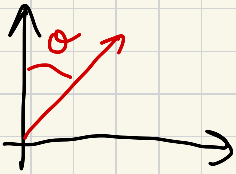
$$E_R e^{-j\beta_R L} + E_L e^{-j\beta_L L}$$

Now come back to x, y and the output is:

$$e^{j\frac{1}{2}\underbrace{(\beta_R-\beta_L)L}}$$

$$\theta = \frac{\omega}{2}\sqrt{\frac{\mu_0\varepsilon_0}{\varepsilon_\perp}}\,\delta\varepsilon L$$

θ is the rotation of the output field w.r.t. the input one. So at output I have E_x but rotate of θ:



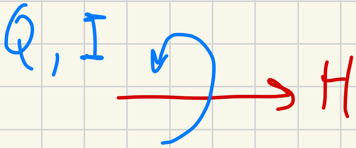
OPTICAL ACTIVITY = ROTATE POLARIZATION
KEEP ELLIPTICITY

θ can be seen as $\delta\varepsilon \propto \vec{H}$:

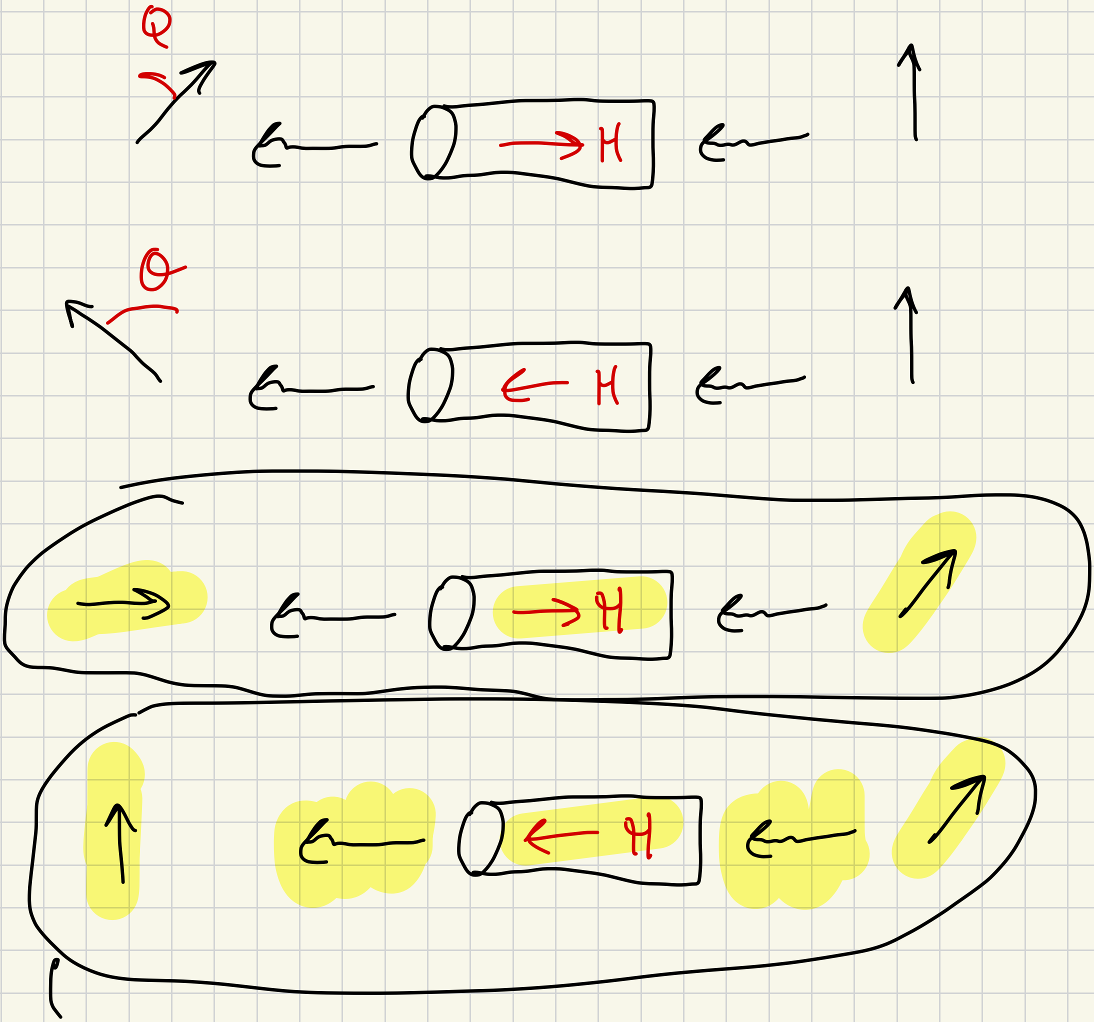
$$\boxed{\theta = \underbrace{V}_{\text{Verdet constant}} \cdot H \cdot L}$$

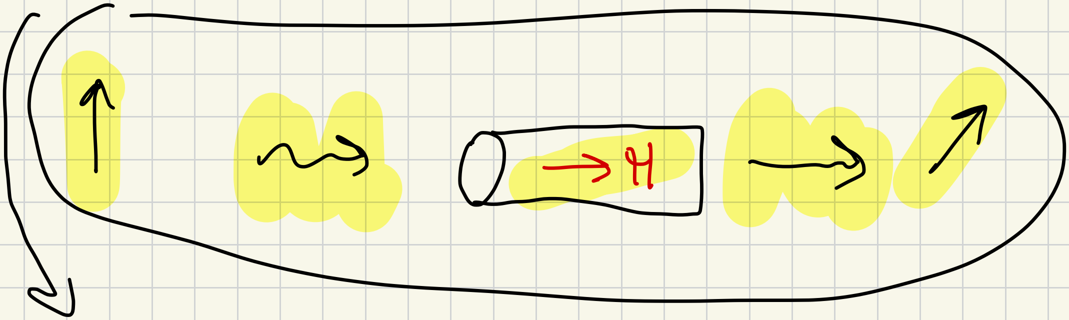
The rotation depends on H so has direction.

↳ Same direction of the current



What if I inverted the direction?





To come back as before I need to change $\vec{H}$ (NON RECIPROCITY), changing only the direction is NOT sufficient.

ISOLATOR → BULKY (NOT POSSIBLE IN INTEGRATED OPTICS)

CIRCULATOR → 3/4 PORTS CIRCUIT

BRAGG GRATINGS

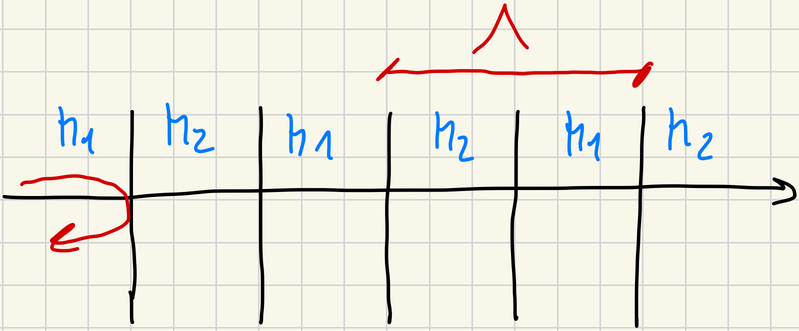
Periodical perturbation of something

⇓

Here in the direction of prop.

↓

Like periodic variation of Δn to induce a periodic variation of $\Delta \phi$.

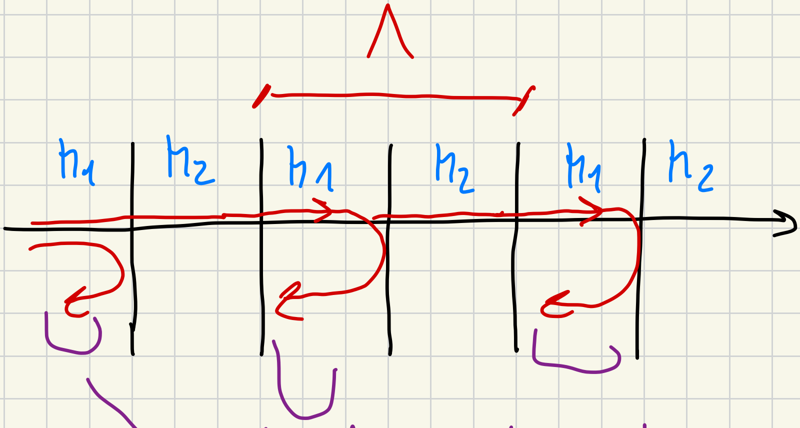


$$r = \frac{n_2 - n_1}{n_1 + n_2}$$ due to the discontinuity of n

If $n_2 > n_1 \Rightarrow$ phase of reflected wave is positive

If $n_2 < n_1 \Rightarrow$ " " " negative

If $n_2 \simeq n_1$, r is small, most of the wave is transmitted:



If it' reflected with the same fase they interfere, the three r "sum" in fied

So the phase of the second must be:

$$\frac{\cancel{2\pi}}{\lambda}\bar{n}\Lambda\cdot 2=\cancel{2\pi}$$

$$\boxed{\lambda_B = 2n\Lambda}$$

BRAGG WAVELENGTH

$$\bar{n}=\frac{n_1+n_2}{2}$$

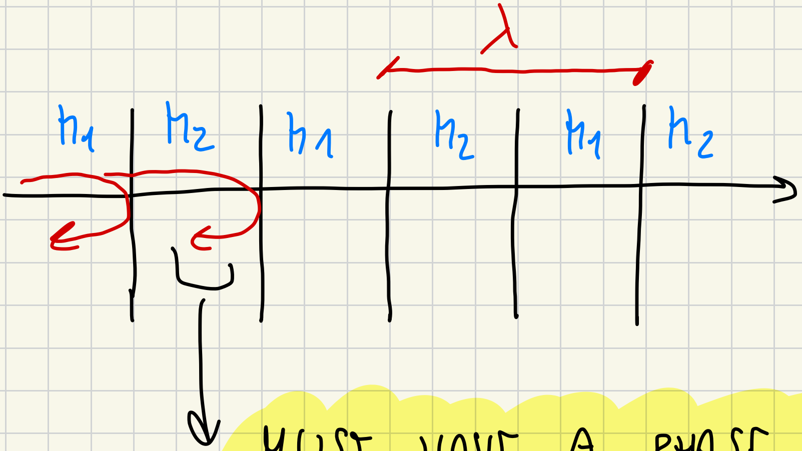
The third reflection must have a phase:

$$\frac{2\pi}{\lambda}\bar{n}\Lambda\cdot 4=4\pi$$

So the total reflection can be very large even with small r ($n_2 \simeq n_1$), because they pile up.

↳ It's a reflector wavelength dependent.

And reflections between $n_2|n_1$?



MUST HAVE A PHASE OF π $(r<0)$, IT'S CORRECT BECAUSE IT'S HALF A PERIOD Λ

$$\frac{\cancel{2}\cancel{\pi}}{\lambda}\bar{n}\frac{\Lambda}{\cancel{2}}+\pi=2\pi$$

SAME!

Every small reflection add up!

Order of Λ

$\lambda = 1,5 \ \mu m$, $n = 1,5$

$$\Rightarrow \Lambda = \frac{\lambda_B}{2n} = 0,5 \ \mu m$$

$$\delta n = n_2 - n_1 = 10^{-5} \div 10^{-3}$$

$$\Downarrow$$

$$r = \frac{10^{-4}}{3} \rightarrow \underbrace{R}_{\text{in power}} = 10^{\underbrace{-9}}$$

I NEED MANY OF THEM

to integrate lasers

Bragg reflector are used to create the mirror of laser, filters, dispersion compensation, equalization filter after amplifier ...

In a Fabry Perot the mirror (100% almost and 95%) play the role of K in the ring, at resonance all the light exit, but the BW is decided on mirrors

↳ In a Fabry Perot the finesse depends on the mirror reflectivity ↑↑r, ↑↑F, the light jumps alot inside it, but at λ_0 everything exit.

↳ $FSR = \frac{c}{n_g 2 L_{CAVITY}}$

Add Ge in gass on SiO_2, perturbe the lattice

↳ Wanted defect → $\delta n = n_2 - n_1$

↓

Not stable, bonding can be broke with UV light to change property

↓

Illuminating Ge - SiO_2 create $Ge e^-$ that is more stable than before

↳ SO PERMANENT VARIATION OF n_{eff}

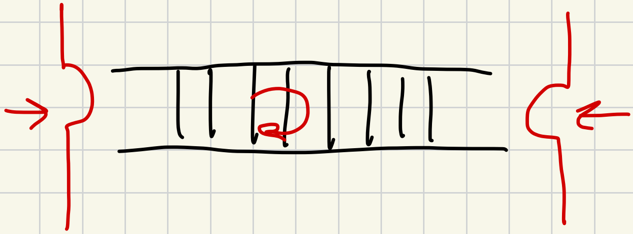
↓

You notice because you change the absorbtion spectrum of the material

↳ SINONIMO DI CAMBIO DI neff

Now you have produce a long chain of this variation.

I want the transmission matrix



I know that the device reflect, so can I find a relation between the two mode, one propagating in z and the other counter propagating?

