

# Electronic Systems Exercise Sessions Notes

Noise, advanced Operational Amplifiers and circuits,  
Sample&Hold circuits, advanced DAC and ADC converters

Franco Zappa

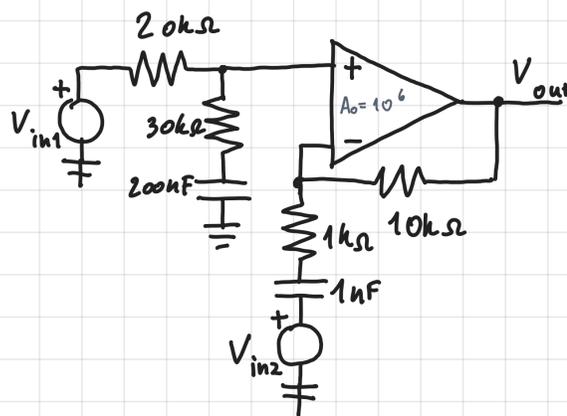
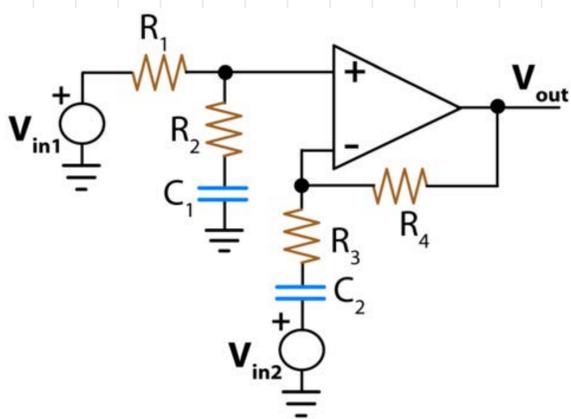


SOCIETÀ EDITRICE  
CULAPIO

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- OpAmp STAGES
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- MUX and DIGPOT
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- EXAMS examples

①

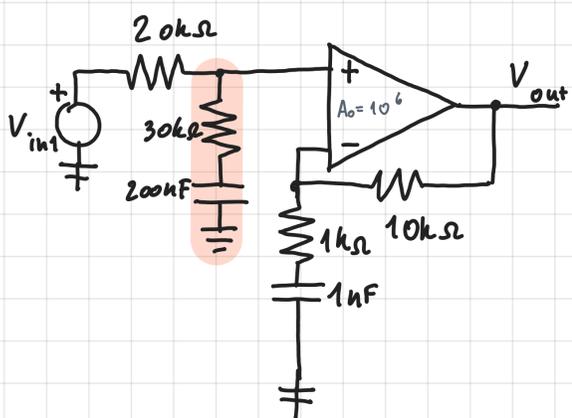


$R_1=20k\Omega$   $R_4=10k\Omega$   $R_2=30k\Omega$   $C_1=200nF$   $R_3=1k\Omega$   $C_2=1nF$   
 $A_0=10^6$   $I_B=10nA$   $f_0=10Hz$

a) Plot the ideal gain  $V_{OUT}(f)/V_{IN1}(f)$

b) Plot the ideal gain  $V_{OUT}(f)/V_{IN2}(f)$

a)



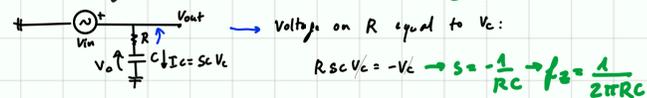
• at DC:  $C_1, C_2$  OPEN  
 [0 Hz]

$\hookrightarrow G_{DC} = \frac{V_{out}}{V_{in}} = 1$

• Zeros and poles

• ZERO 1:  $\rightarrow$  RC-series hanging at a node case

$\hookrightarrow$  3 RC-series hanging at a node  $\Rightarrow$  a finite zero

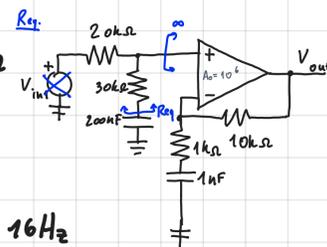


- for  $C_1$

$\rightarrow zero_1 = \frac{1}{2\pi \cdot 30k \cdot 200n} = 26Hz$

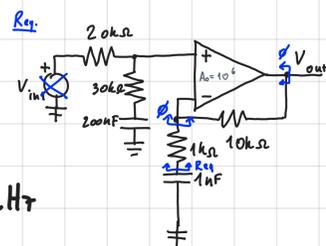
• POLE 1:  $\rightarrow R_{eq1} = 20k\Omega + 30k\Omega = 50k\Omega$

$\rightarrow pole_1 = \frac{1}{2\pi \cdot 200n \cdot 50k} = 16Hz$



• POLE 2:  $\rightarrow R_{eq2} = 1k\Omega$

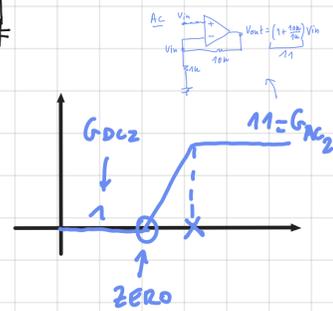
$\rightarrow pole_2 = \frac{1}{2\pi \cdot 1n \cdot 1k} = 160kHz$



- for  $C_2$

• ZERO 2:  $\rightarrow$  computed through Bode analysis of the stage with  $C_2$

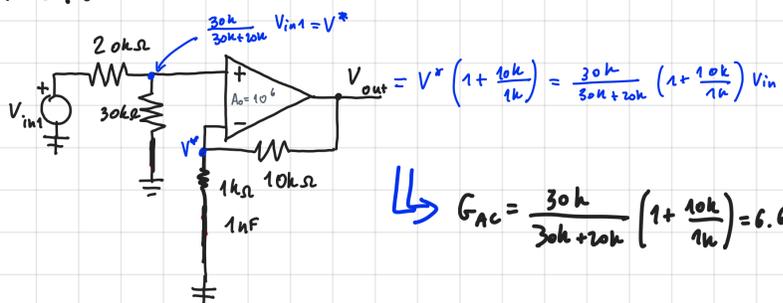
$\rightarrow zero_2 = \frac{pole_2}{\frac{G_{AC2}}{G_{DC2}}} = \frac{pole_2}{11} = 14.5kHz$



$\hookrightarrow$  Note: We can compute the zero of a capacitor that is series to a non-inverting op-amp with gain can be computed as:

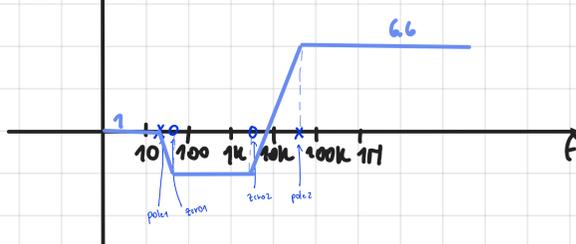
$zero_2 = \frac{1}{2\pi C_2 (1k + 10k)}$

• at AC:  $C_1, C_2$  short circuit  
 [∞ Hz]



$\hookrightarrow G_{AC} = \frac{30k}{30k+20k} \left(1 + \frac{10k}{1k}\right) = 6.6$

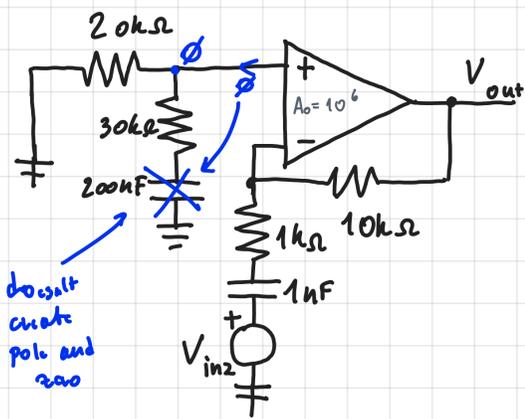
Bode:  $\left| \frac{V_{out}}{V_{in}} \right|$



$\rightarrow$  Note: this Bode is due to wrap-around. It would have been better this type of characteristic