↳ Coupled mode theory

$$\psi = \underbrace{A e^{-j\beta_+ z} + B e^{-j\beta_- z}}_{\text{Real modes}}$$

$$\simeq \overbrace{A(z)\underbrace{e^{-j\beta_0 z}} + B(z)\underbrace{e^{-j\beta_0 z}}}^{\text{Approximated modes coupled}}$$

Not real mode (they exchange power) but approximate with A and B varying in z

Modes of single Wg unperturbed

Difference with couplers

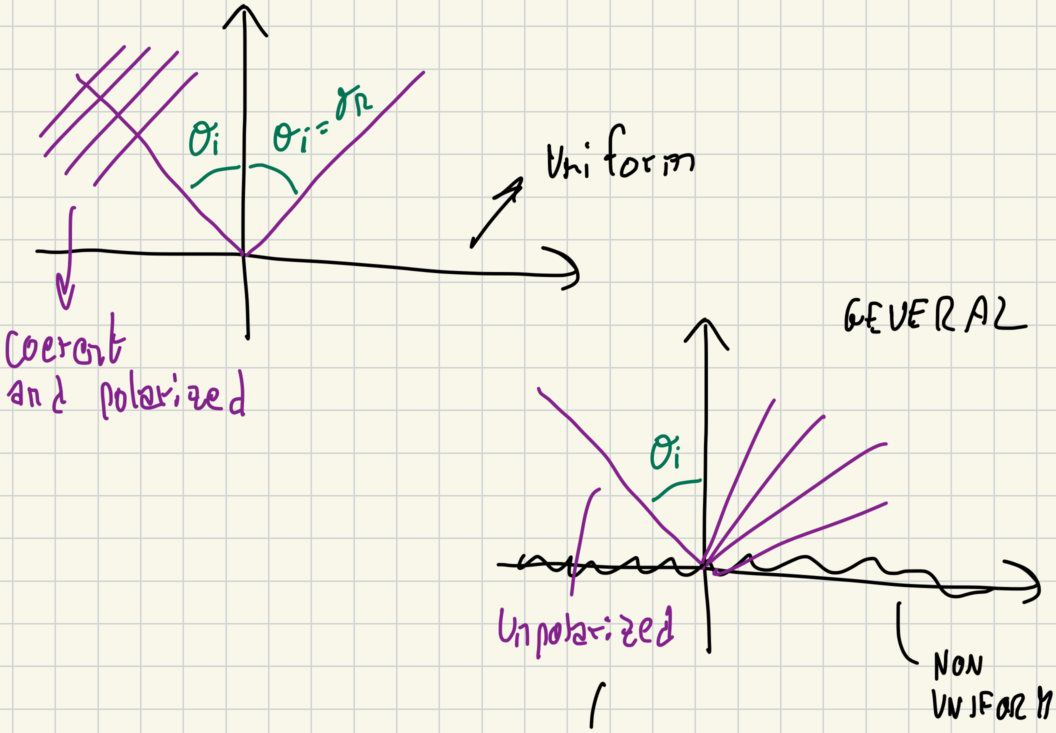
$$\Delta\beta \to 2\beta_0$$

$C \to$ SOMETHING $\propto z$, periodical

$\Delta\beta$ is huge compared to C and c is periodical in z.

↳ There will be difference!

Reflection on a mirror



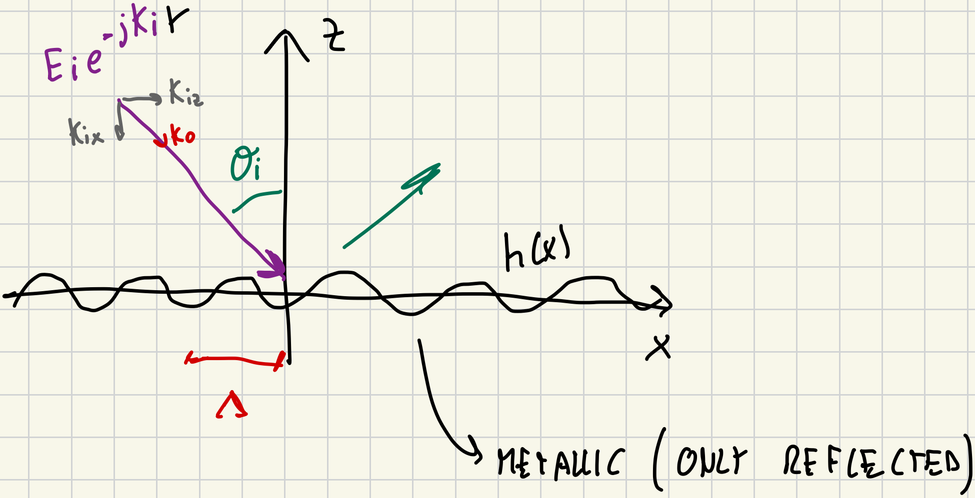
↓

SCATTERING

The scattered field can be represented as:

$$E_R = \int \underbrace{e^{-jk_R\bar{r}}}_{\text{INPUT PLANE WAVE}}$$

Now the surface is periodical



The reflected one :

$$\vec{E}_i\, e^{-jk_{ix}\hat{x}}\, e^{-jk_{iz}\hat{z}} + \underbrace{\vec{E}_R(x,z)}_{\propto\, e^{-jk_{i_x}\hat{x}}\; g(x,\underbrace{z}_{\big\downarrow})} = \begin{cases} 0 & \text{metal} \\ \overline{E_T} & \text{other} \end{cases}$$

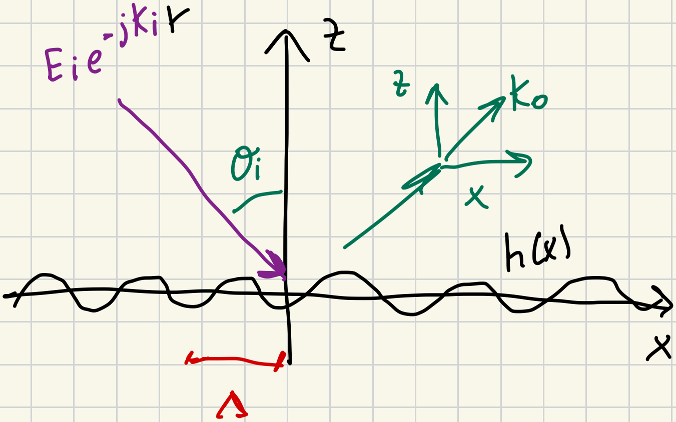
$$\underbrace{h(x)}_{\text{PERIODIC}}$$

Also $\vec{E}_R$ will be periodic in Λ, so I can represent with a Fourier expansion (Not integration)

$$E_R, E_T \propto e^{-jk_{i_x}\hat{x}} \sum_{\underbrace{m}_{\text{order of harmonic}}} R_m\, \underbrace{e^{-j\frac{2\pi}{\Lambda/m}\hat{x}} \cdot e^{-jk_z^{(m)}\hat{z}}}_{\text{Harmonic in } x \text{ e } z}$$

For the m harmonic the phase :

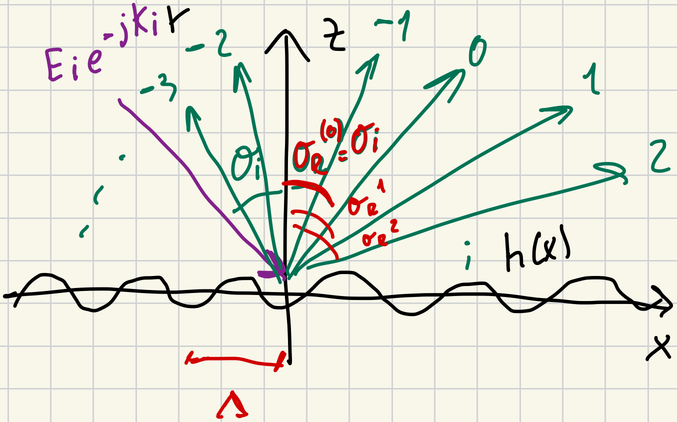
$$\left[K_{ix} + \frac{2\pi m}{\Lambda}\right]^2 + k_{Rz}^{(m)\,2} = \underbrace{k_0}_{\text{of the reflected}}{}^2$$



$$\begin{cases} K_i = \sin\theta_i \\ K_{Rz} = k_0 \sin\theta_R^{(m)} \\ K_0 = \dfrac{2\pi}{\lambda} \end{cases}$$

(metal)

$$\boxed{\sin\theta_R^{(m)} = \sin\theta_i + \frac{m\lambda}{\Lambda}}$$



If $\sin\theta_i + \frac{m\lambda}{\Lambda} > 1$, $\underbrace{\theta_R \text{ is Imm}}_{\text{Doesn't carry real power}}$

Rm is the intensity, depends on the shape of surface

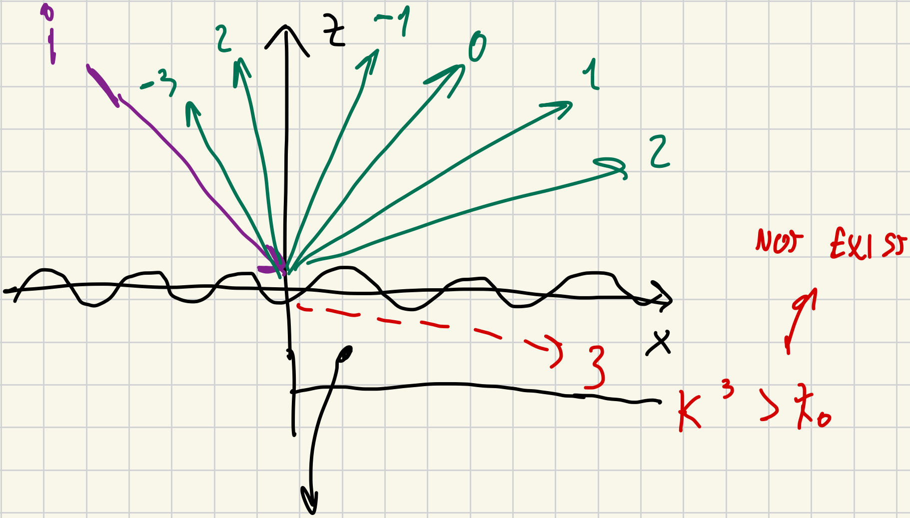
LEZIONE 26 AUDIO 29:00

THIS IS A DIFFRACTION GRATINGS

↳ LIKE DVD OR CD

Applications

Look at this :



But if this is a wg with

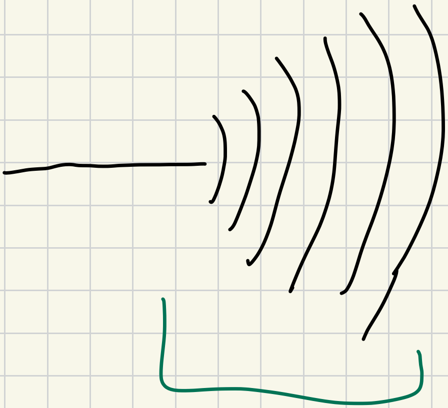
$\beta = k^3$

(

↓

I can have coupling in the waveguide of the modes.

So it's a way of coupling with fibers, not entering orizontally, but shining at θ_i to the integrated wg.

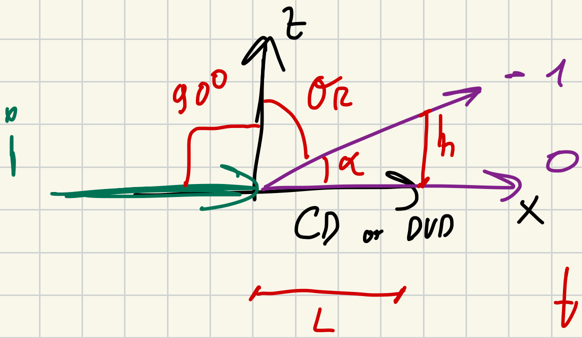


Grating used in Si photonic to couple with fibers (same idea of diffr. grat.)

Used for testing also and good efficiency in Si (2 dB/cm of losses, 30 nm of BW for good coupling).

last application

$\lambda = 532$ nm (green laser) arrive with a beam tangent to surface



$$\boxed{\operatorname{tg} \alpha = \frac{h}{L}}$$

le misuro

$$\sin \theta_R^{(m)} = 1 + \frac{m\lambda}{\Lambda}$$

$$\sin \theta_R^{-1} = 1 - \frac{\lambda}{\Lambda} \hookrightarrow \boxed{\Lambda = \frac{\lambda}{1 - \sin\left(\frac{\pi}{2} - \alpha\right)}}$$

Now I measure θ_R and I can find Λ

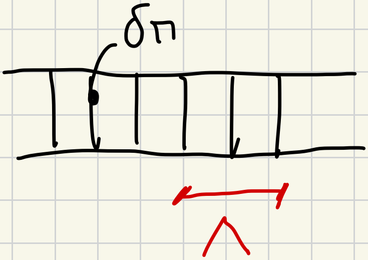
$\Downarrow$

OPTICAL SPECTRUM ANALYZER

↳ A diff. grat. with light goes around, I place many photodiode around the plating and depending on wher light go you Know the angle, so Λ. Or the opposite: I Know Λ and I "read" what λ are inside the optical source.