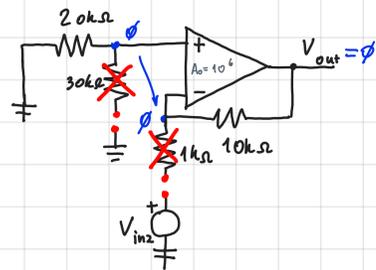


b)



• at DC:  $C_1, C_2$  OPEN  
[0 Hz]

$\hookrightarrow G_{DC} = \emptyset$



• Zeros and poles:

• ZERO 2:  $\rightarrow$  Capacitor along the signal path case

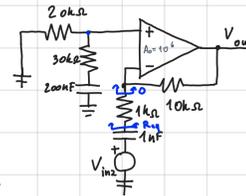
Capacitor along the signal path  $\Rightarrow$  ZERO at the origin

$i_c(t) = C \frac{dV_c}{dt} \xrightarrow{\mathcal{L}} I_c(s) = sC V_c(s) = \emptyset$   
 $\rightarrow I_c$  must be  $\beta$ :  $s=0 \rightarrow f_z=0$

$\rightarrow z_{cv02} = \emptyset$

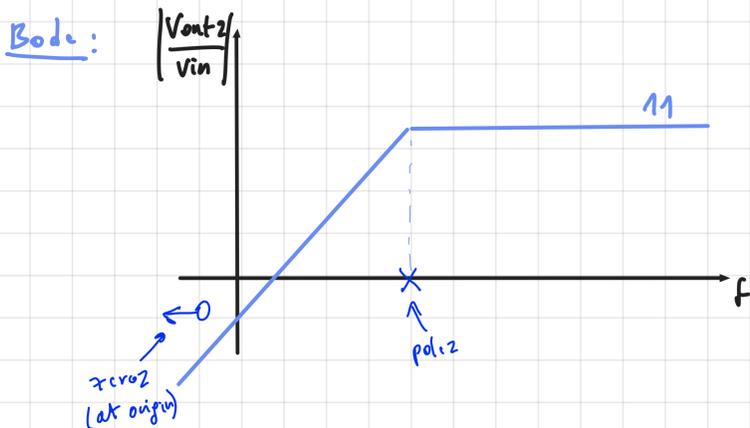
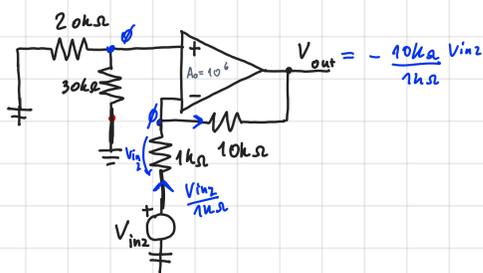
• POLE 2:  $\rightarrow R_{eq} = 1 \text{ k}\Omega$

$\rightarrow \text{pole 2} = \frac{1}{2\pi \cdot 1 \text{ k}\Omega \cdot 1 \text{ nF}} = 160 \text{ kHz}$

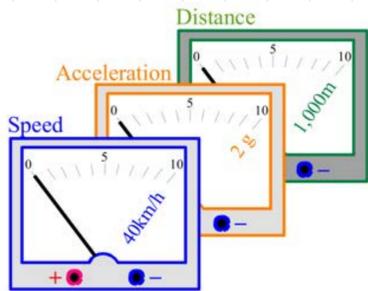


• at AC:  $C_1, C_2$  short circuit  
[ $\infty$  Hz]

$\Rightarrow G_{AC} = -11$   
 $|G_{AC}| = 11$



2



Calvin measures his bike's speed  $S$ , by employing Hobbes' speedometer ( $v_s = S \cdot 50 \text{mV/m/s} + 200 \text{mV}$ ) and 10V FSR voltmeters

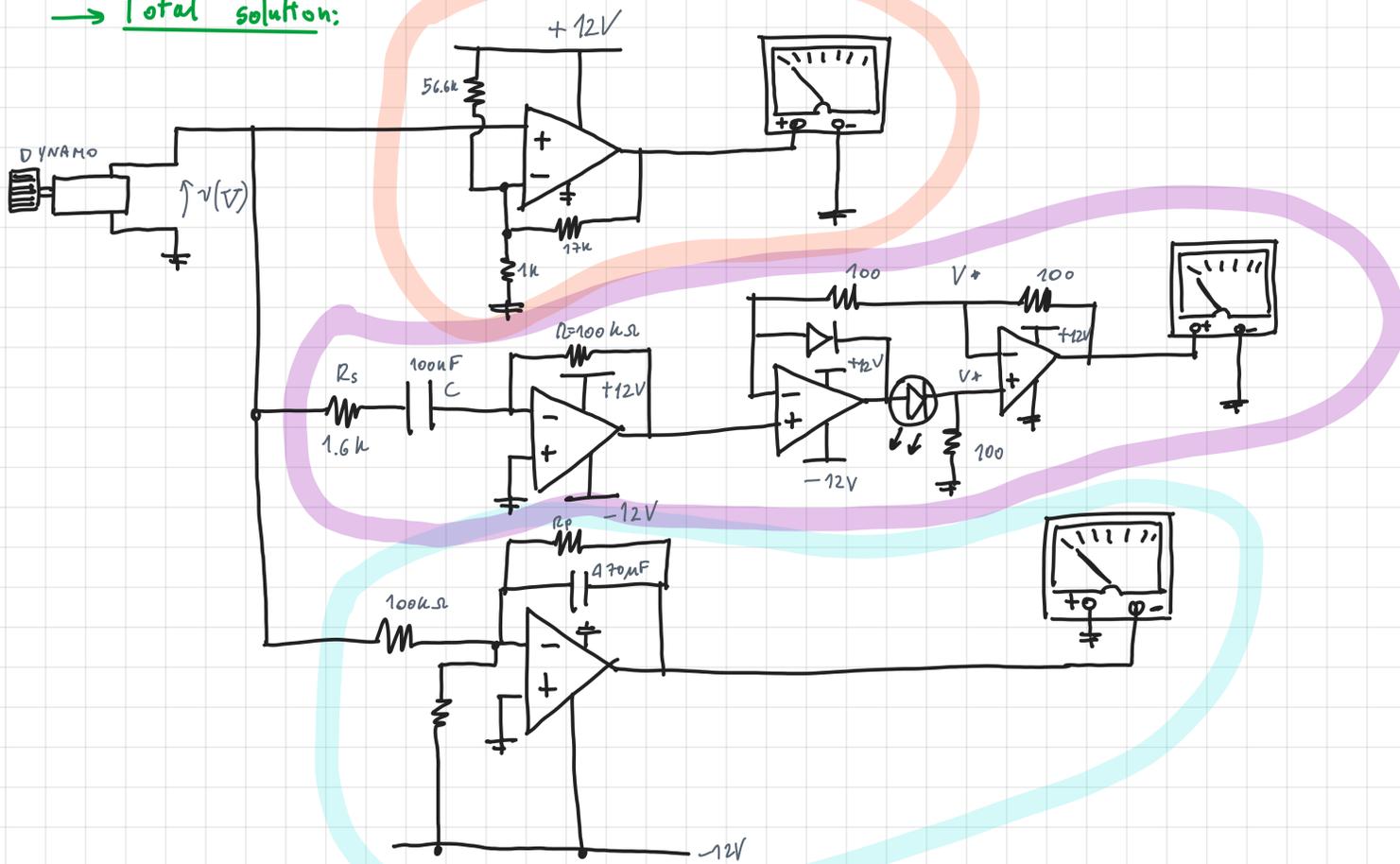
- a) Design a circuit for displaying the speed, up to 40km/h, with 3s smoothing
- b) Display acceleration/deceleration, up to 2 g ( $g=9.81 \text{m/s}^2$ )
- c) Measure and display the ride distance, up to 1km

We want to express speed, acceleration, distance scale with a voltmeter with max value 10V.