Bragg gratings come back

The modes have different sign of β :



Every field inside this structure can be described by backward and foward mode (but they are not mode of the structure):

$$A(z)\,e^{-j\beta_0 z} + B(z)\,e^{+j\beta_0 z}$$

Put this assumption in the wave equation:

$$\begin{cases} \dfrac{dA}{dz} = -j\,C_{11}\,A(z) - j\,C_{12}\,B(z)\,e^{-j(\beta_1-\beta_2)z} \\ \dfrac{\partial B}{\partial z} = \dots \end{cases}$$

$$C_{ij} = \frac{k_0^2}{\beta_i} q(z) \iint \psi_i \psi_j \delta n(x,y)$$

$$= C_{ij}' \underbrace{q(z)}_{\text{periodic, Fourier}}$$

$$= C_{ij}' \underbrace{\sum_m{}' q_m e^{jm\frac{2\pi}{\Lambda}z}}_{\text{IF sinusoidal perturbazion}\ m=1}$$

Solve the system :

$$A(z) = A(0) - jC_{11} \cancel{\sum q_m \int_0^z e^{jm\frac{2\pi}{\Lambda}v} A(v)\,dv}$$

$$- jC_{12} \sum_m{}' q_m \int_0^z e^{j\left(m\frac{2\pi}{\Lambda} - (\beta_1 - \beta_2)\right)} B(v)\,dV$$

So $A(z)$ is a variation of the 1st "mode" due to the second.

Now $e^{jm\frac{2\pi}{\Lambda}v}$ rotate every period, so every round trip it comes back, so it cancel itself (it continues to change phase).

Also the second is a changing phase term summed up, but if the phase is null, it doesn't cancel anymore ($e^{j0} = 1$, NOT changing phase) and the integral is NOT negligible.

$$\underbrace{m}_{\substack{m=1 \\ \text{if sinusoidal} \\ \text{perturbation}}}\frac{2\pi}{\lambda} - \underbrace{(\beta_1 - \beta_2)}_{2\beta_0} = 0$$

PHASE MATCHING CONDITION

$$2\beta_0 = 2\frac{2\pi}{\lambda}n_{eff}$$

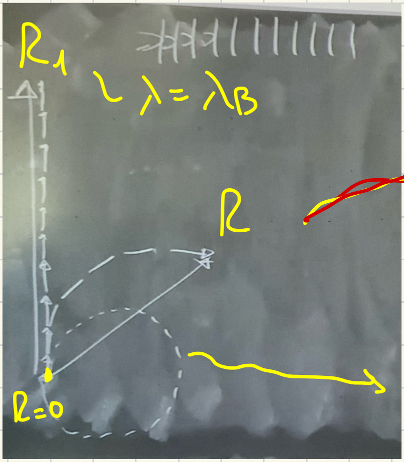
$$m\frac{\cancel{2\pi}}{\Lambda} - 2\frac{\cancel{2\pi}}{\lambda}\underset{\text{refractive index}}{n} = 0$$

$$\boxed{\lambda_B = 2n\Lambda/m}$$

Bragg λ
(same as above)

So the second integral is important:

$$\left(m\frac{2\pi}{\lambda} - 2\beta_0 = \frac{\pi}{2}\right)$$



For $\lambda \neq \lambda_B$

R become lower

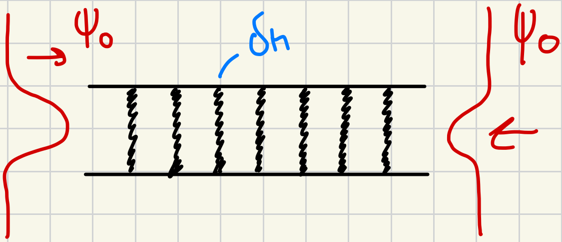
if $m\frac{2\pi}{\lambda} - 2\beta_0 = 2\pi$

$R = 0$

Inside I have reflectivity, but outer is cancelled by all contribution along z

Now C_{12} has $q(z)$ inside.

It's possible to demonstrate for even δn structure:



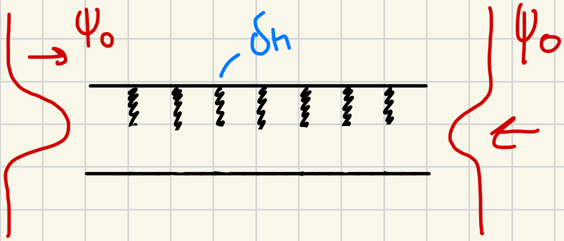
$$C_{ij} = \frac{k_0^2}{\beta_i} q(z)\, \delta n(x,y) \underbrace{\iint \psi_i \psi_j}_{1}$$

$C_{ij} \boxed{\neq 0} \Rightarrow$ The two modes couple.

$$\underbrace{C_{ij} = \frac{\pi\, \delta n}{\lambda}}$$

if $q(z)$ sinusoidal

If δn odd :



$$C_{12} \rightarrow \iint \underbrace{\psi_0}_{\text{Even}} \underbrace{\psi_0}_{\text{Even}} \underbrace{\delta n}_{\text{odd}} = 0$$

So condition to have reflections :

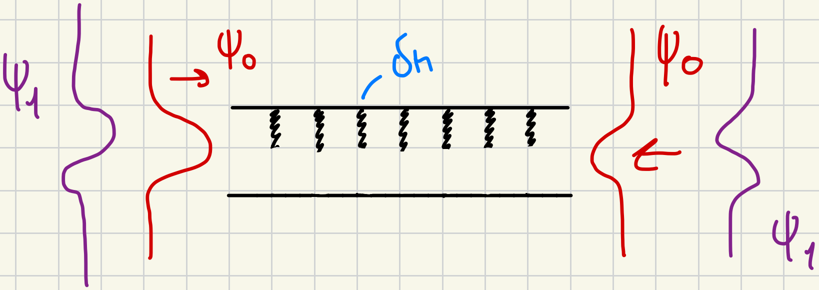
$$C_{12} \neq 0 \qquad \lambda_B = 2n \frac{\Lambda}{m}$$

How much is the total R ?

Depends on z

What if the guide is bimodal?

Now with δn odd :

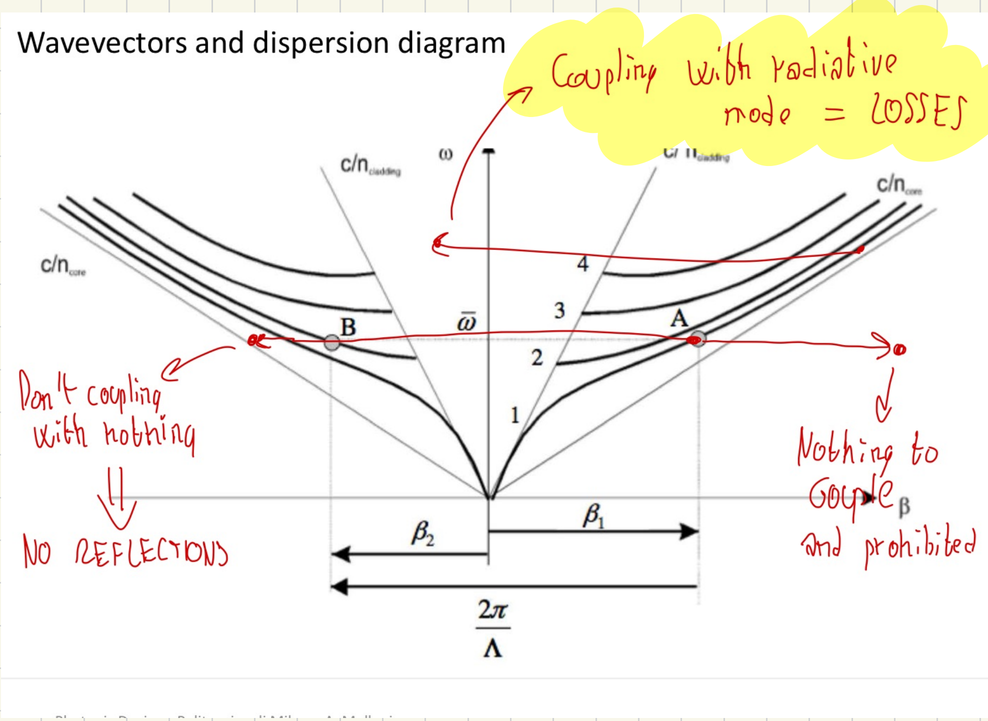


$$C_{12} \rightarrow \iint \underbrace{\psi_0}_{\text{Even}} \underbrace{\psi_0}_{\text{Even}} \underbrace{\delta n}_{\text{odd}} = 0$$

$$\hookrightarrow \iint \psi_1 \psi_1 \delta n \neq 0$$

Only valid for δn small.

$\omega - \beta$ diagram ↳ let you see coupling for a given $\bar{\omega}$



Wavevectors and dispersion diagram

Coupling with radiative mode = LOSSES

Don't coupling with nothing

⇓

NO REFLECTIONS

Nothing to couple and prohibited

Also valid for multimode wg (funtamental foward and backward second order), also between two foward modes

↳ LPG.

The phase matching alone is not enough ($c_{12} \neq 0$)!

$q(z)$?

If sinusoidal c_{12} has a precise value, if there are highe harmonics, I will couple with higher order modes

KEEP TRANSITION OF δn SMOOTH TO AVOID DISCONTINUITIES

The matrix

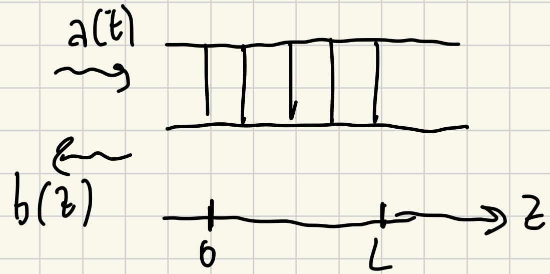
Connect the amplitude of forward and backward

$$\begin{bmatrix} a(0) \\ b(0) \end{bmatrix} = \Gamma_q \begin{bmatrix} a(L) \\ b(L) \end{bmatrix}$$

Uni Form BG

↳ δn const.

↳ Λ const



Assume to have forward-backward wave:

$$\begin{cases} \dfrac{da}{dz} = -j\,\sigma\, a(z) - j\,K\,b(z) \\[2ex] \dfrac{db}{dz} = +j\,\sigma\, b(z) + j\,K\,a(z) \end{cases}$$

$$K = \frac{\pi\,\delta n}{\lambda} \qquad \sigma = \underset{\text{very small}}{\cancel{K_{11}}} + \beta - \frac{\pi}{\Lambda} = \frac{2\pi}{\lambda} n_{eff} - \frac{\pi}{\Lambda}$$

At λ_B $\sigma = 0$ and $\frac{da}{dz}$ depends only on $B(z)$ and viceversa.

From this:

$$T_G = \begin{bmatrix} \cosh(\delta L) - jR\,\text{senh}(\delta L) & -jS\,\text{senh}(\delta L) \\ jS\sinh(\delta L) & \cosh(\delta L) - jR\,\text{senh}(\delta L) \end{bmatrix}$$

$$R = \frac{\sigma}{\delta}, \quad S = \frac{k}{\delta}, \quad \delta = \sqrt{k^2 - \sigma^2}$$

$$S^2 - R^2 = 1 \qquad \det(T_G) = 1$$

Iperbolic functions are NOT periodical, so you don't have periodical transfer of energy between the two, once reflected is reflected.

At λ_B $\delta = K$, $S = 1$ and $R = 0$

↳ But for $\lambda > \lambda_B$ it changes a lot, δ is immaginary.

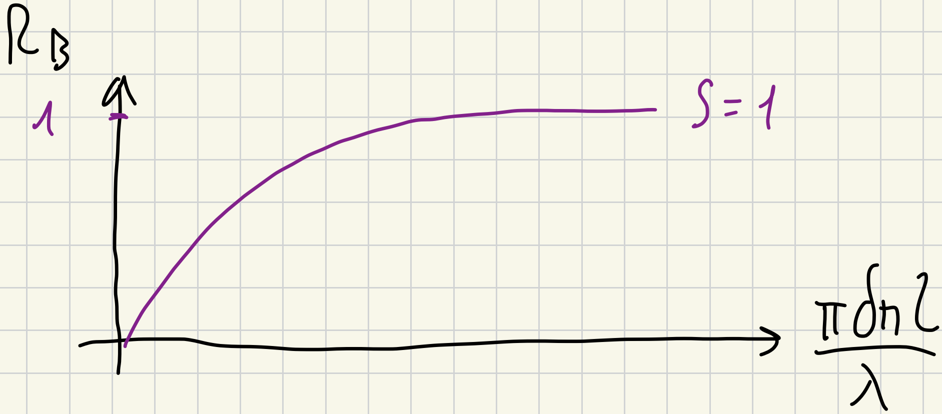
$$\begin{bmatrix} a(L) \\ b(L) \end{bmatrix} = T_G \begin{bmatrix} a(0) \\ b(0) \end{bmatrix}$$

$$R = \left| \frac{b(0)}{a(0)} \right|^2 = \left| \frac{T_{G21}}{T_{G22}} \right|^2 = \frac{\sinh^2 \delta L}{\cosh^2 \delta L - \left(\frac{R}{S}\right)^2}$$

↳ At λ_B :

$$\boxed{R_B = tgh^2\left(\frac{\pi \delta n}{\lambda} L\right)}$$

$$\underbrace{\text{Perfect}}_{C_{12} \neq 0} \text{ phase matching}$$



The limit is 1 so:

$$R_B \begin{cases} 1 & L \to \infty \\ \underbrace{\frac{\pi \delta n}{\lambda}}_{K} L & L < \infty \end{cases}$$

So for BG R_B never arrive to 1, but can be high, for the right λ_B.

What if $\lambda >> \lambda_B$ or $\lambda << \lambda_B$?

- δ is imaginary
- $R \to 1$
- $S \to 0$

So

$$\Gamma_c = \begin{bmatrix} e^{-j\delta L} & 0 \\ 0 & e^{-j\delta L} \end{bmatrix}$$

So the wave are not couple anymore and they acquire phase while propagating, like normal wg.

↳ So only phase shift.

$$\operatorname{senh}(x) = \frac{e^{x} + e^{-x}}{2}$$

$$\operatorname{sen}(x) = \frac{e^{ix} + e^{-ix}}{2}$$

So strong BG is for $\kappa L = 2 \div 3$.

↙ L change R but NOT the BW

↘ change K to change R and BW

BW of reflectivity

From max to the first zero, so for strong grating:

$$\boxed{BW = \frac{\Delta\lambda}{\lambda_B} = \frac{\delta n}{n_g}} \rightarrow \text{Approx}$$

$$\boxed{\lambda_B = 2 n_{eff} \Lambda}$$

WEAK $\Delta\lambda = \frac{\lambda_B^2}{n_{eff} L}$ → NOT SO USED!

↳ $BW \propto L$

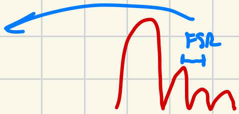
Design

① λ_B → given by technology and Λ
② BW → change δn
③ R → choose L that maximise it

All of this is from aprox of coupled theory (only two mode inside it)

↳ Works well for fibers optics, in integrated optics is a little more difficult (because we change w to reach δn)

FSR of spurios reflection $= \dfrac{c}{n_g L}$



Group delay

Chromatic dispersion cause the change of the shape of a pulse, when $\lambda\uparrow\uparrow$ are less confined and $\lambda\downarrow\downarrow$ more confined.

↳ You see it in time domain.

And in simulation 27 1:15:00 you see light trapped in the BG bouncing up and down. It's like a Fabry Perot, quasi.

⇓

It means huge group delay

The pulse mantain the shape when reflected (at λ_B), what is transmitted is distorted.

The time that it takes to the light to exit from BG once it's entered is the group delay and I can define the penetration length :

$$L_p = \frac{c}{n_g} \cdot \frac{\tau_g}{2} \Big] \rightarrow \text{Up and down}$$

$$= \frac{\sqrt{R_b}\,\lambda_B}{2\pi\,\delta n} \xrightarrow[R \to 1]{} \frac{\lambda_B}{2\pi\,\delta n}$$

(SiO_2)

For optical fiber $\delta n \simeq 10^{-4}$:

$$L_p = 2,5 \text{ mm}$$

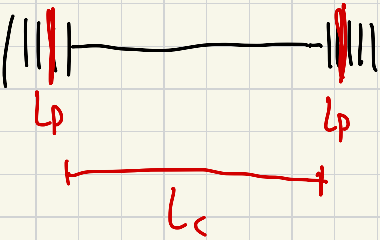
$$\boxed{@\ \lambda = 1,55\ \mu m}$$

In Si $\delta n \simeq 0.1$:

$$L_p = 0,25\ \mu m$$

↳ In Si the reflection happens in very short distance compared to glass (Fiber).

So Now I can design a Fabry Perot in Si, with given finesse and FSR and the design of mirror, knowing the right δn to not have L_p big and modify the resonance of the cavity.



Transmission at λ_B

$$T_B = 1 - R_B = \operatorname{sech}^2\left(\frac{\delta n \pi L}{\lambda}\right)$$ [power]

l_p and τ_q

① At λ_B I SEE REFLECTIONS

$\Downarrow$

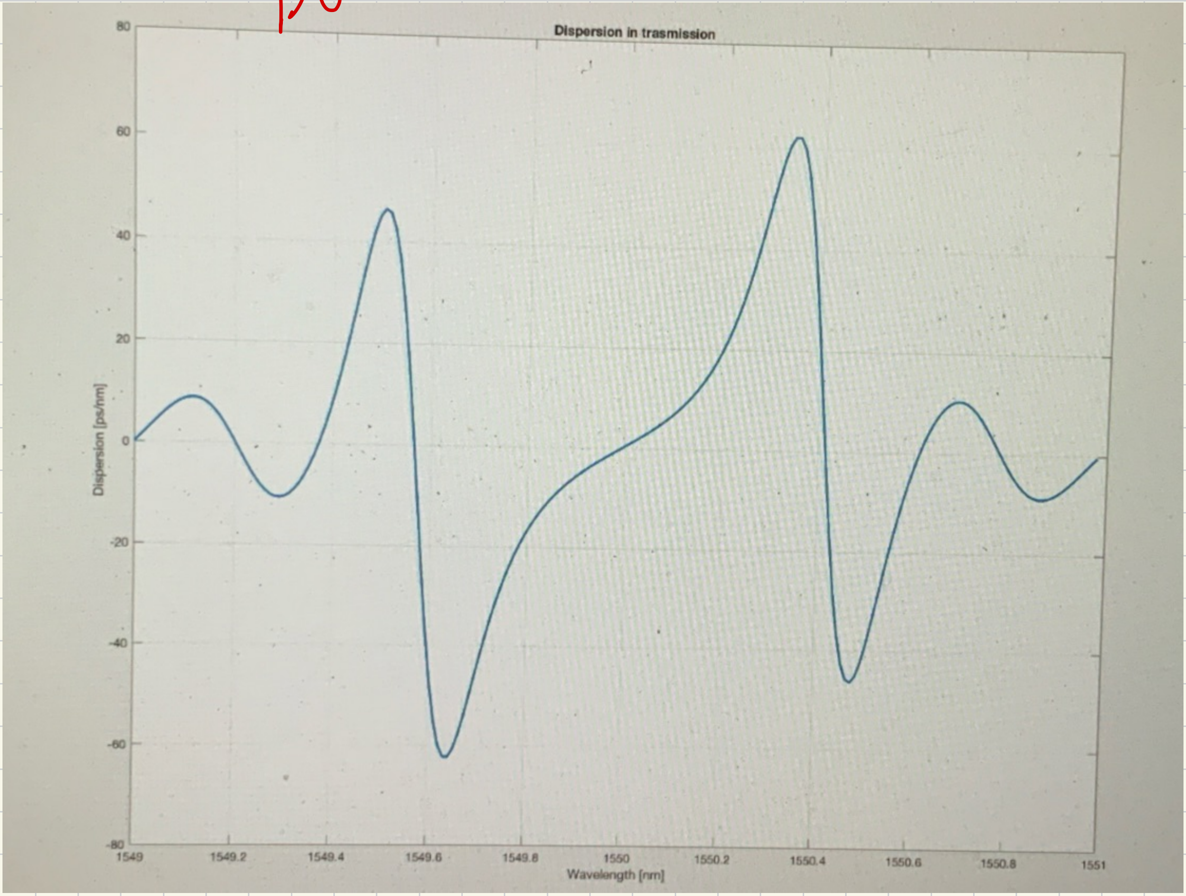
- l_p is more important
- $\tau_q = \frac{l_p}{2 c / n_q}$
- D_G is 0 (good)

② At $\lambda \neq \neq \lambda_B$ I see TRANSMISSION

$\Downarrow$

- $\tau_q = \frac{\overbrace{L}^{\to \text{length of the grating}}}{c / n_q}$ (bigger)
- l_p play NO ROLE
- D_G has peak near λ_B, then $\to 0$

DG


Dispersion in trasmission
Dispersion [ps/nm]
80
60
40
20
0
-20
-40
-60
-80
1549
1549.2
1549.4
1549.6
1549.8
1550
1550.2
1550.4
1550.6
1550.8
1551
Wavelength [nm]

$\tau_g \sim$

NON UNIFORM BG

Uniform gives a BAD out of band response

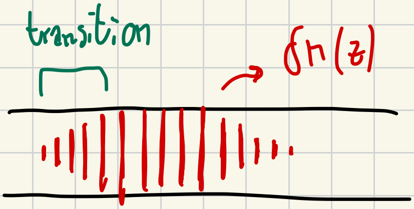
⇓

Remove discontinuity in δn
Match the impedance of the
incoming wave with that of the wg

↓

Apogized grating

δn NOT const. in z, there are transition zone



The light want bounce up and down anymore, there isn't anymore the spurius FP like uniform.

$$\Downarrow$$

But l_p ??

→ DG similar to uniform, but less peak!

Every time I have a pick in $\tilde{c}_q$, the field intensity is stronger (accumulation of energy)

↳ Apogized hele here.

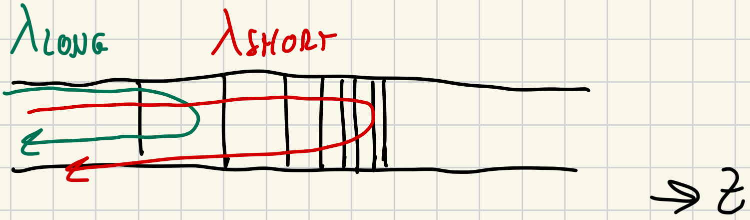
We don't have closed form Formula, but rule of thumb for gaussian apogized:

$$\underbrace{L} = 3\,l$$

$$\left\{\begin{array}{l} 2\cdot\frac{1}{3}\ \text{transition} \\ \frac{1}{3}\ \text{uniform part}\end{array}\right.$$

Chirped grating

$\delta(n)$ const. but Λ depends on z



Used for very large Ω BW.

If Λ change also λ_B change, cascading it reflect more λ

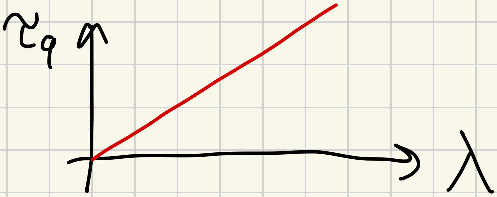
↳ light reflect where λ match Λ

↳ short λ, big L_p, big τ_g

↳ long λ, low L_p, low τ_g

Chromatic dispersion compensation

$$\Lambda(z) = \Lambda_0 \pm Cz \rightarrow \lambda_B(z)$$



So the BW :

$$B = \lambda_{B_L} - \lambda_{B_S} = 2\pi n_{eff} \underbrace{C}_{\text{Chirped factor}} L$$

NO more related only to δn

$$\tau_g = \frac{z}{c/n_g} = \frac{\lambda - \overbrace{\lambda_{short}}^{\text{or } \lambda_{long}}}{c\,\underbrace{C}}$$

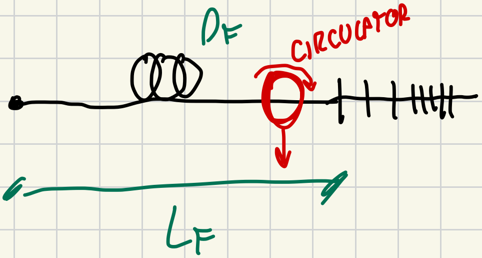
So the dispersion:

$$\frac{\partial \tau_g}{\partial \omega, \lambda} = \pm \frac{1}{cC} = D_G$$

→ giro al contrario il chirped BG, cambia segno

Example in fiber

Mode not well confined, quasi a plane wave



Total ch. disp.:

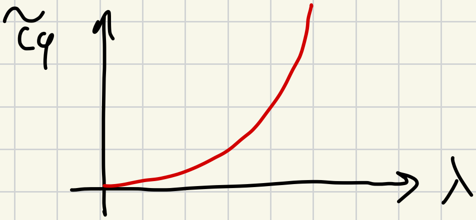
$$D_F \cdot L_F = D_G$$

$$D_F \sim -\frac{20\ ps}{nm}$$

OCCHIO CHE I D_G, D_F HANNO SEGNO

$L_F \sim$ Km (more)

This is only useful for first order D, while higher order i need different chirp :



The BG in fibers + circulator are POL. INDIP.

↳ Unless you need varying R (difficult that L_F change).

Why in reality in τ_g there are ripples?

They cause ISI, they born from discontinuity in the single section of the BG chirped.