We use a dynamo sensor that converts speed in volts (Hobbes speedometer) following this relation:

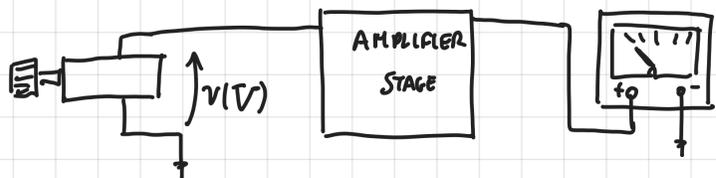
$$v \text{ (V)} = 50 \frac{\text{mV}}{\frac{\text{m}}{\text{s}}} \cdot V + 200 \text{mV}$$

→ Total solution:

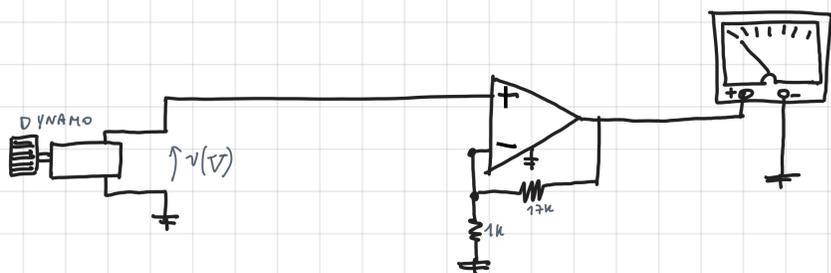


→ Solution analysis and derivation:

a) First consider the simple scheme:



↳ e.g. Amplifier design in inverting config.:



• Gain computation: We convert the known max speed ( $40 \frac{\text{km}}{\text{h}}$ ) in volts through this equation, and then see the gain proportion with the known max voltage (10V)

$$v \left( 40 \frac{\text{km}}{\text{h}} \right) = 50 \frac{\text{mV}}{\frac{\text{m}}{\text{s}}} \cdot \frac{40 \cdot 1000 \text{ m}}{3600 \text{ s}} + 200 \text{mV} = 555 \text{mV}$$

$$\Rightarrow G = \frac{10 \text{V}}{555 \text{mV}} \approx 18$$

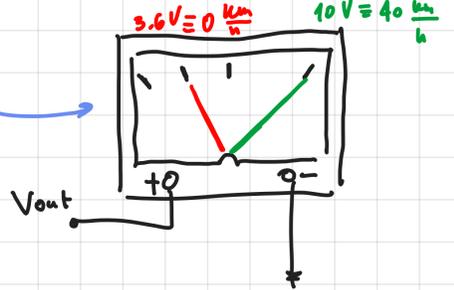
→ But in this case when we are at 0 speed we'll still have an offset on the scale due to the voltage offset of 200mV

↳ We need to subtract that offset:

For 0 speed  $v_{in} = v(0 \frac{km}{h}) = 200mV \rightarrow$  at the output of amplifier  $V_{out} = v_{in} \cdot 18 = 3.6V$

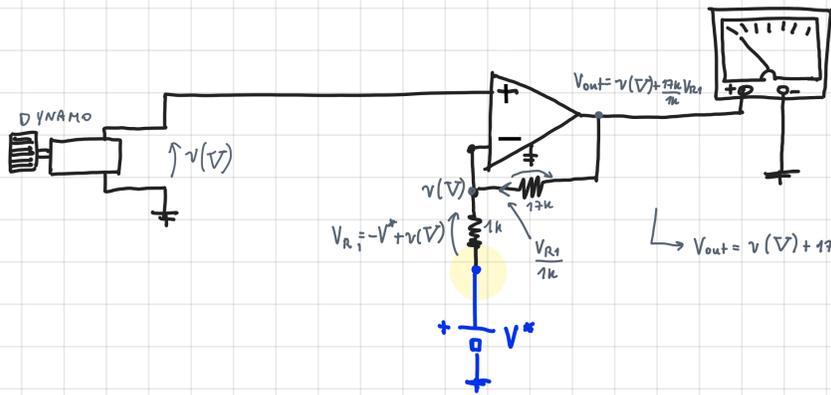
Waste of the scale resources

↳ speed not at the beginning



1st method

We can put this node not to ground, but to some voltage in such way that multiplied for the gain of 17 equals the 3.6V to subtract