⇓

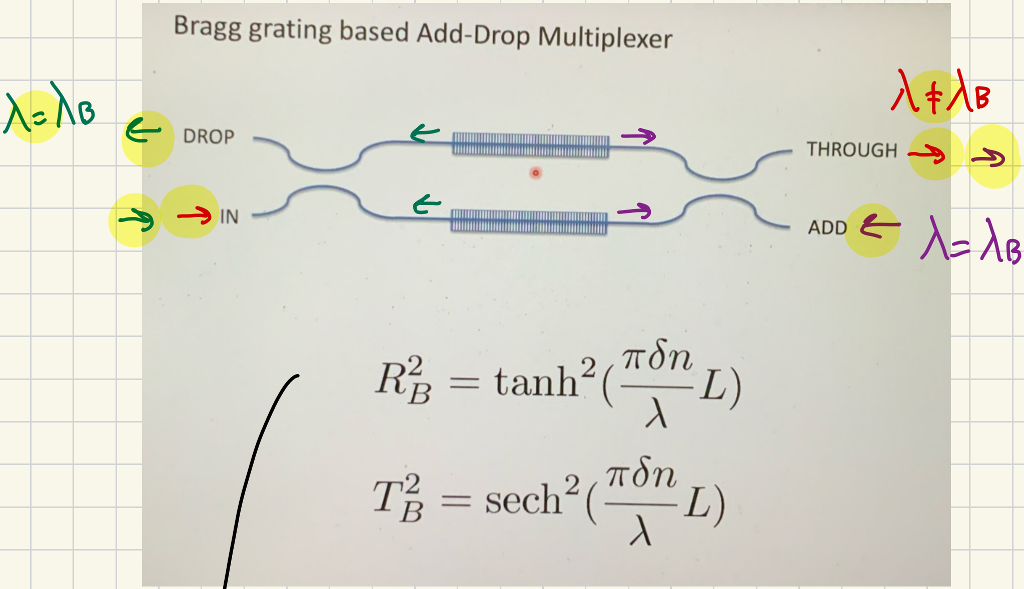
USE APOGIZED + CHIRPED GRATINGS

Less ripples, less ISI.

Applications of BG

- CDC (above)
- ADD - DROP
- Sensors
- Fake isolator

Add-drop :



Bragg grating based Add-Drop Multiplexer

$$R_B^2 = \tanh^2\left(\frac{\pi\delta n}{\lambda}L\right)$$

$$T_B^2 = \mathrm{sech}^2\left(\frac{\pi\delta n}{\lambda}L\right)$$

Important to match L_p, otherwise the field reflected arrives to the coupler phase shifted to the other and can exit in IN/ADD port.

Only for one λ ($= \lambda_B$)

FOR ADD DROP BETTER AWG - RING

Field inside in BG

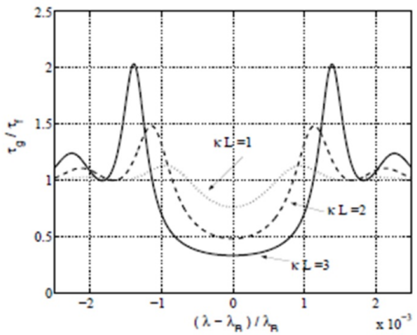
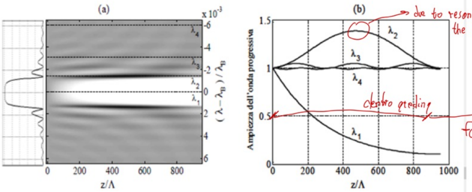


Figure 6.14: Group delay normalized to the uniform Bragg gratings having reflectivity spectrum of Fig. 6.13 (b).

Even if is totally transmitted, in the BG the intensity is bigger

all the reflection built up inside and cancel after L



due to resonance of the cavity

dentro grating

fuori

Figure 6.15: Behaviour of the field inside a uniform grating.

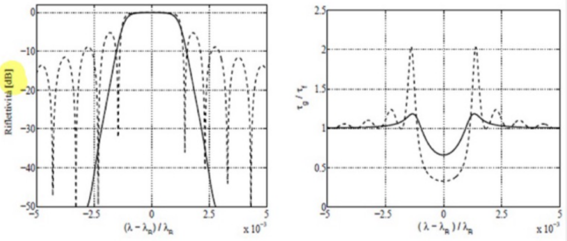
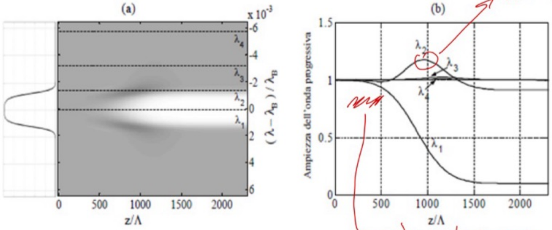


Figure 6.17: Reflectivity spectrum (a) and normalized group delay (b) for a uniform grating (Dotted line) and an apodized grating with Gaussian profile (continued line).

less resonance



λ_1 enters more

Figure 6.18: Field behaviour inside an apodized grating.

Very small imperfections in the center of the BG (causing it to "divide" in two equivalent, quasi) I have a Fabry Perot ^ (NOT BG anymore).
cavity

Fake isolatore → solution for integrated optic ?

↳ NO!

MODULATOR

Recive the info to transmit and lad it on the carrier.

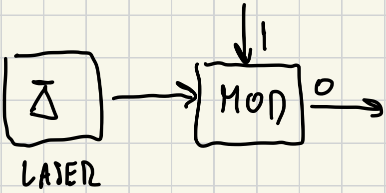
↳ Intensity

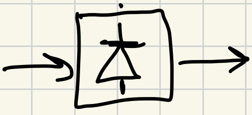
↳ Phase
↳ Frequency
↳ Polarization

And then DEMODULATOR (after link with losses, noise and dispersion).

The best? Laser + MODULATOR

↳ Intensity
↳ Phase
} QAM
↳ Polarization (abandoned)
↳ But NOT frequency MOD.
(λ fixed, difficult to act on laser)



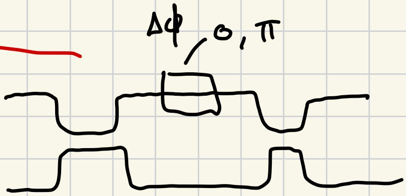


(NOT SIMPLE FOTODIODO)

Why only 5 years ago the first photonic QAM?

Because you need the correct modulator.

- ONLY INTENSITY ON-OFF → MZ



- QAM → MORE DIFFICULT

I need very fast Δφ change → HEATER CANNOT DO THAT (1 ÷ 10 μs, few MHz of modulation)

THIS IS DIFFICULT

✓

What material can do this?

The signal is electric, the "medium" is optic

⇓

ELECTRO-OPTIC

- **electrorefractive material**
  - **Δn** — Si + PN j; $LiNbO_3$
  - ↓ $\Delta \varphi$
  - PHASE MODULATOR
- **electroabsorbing material**
  - **$\Delta \alpha$** — Si / InP (Better) + PN JUNCTION: TO CHANGE THE NUMBER OF CARRIER INSIDE BY BIAS
  - ⇓ INTENSITY MODULATOR
  - ΔIL

Problem

In those materials if you change Δn, you also change $\Delta \alpha$ and viceversa.

↳ So there's NOT the best one

If I want to induce $\Delta\alpha$ to switch on or off the light, it's detrimental to have also a phase shift (modulation) due to Δn. It's difficult to NOT have Δn after $\Delta\alpha$. So these material are used rarely for AM. Only InP is used for short link AM

↳ It's easier to change Δn without change $\Delta\alpha$ too much in the right material

↳ PM is more feasable

The dream material is $LiNbO_3$, a crystal where $\vec{E}$ produce Δn without $\Delta\alpha$

Si + PN junction can have Δn (also $\Delta\alpha$, but small) and short wg are used in Si photonics). In general less efficient than $LiNbO_3$.

Intensity modulator with PM

↳ With $LiNbO_3$ + MZ
(or Si+PN junction)

$LiNbO_3$ is extremly fast $\gg \underbrace{50 \div 60 \text{ GHz}}_{\text{Si + PN j}}$

↓

It's a microwave problem
how to design electrodes for $\vec{E}$
at that frequency.

ELECTRO-OPTIC MATERIAL ($LiNbO_3$)

$\vec{E} \Rightarrow \Delta n$

↓ Crystal, really fast time response

With $\vec{E}$, the molecule of the crystal change a little bit the orientation or the polarizability and the molecule are oriented in the same way, overal the litte change produce Δn.

↳ Glass = Amorphous cannot do this, the overall effect is at average null.

↳ You need crystal

↳ Also for magneto-optic

Si is simmetrical w.r.t. center, if you induce a rotation of molecules in the lattice you don't se anything

↳ $LiNbO_3$ is asymmetrical crystal

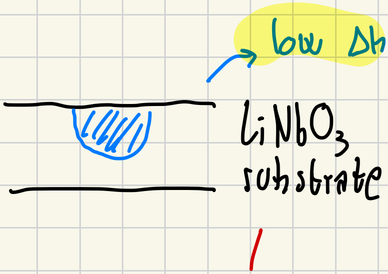
EO EFFECT = CRYSTAL ASYMMETRIC

With interferometer + $LiNbO_3$ you can have AM.

How to build wg in $LiNbO_3$?

It's a crystal, cannot be deposited. So I have to start from the crystal ($\underbrace{\text{a wafer}}_{\text{substrate}}$) and construct on it.

Ti Diffused

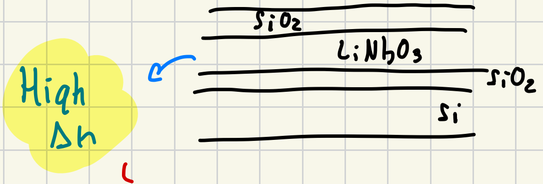


Thin Film

$Si + SiO_2$ substrate

+

WAFER BONDING (DIFFICULT



↓

Best move for modulators (high frequency) → More compact

↓

But high loss

$R_{MIN} = 100\ \mu m$

$\Delta n = 2{,}2$

↓

But I cannot diffuse to much Ti otherwise losses ↑↑ (yes good confinement).

$$R_{\substack{min \\ DIFFUSE}} \geq 5\ cm$$

$$\Delta n_{DIFFUSE} = 0{,}2 \div 0{,}4$$

How it works?

$$\vec{P}(E) = \varepsilon_0 \underbrace{\chi}_{(\varepsilon_r - 1)} \vec{E}$$

Polarizability

$$D(E) = \varepsilon_0 E + \underbrace{\vec{P}(E)}$$

$\vec{E}$ from now on is external, not that of the optical mode

↳ in general depends on $\underbrace{E}$, $\underbrace{E^2}$, ...

POCKELS EFFECT → I can increase or decrease Δn

KERR EFFECT → I can only increase Δn

So ε is a tensor:

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}$$

If $\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} \rightarrow$ MATERIAL ISOTROPIC

" $\varepsilon_{xx} = \varepsilon_{yy} \neq \varepsilon_{zz} \rightarrow$ " ANISOTROPIC UNIAXIAL

" $\varepsilon_{xx} \neq \varepsilon_{yy} \neq \varepsilon_{zz} \rightarrow$ " " BIAXIAL

$LiNbO_3$ is uniaxial, along X and Y has the same n_{eff}, along Z is different.

How can the wave propagate?

ε is NOT scalar in the wave equation. Then with external $\vec{E}$ I change ε, so n_{eff}.

But when I change $\vec{E}$, maybe I change $\varepsilon_{xx}(E)$ ok? But then I change also the other coeff. in the matrix

↳ I cannot say: apply $\vec{E}$ to change neff in this direction, it changes in all directions.

Any material has its own Δn_k matrix (dispense), $\underbrace{r_{33}}$ is the big in $LiNbO_3$.

$\hookrightarrow \frac{\text{meter}}{\text{Volt}} = 30.8 \cdot 10^{-12} \frac{m}{V}$

For strong Δn I have to work with $\underbrace{r_{33}}$.

It's constant only for $\vec{E}$ modulating at high frequency

$\vec{E}$ low frequency chang the lattice

(r is the biggest)

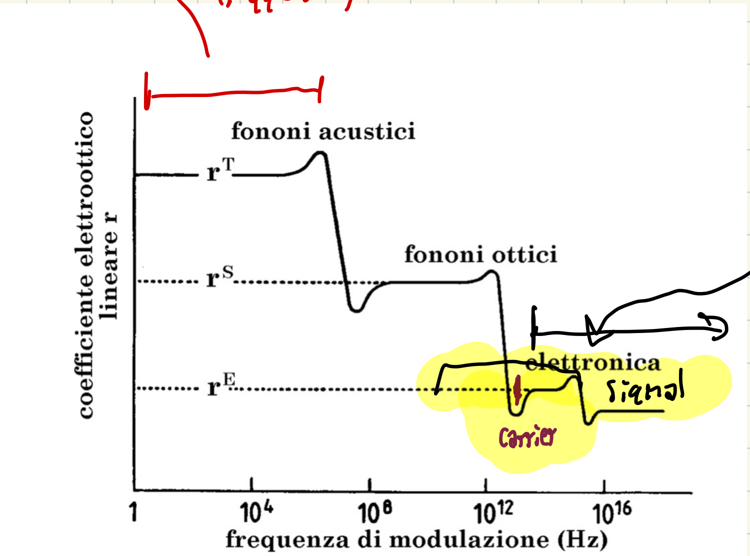


Figure 5.3: Dependence of the electro-optical coefficients on the frequency of the applied electric field.

I can stretch the molecules,

r is bigger

With electric signal at 10 GBit/s $(\sim \overbrace{10\ \text{GHz}}^{\text{from } 0})$

I excite almost all of these three

$\Downarrow$

SO SOME PART OF THE SPECTRUM IS MODULATED BETTER THAN OTHER PART

The carrier is a 200 THz.

Directions

If we propagate on a generic direction, the wave will depends on the entire $\Delta n K$. But if we propagate along x or y or z, there is only the transverse characteristic of the wg to play any role.

↳ So polarization dependent

↳ Use the wg along one axis.

Then the $\vec{E}$ applied can (comunque) produce charge along other axis, so use an $\vec{E}$ with a direction to exploit r_{33}.

↳ $\vec{E}$ along z.

LIGHT ALONG X TE POLARIZED
$\vec{E}$ CONTROL ALONG Z

↓

Now place electrodes to induce E_z and:

$$\Delta n_z \propto r_3 E_z$$

z-cut

Δn_y and Δn_x will change also (via r_{13}) but they don't play any role, because the light is polarized TE (doesn't see $n_x | n_y$).

Other case X-CUT, TE (horizontal pol.)

$\vec{E}$ always along z and wave $\underbrace{\text{TE}}_{\text{pol. along } z}$ along x

$$\Downarrow$$

Change position of electrodes

What if I enter with TM (vertical polarized)?

E_z will produce Δn_x and Δn_y with r_{13}:

$$\text{TM} \propto r_{13} << r_{33}$$

There will be a (varying) birefringence.

↳ X-cut

The z-axis is the optical axis (⊥ to the substrate).

The phase along the wg in the two case is:

$$\Delta\varphi = \frac{2\pi}{\lambda}\frac{n^3}{2}\underbrace{r_{33}E_z}_{\Delta n}L$$

Semiconductor

In PN junction you can change the number of carrier to change Δn (also Δα though), via the biasing.

↳ For every Δn change I pay also $\Delta\alpha$, so keep small variation

↙

How much can i reduce the carrier in the core?

↳ 10^{18} cm^{-3}

Remember: this produce a $\Delta\varphi$, but it's NOT electro-optic effect but plasma dispersion.

Slide 15

I have more Δn than $\Delta\alpha$ in Si, so it's easier to built PM than AM in Si. But it's NOT possible to reach a pure PM, always a residual one ($\Delta\alpha$ cause a chirp, distortion of the pulse).

↳ NOT SO EFFICIENT FOR ON-OFF KEYING

So applying V I can have $\Delta\varphi = \pi$, but also $\Delta IL = 20$ dB.

↳ I DON'T WANT CHIRP, SO I HAVE TO CHANGE THE DEVICE.

NO MZ, BUT RING ←

Come back to $LiNbO_3$

Photoresist → pattern mask → litography → etch

↳ Deposit Ti all over (10 nm thick)

↓

Remove the photoresist + Ti

↓

Increase T and Ti diffuse more and more while time pass

←

Ti in $LiNbO_3$ change n_{eff} for two reason

↓ Substitute Ti larger tha $LiNbO_3$

→ Ti is larger and induce a stress in the lattice.

DIFFUSE WAVE GUIDE

→ smooth Δn

→ Graded index wg, Not step index

$\Delta n \simeq 0,3\%$ → low losses

↳ large mode

↳ Weak confinement

$R_{min} = 5cm$ ← ↳ Poor bending capability

↳ low density (device are large).

Now apply $\vec{E}$

Electrode are L ~ less than mm and $\vec{E}$ must be $\underbrace{\text{horizontal}}_{x\text{-cut}}$ or $\underbrace{\text{vertical}}_{z\text{-cut}}$ field in the wg.

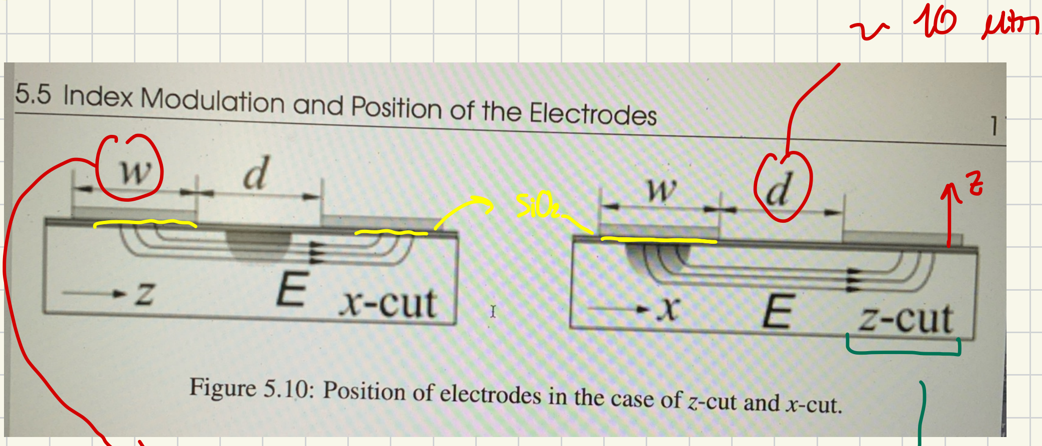
↳ So $\Delta n = \frac{n^3}{2} r_{33} \underbrace{E_z}$

VOLTAGE

DISTANCE

BETWEEN ELECTRODES

So to have low voltage (high frequency is NOT so feasable to have high power), so small d, but NOT too close to NOT attenuate the optical field.



5.5 Index Modulation and Position of the Electrodes

Figure 5.10: Position of electrodes in the case of z-cut and x-cut.

$w > d$

Metal - SiO_2 - $LiNbO_3$

If the thickness of SiO_2 is smaller, the attenuation is larger and $\vec{E}$ larger

↳ TRADE-OFF

SiO_2 serve only as isolation so not so thick.

Electrode

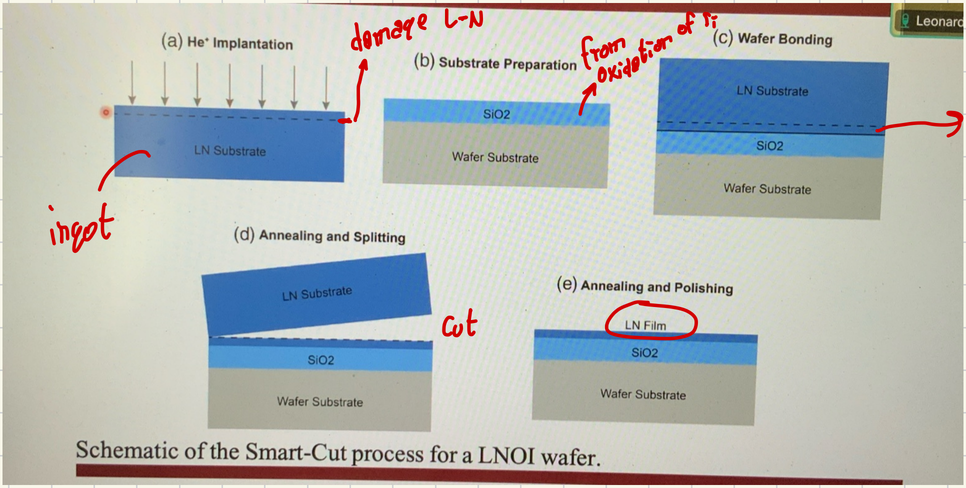
Are microstrip line because of fast signals.

After made the wg → Glue, so a few nm of Cr are deposited → Deposite Au → Deposit photoresist → Remove litography all unless where it's protected → Enlarge the thickness of Au $\underbrace{\text{(small R)}}$
(ion deposition)

At least 3÷4 μm (1 GHz, skin effect limits).

Cr colla tra Au e $LiNbO_3$

How to make thin film L-N



Schematic of the Smart-Cut process for a LNOI wafer.

Thin film → 3 ÷ 700 nm

SiO_2 substrate → LNOI ($LiNbO_3$ on Insulator)

Cladding air ↗

↳ Δn ↑↑

↳ Rmin ↓↓

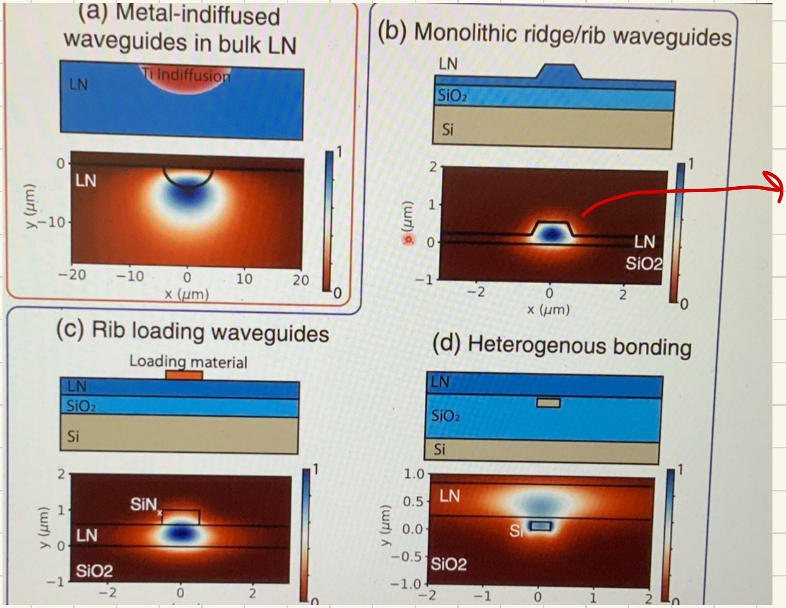
↳ Density ↑↑

↳ Losses ↑↑ and backscatter ↑↑

With wg smaller, electrodes can be closer to the wg

→ Volt ↓↓ → Higher frequency

Everyone is trying with LNOI instead of $LiNbO_3$ (used because simple to produce in mass scale)



1 order of magnitud smaller

How to apply the high freq. signal?

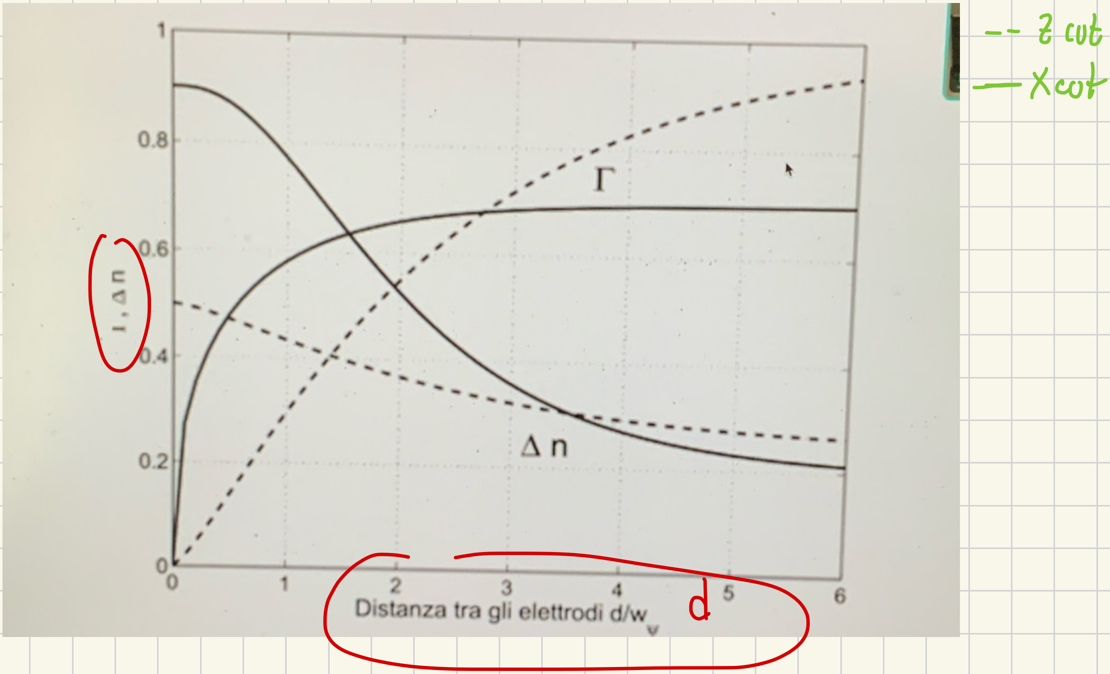
In the figure of x-cut and z-cut you see that $\vec{E}$ is not exactly oriented in the direction of the optical axis z, so I have to consider the overlap between optical and external field:

$$\Gamma = \frac{d}{V} \underbrace{\int}_{\text{Over the cross section}} E \, |\psi|^2 \, d\sigma \leq 1$$

$\Gamma = 1$ in case of perfect overlap, so:

$$\Delta n = \frac{n^3}{2} r_{33} \, \Gamma \, E$$

Then I want Δn as large as possible:



- d ↑↑ → Γ ↑↑, until const
- d ↓↓ → Γ ↓↓, $\vec{E}$ tends to stay between electrodes and doesn't prenetrate the wg

- d ↑↑ → Δn ↓↓
- d ↓↓ → Δn ↑↑

X-cut gives larger Δn than z-cut until the electrodes are $3 \div 4$ times the dimension of the optical wg (W), then it's better z-cut

$$\Downarrow$$

X-cut is to prefer if the two electrodes are not too far away from each other

So SiO_2 in z-cut matter to not have $\Delta n \simeq 0$.

I don't apply high Voltages ($1 \div 2$ volts) over distances of μm, so $|\vec{E}|$ is huge.