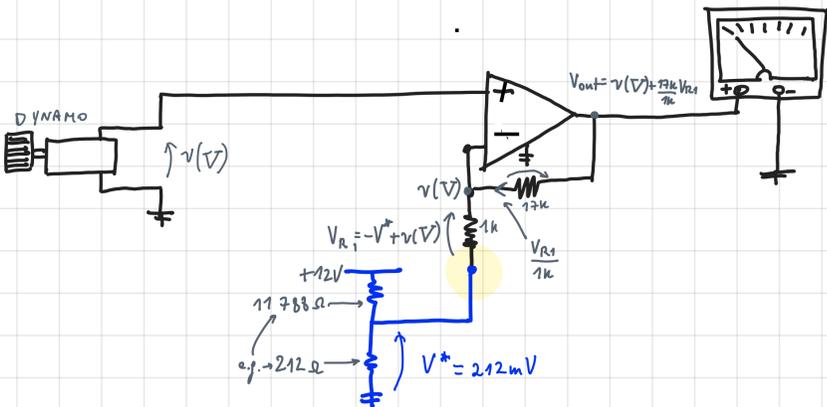


$$V^* \cdot 17 = 3.6V \rightarrow V^* = 212mV$$

↳ Indeed:  $V_{out}(0 \frac{km}{h}) = 3.6V - 17 \cdot 212mV = 0V$

↑ beginning of the scale!

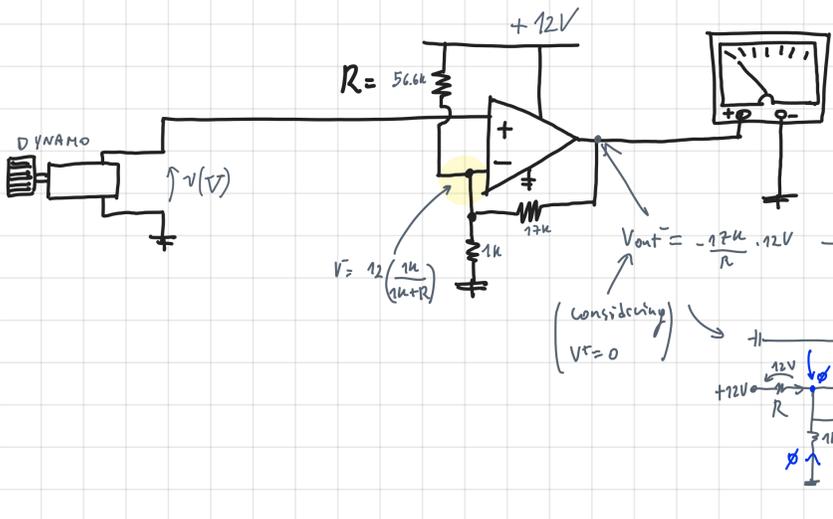
Note: Instead of buying a peculiar battery of 212mV we can obtain this voltage through a proper partition



2nd method

We could also use the OpAmp P.S. to inject current at this node (through a resistor) and choose R such that we have -3.6V at the output. In this way if we apply the superposition principle for:

(FINAL for speed)



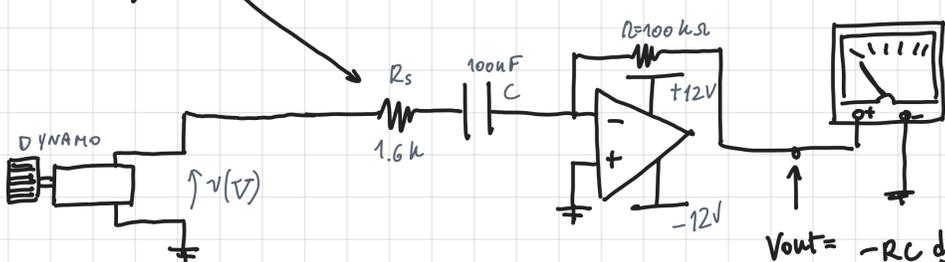
$$\begin{aligned} \bullet V_{in}^+ = 200mV &\rightarrow V_{out}^+ = 3.6V \\ \bullet V_{in}^- = 12 \left( \frac{1k}{1k+R} \right) &\rightarrow V_{out}^- = -3.6V \end{aligned} \rightarrow V_{out} = V_{out}^+ + V_{out}^- = 0$$

$$V_{out}^- = -\frac{17k}{R} \cdot 12V = -3.6V$$

$$\hookrightarrow R = 12V \cdot \frac{17k}{3.6} = 56.666\Omega$$

b) For the acceleration we can use the volt/velocity input and derivate it to a derivator