↳ DO NOT REACH BREAKDOW

Small electrodes

$$C = \varepsilon_{eff} \frac{A}{d} \rightarrow \underbrace{C \downarrow\downarrow , \ d \uparrow\uparrow}_{\text{HIGH BW}}$$

But $d \uparrow\uparrow$, weak $\vec{E}$, so small Δn, so I achieve $\Delta\varphi$ by increasing the length L:

$$\boxed{\Delta\varphi = \frac{2\pi}{\lambda} \frac{n^3}{2} r_{33} \Gamma \frac{V}{d} L}$$

Depend on frequency (see later)

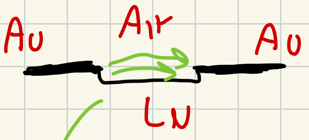
$$f_{\substack{cut \\ OFF}} = \frac{1}{R C}$$

But increase L → NO more lumped.

$$\varepsilon_{r_{LN}} = 28 \text{ along } x$$

$$= 43 \text{ along } z$$

LN is anisotropic



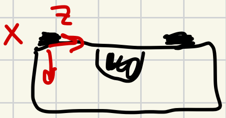
$$\varepsilon ff_{air-LN} = 20 \ (z)$$

$$= 15 \ (x)$$

(average)

C DEPENDS ON THE CUT

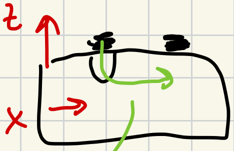
X-cut



$\varepsilon_{r_z} = 43$

$C_x = \varepsilon_{ff_x} \dfrac{A}{d}$

z - cut



most of the material is crossed along X

$$\varepsilon_{r_x} = 20$$

$$C_z = \underbrace{\varepsilon_{ff_z}}_{15} \frac{A}{d} < C_x$$

So from the graph I say X is more efficient, but it can be slower.

z - cut is more efficient because C is smaller

X - cut is more efficient because Δn is bigger

?!

L PLAY A ROLE

$l \sim 1$ cm for $V \sim <10$ V

Speed of modulation

The V voltage has a λ:

$$\lambda = \frac{c}{\sqrt{\varepsilon_{r_{LN}}}\, f} \gg \underbrace{L}_{\text{to use the lumped model}}$$

Max BW ~ 1 GHz

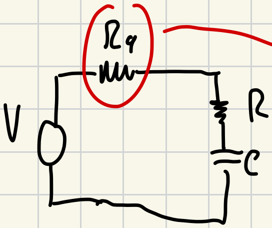
↳ For more you decrease L, and increase V (d cannot be touched anymore)

Special case : Si + PN j

The Si-Ring is the way to build modulator in silicon.

Here I can use the lumped electrodes also for higher BW

The ring is a C (or C with R if losses).



matched with Z of the electrodes to not have reflections

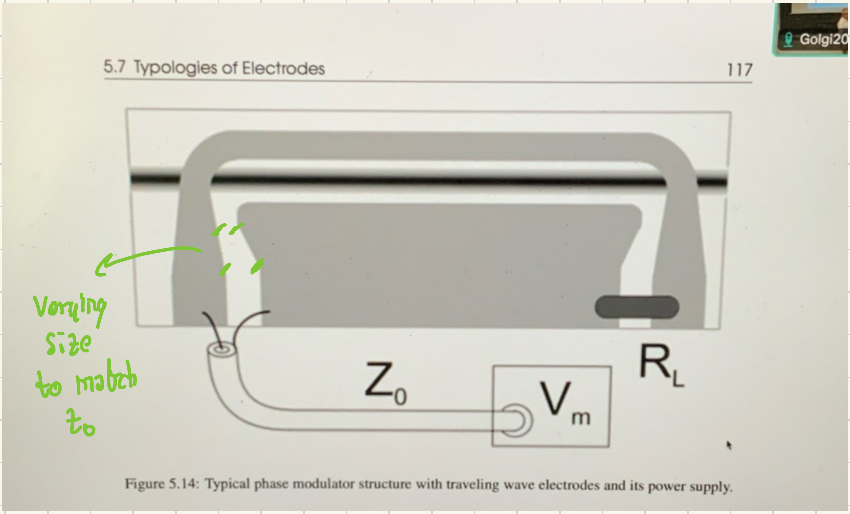
OTHERWISE ISI

In $LiNbO_3$

$$L \simeq \lambda$$

USE TRAVELLING WAVE ELECTRODES TO GO FASTER AT LOW V

I use a coax between generator and electrodes, with matched impedances.



5.7 Typologies of Electrodes 117

Figure 5.14: Typical phase modulator structure with traveling wave electrodes and its power supply.

$$Z_0 = Z_e = R_L$$

MUST !

But Z_e depends on d, that play a role in Δn and γ !

To have $Z_e = Z_0 = 50\ \Omega$ you have to set d

Not so short unless you vary the size of elect. or charge to.

Now I have a wave that travel in te and modulate the light that travel in the wg

↳ A wave perturb sn with a certain time response and light read sn at different speed

⇓

VELOCITY MISMATCH

modulating

$$V_0 = \frac{c}{n_{LN}} = \frac{c}{2,2} \qquad V_m = \frac{c}{\sqrt{\varepsilon_{r_{eff}}}} = \frac{c}{\sqrt{15}} \simeq \frac{c}{4}$$

There is a factor of 2, the optic is faster than the electrical signal, in $LiNbO_3$.

For Si, InP:

$$V_0 = \frac{c}{n_{si}} = \frac{c}{3,5} \qquad V_m = \frac{c}{\sqrt{\ }} = \frac{c}{\sqrt{4}} = \frac{c}{2}$$

In Si the electrical signal is faster than the optical signal.

↳ Also here a mismatch though.

Velocity mismatch

→ NOT CONST. IN L

$$\Delta\varphi = \frac{2\pi}{\lambda}\int_0^L \underbrace{\Delta n}\, dl$$

$\hookrightarrow \Delta n \propto E, V$

It's interestic to see what voltage sees the light, nor the electrodes.

The modulating voltage :

NOT OPTICAL AXIS → PROPAGATION ALONG X or Y

$$V_m(z,t) = V_0 \sin\left(\frac{2\pi f_m}{c}\sqrt{\varepsilon_{r_{eff}}}\, z - \omega_m t\right)$$

Change the reference (the signal is like not moving, while the wq is moving) :

$$V_m(z,t_0) = V_0 \sin\left(\frac{2\pi f_m}{c}\left(\sqrt{\varepsilon_{r_{eff}}} - n_{eff}\right)z - \omega_m t_E\right)$$

I'm interest in the phase velocity, not group one. Now I have V, find E and Δn :

$$\Delta\varphi = \frac{2\pi}{\lambda} - \frac{n^3}{2} r_{33} \frac{V_0}{d} \Gamma \int_0^L \sin\left[\frac{2\pi f_m}{c}\left(\sqrt{\varepsilon_{r_{eff}}} - n_{eff}\right) z \overset{-\omega t}{\,}\right].$$

↓
CHISSENE

$$\cdot\, \underbrace{e^{-\alpha_m z}}_{}$$

Attenuation of the voltage along the electrodes

Approx $\alpha_m = 0$, now I find the phase modulation induced due to velocity mismatch and then I don't consider the vel. mism. but $\alpha_m \neq 0$, so I have the two limit:

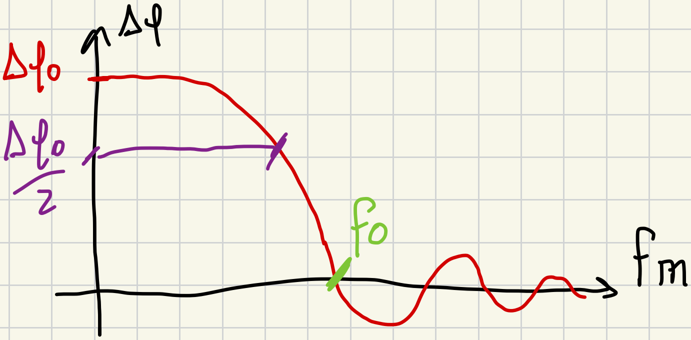
$$\Delta\varphi = \Delta\varphi_0 \cdot \frac{\sin \frac{\pi f_m}{f_0}}{\pi f_m / f_0} = \Delta\varphi_0 \, \mathrm{sinc}\left(\pi \frac{f_m}{f_0}\right)$$

$\alpha_m = 0$

where

$$f_0 = \frac{c}{L\underbrace{(\sqrt{\varepsilon_r} - n_{eff})}_{}} \longrightarrow \text{This must be small}$$

$$\Delta\varphi_0 = \frac{2\pi}{\lambda} \frac{n^3}{2} r_{33} \frac{V_0}{d} \Gamma L$$



$$\boxed{BW = f_0 \cdot \frac{2}{\pi}}$$

BW of modulator

The light and the electric beam starts together, the e-beam modulate the light, they continue to do this at some point the arrive in opposition of phase and the e-beam cancel the modulation beam of the first part.

$\Downarrow$

THE DIFFERENT IN VELOCITY IS THE MAX REASON THAT LIMIT THE BW OF MODULATORS

AND THE LENGTH

If $L \uparrow\uparrow$, $f_0 \downarrow\downarrow$, $BW \downarrow\downarrow$

$\Downarrow$

You want to go fast? $L \downarrow\downarrow$, but then $V \uparrow\uparrow$, cost $\uparrow\uparrow\uparrow\uparrow$

In L-N → $f_{0_{MAX}} = 15$ GHz

With attenuation

There is the skin effect increase at high freq, so increases losses.

Now $\sqrt{\varepsilon_r} - n_{eff} = 0 \rightarrow \sin(\omega t)$ exit integral.

$$\Delta\varphi = \Delta\varphi_0 \int_0^L e^{-\alpha_m z}\, dz$$

The mod. signal continues to decrease while propagating along electrodes.

$\alpha_m \propto \sqrt{f}$ because of skin effect

	Au	1 GHz	10 GHz
thickness of current	δ	2,4 μm	0,76 μm
	α_m	0,5 $\frac{dB}{cm}$	1,58 $\frac{dB}{cm}$

$$a_m \sim 0{,}5 \sqrt{\underbrace{f}_{\text{in GHz}}}\ \frac{dB}{cm}$$

So :

$$\Delta\varphi = \Delta\varphi_0 \left(\frac{1 - e^{-\alpha_m L}}{\alpha_m L} \right)$$

$\|$

V

REDUCTION OF BW DUE TO ATTENUATION

For $1 \div 2$ dB/cm → $BW_{\alpha m} \simeq 10$ GHz

If you have also velocity mismatch, you combine the two effect.

THE PROBLEM IS L, IF YOU REDUCE IT, ↑↑ POWER

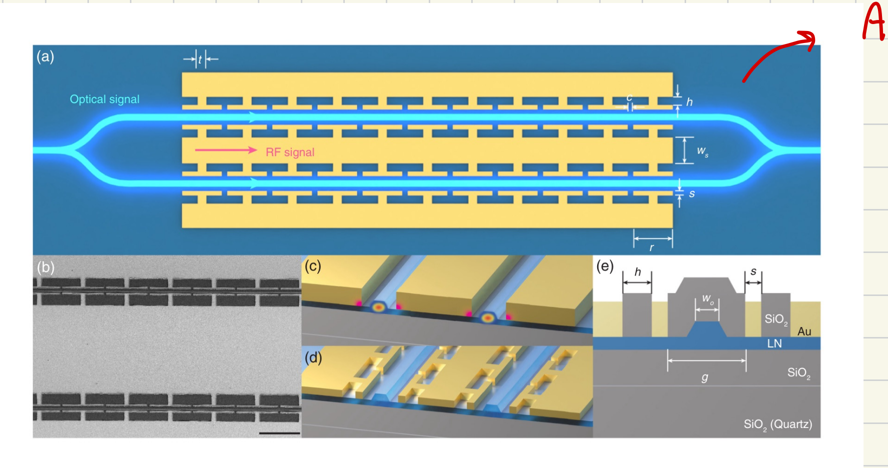
In L-N → 0 faster than E, you cannot due much unless shrink L and V ↑↑