↳ e.g. real derivator

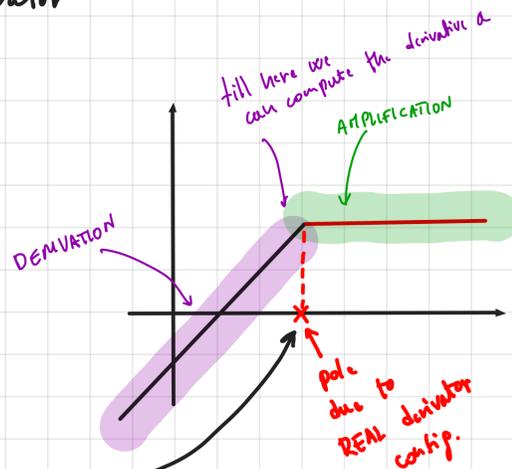


$$a = \frac{dV}{dt}$$

$$V_{out} = -RC \frac{dV_{in}(V)}{dt} = -RC \cdot 50 \frac{mV}{\frac{m}{s}} \cdot \frac{dV}{dt}$$

↳ 1st constraint:

↳ the pole will be at  $\frac{1}{2\pi R_s C} \rightarrow$  e.g. max freq. 1Hz (1 acc. per second)



• 2<sup>nd</sup> constraint: We have to consider the max for the scale

max acc. value:  $2g = 2 \cdot 9.81$

$V_{out,max} = 10V$

$$V_{out} = -RC \frac{dV_{in}(V)}{dt} = -RC \cdot 50 \frac{mV}{\frac{m}{s}} \cdot \frac{dV}{dt}$$

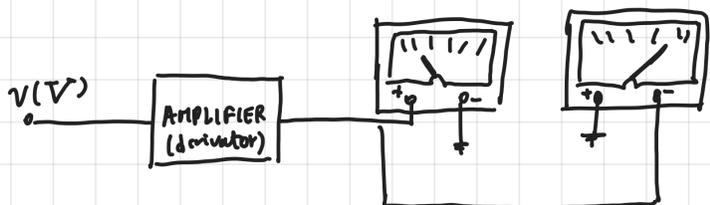
negative sign

$$\hookrightarrow -10V = -RC \cdot 50 \frac{mV}{\frac{m}{s}} \cdot 2 \cdot 9.81 \frac{m}{s^2} \quad \rightarrow RC = \frac{10V}{50mV \cdot 2 \cdot 9.81} \approx 10s$$

Accounting for the constraints we can choose for ex. 
 $C = 100\mu F$   
 $R = 100k\Omega$   
 $R_s > 16Hz$ 
} 2<sup>nd</sup> constr.  
 } 1<sup>st</sup> constr.

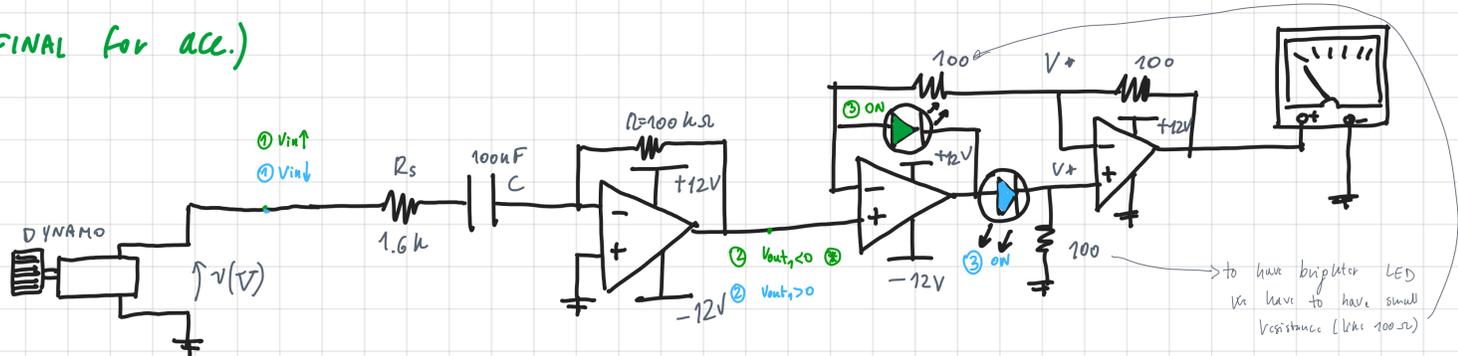
→ To take in account also the fact that the acceleration can be negative

• 1<sup>st</sup> method we can attach a scale display with inverted + and - links



• 2<sup>nd</sup> method We could also use a double-rectifying circuit → to measure the absolute value of the acc.