In Si, InP → E faster than O:

$$v_m = \sqrt{\frac{1}{LC}}$$

$$z_0 = \sqrt{\frac{L}{C}}$$

I can decrease E by increasing C, so use periodical loaded electrodes



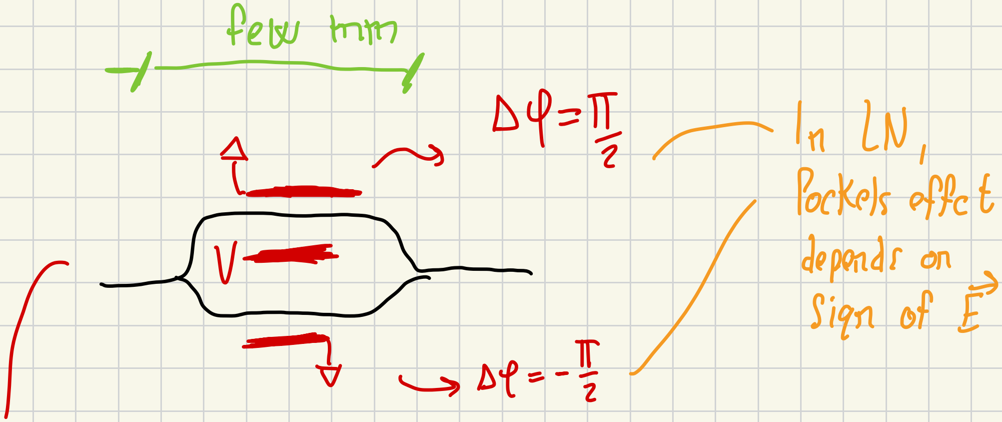
All at z_0

POSSIBILITY FOR VELOCITY MATCHING IN SEMICONDUCTOR

INTENSITY MODULATION FROM PM

$\Downarrow$

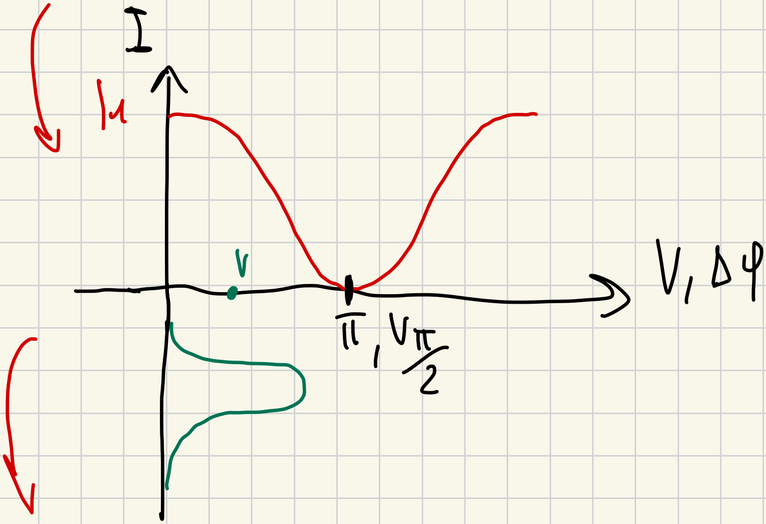
Interferometer → MZI



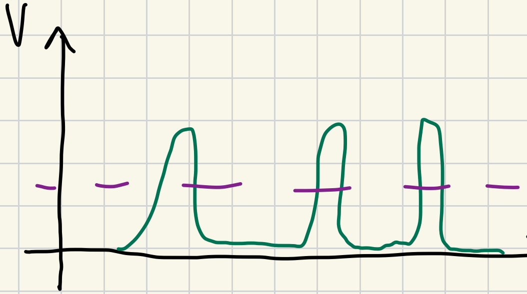
The voltage to produce $\Delta\varphi = \pi$ is called V_π, with two $\Delta\varphi$

↳ PUSH-PULL CONFIGURATION FOR ELECTRODES, USE LESS POWER $\left(\frac{1}{4}\right)$ HALF VOLTAGE, SO COOL DOWN THE DEVICE

$\frac{V_\pi}{2}$ produce π



So the RF signal :



ZERO MEAN SIGNAL

FAST

ELECTRONIC DOESN'T LIKE IT

I want to remove the DC component, I have to polarize the modulator to stay a $\frac{\pi}{2}$ at DC

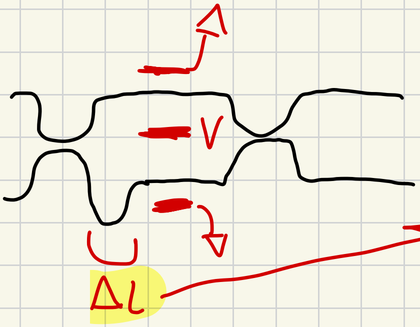
$\Downarrow$

BIAS

But a constant V dissipates a lot

$\Downarrow$

Unbalance the MZ



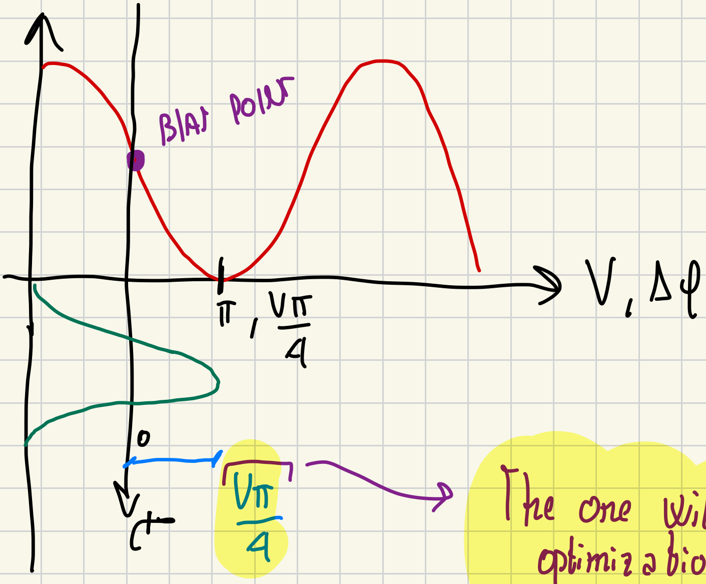
$\frac{\lambda}{4} = 250$ nm, good!

So without signal $\Delta\varphi = \frac{\pi}{2}$.

Now V can be 4 time less, I need to move only of $\pm\frac{\pi}{2}$, so $\frac{V}{4}$

Another factor of 2 in POWER

✓



The one without optimization

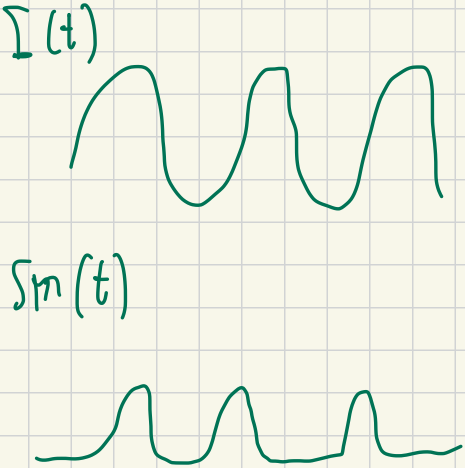
$$V_\pi = \frac{\lambda}{n^3 r_{33} \Gamma} \frac{d}{L}$$

So from simple Mz to optimized $\frac{1}{16}$ of power is used.

Attention

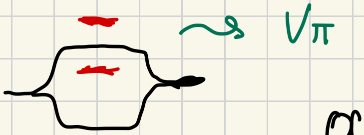
- If RF signal is sm(t), I(t) is NOT just a copy of sm(t), pass throught a MZ, so

$$I = \cos^2(\mathrm{sm}(t))$$

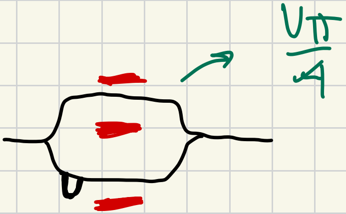


→ There's distortion

Push-pull vs simple

- 

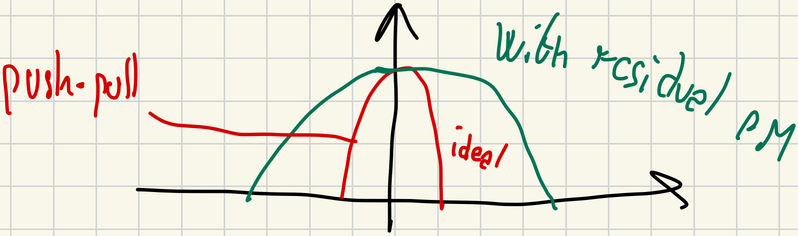
$$OUT = 1 + e^{-j\Delta\varphi(t)} = \underbrace{\boxed{e^{-j\frac{\Delta\varphi(t)}{2}}}}_{\text{CHIRP}} \cos\left(\frac{\Delta\varphi(t)}{2}\right)$$

- 

$$OUT = e^{-j\Delta\varphi(t)} + e^{+j\Delta\varphi(t)} = \cos\left(\frac{\Delta\varphi(t)}{2}\right)$$

The push-pull generate the correct signal to the output, without a residual phase modulation like the simple one.

↳ PM cause spectral regrowth



↓

UNWANTED IN WDM AND
I NEED FILTER WITH LARGHE
BW (COST ↑↑)

↓

Also the chromatic dispersion $D = \beta_2 L$
is larger , so greater distortion

⇓

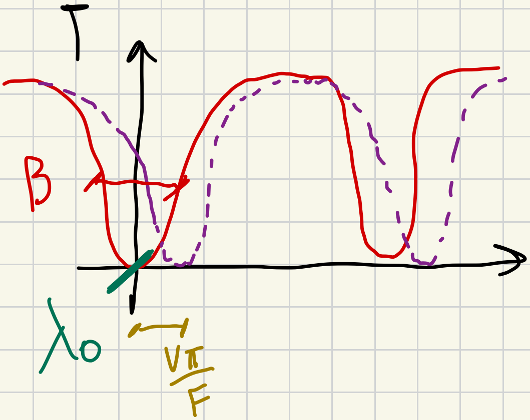
EVERYONE USE PUSH-PULL

RING MODULATOR (Si)

Mt has output intensity sinusoidal and
the extintion ratio is $20 \div 40$ dB , so I'
m able to switch off the Input if It's

enough small (ok).

I can make a modulator with ring. If in critical coupling ($r_1 = \gamma r_2$) I can induce a $\Delta\varphi$, so shift the ring characteristic:



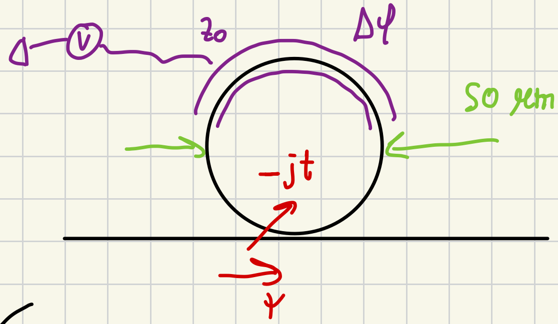
ma vabbe'

in realtà shifterebbe verso sx

So I change the output intensity.

$\Delta n = \bigcirc\!\!\!\!- \ , \dots \Delta N, \Delta P$

In Si



$$\tau_q = \frac{1}{\beta}$$

Molto piú piccolo del MZ.

The shift will be given by $\frac{V_{\pi MZ}}{F}$, so BW

Move the notch to cancel or not the signal.

$\Downarrow$

MANY MODULATOR ON CHIP, LARGE DENSITY AND COMPACT (W.R.T. MZ), SMALL VOLTAGE

Disadvantage

The electrodes are lumped, but the ring can be a pure C, so there will be reflections, the driving circuit must accept it.

What if $F \uparrow\uparrow$?

$B \downarrow\downarrow$, high selective and $V \downarrow\downarrow$ (FSR const.)

$$\Downarrow$$

BUT THE LIMIT IS DUE TO γ, $B \downarrow\downarrow$ CAN'T HAPPEN FOR TOO MUCH

In a modulator you need γ $(= \tau)$.

$$\Downarrow$$

$$\underbrace{\tau_q = \frac{1}{B} \quad \uparrow\uparrow\uparrow}_{} \quad \text{if } B \downarrow\downarrow\downarrow$$

BAD FOR MODULATOR

INCREASED TIME RESPONSE

$B = 1$ GHz, but τ_q too much slow, so the perturbation is wrong.

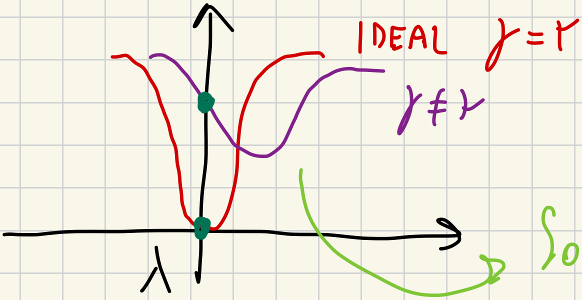
↳ So I need $B \uparrow\uparrow$ to go faster, but to have V low I need FSR $\uparrow\uparrow$ → small $\uparrow\uparrow$

$\Delta f \propto L \Downarrow$

Complexity $\uparrow\uparrow$

What if $r \neq \gamma$?

In Si I change the number of carrier (ΔN or ΔP) to produce Δf, I shift the spectrum but I change also $\Delta\alpha$ (in Si, InP purtroppo), so γ becomes different.



So carrier rejection

$\Delta N \downarrow\downarrow$, $\Delta n \uparrow\uparrow$

Here attenuation help, because the shift is as above but losses increases the transmission and here is wanted (switch on), the $V \rightarrow$ BIAS, $\Delta n \rightarrow 0$, again $T=0$ @ λ_0, so switch off.

Another problem

The phase response is NOT linear

↳ SPURIUS UNWANTED PHASE MODULATION

↙

SO MZ OR RING? CHOOSE
BASED ON THE APPLICATION

↓

After km of ocean is
better MZ without chirp

Related to the
istantaneous frequency

$$if = \frac{d\Delta\varphi}{dt}$$

← Cause D_F and
spurius phase change

← the spectrum of the
signal has different
frequency for PM

Not possible to control chirp in other modulator

⇓

But remember that PM increase
the BW of a signal intensity modulated

Chromatic dispersion

D_F = the difference of time that two harmonics at different ω arrive at destinatio

⇓

My signal in fibers has all the BW, so different λ arrive at different time

Note on Si + PN junction

MZ, push-pull can be implemented in Si photonics, so in Si it's possible to have modulator without chirp.

Ring modulator are good for short distances, like directly on chip or between near chips.