(FINAL for acc.)



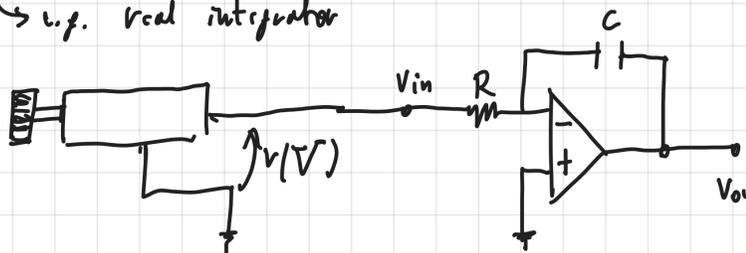
Note: Instead of the diodes we can directly put some LED diodes in this way we can assign a light colour to acceleration/deceleration.

- GREEN DIODE turns ON → ACCELERATION
- BLUE DIODE turns ON → DECELERATION

In this way we can know the sign of the acceleration since the output of the double-rectifier will be the acc. absolute value

c) To measure the distance we can use an integrator

→ e.g. real integrator



$$V_{out}(t) = \int_0^t \left( 50 \frac{mV}{\frac{m}{s}} V + 200mV \right) dt \cdot \left( -\frac{1}{RC} \right) = -\frac{1}{RC} \left[ \frac{50 \frac{mV}{\frac{m}{s}}}{\frac{m}{s}} \int_0^t V(t) dt + 200mV \cdot t \right]$$

RAMP from the volt/speed offset

→ 1<sup>st</sup> constraint: max value dist. = 10 km

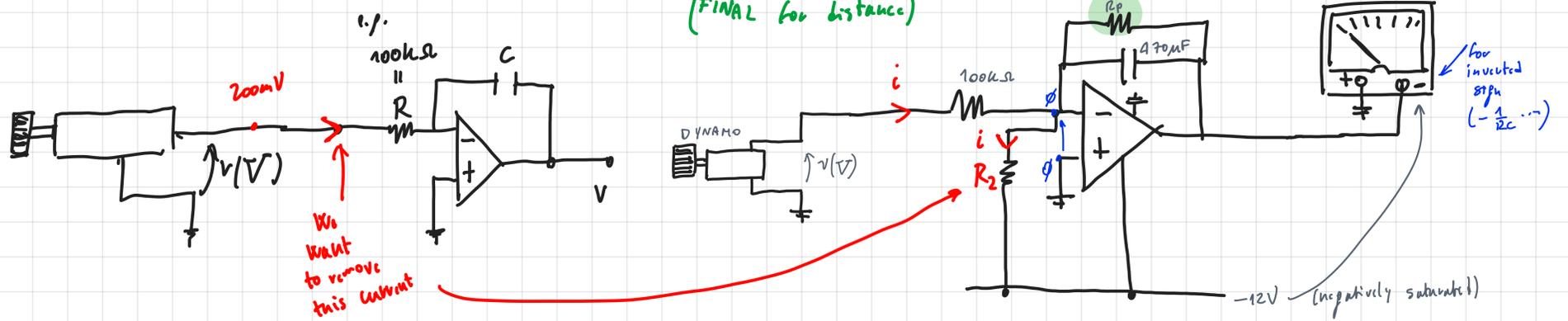
max value volt  $V_{out,max} = 10V$

$$V_{out,max} = 10V = -\frac{1}{RC} \cdot 50 \frac{mV}{\frac{m}{s}} \cdot 10 \cdot 1000$$

$$\hookrightarrow RC = \frac{50mV \cdot 10 \cdot 1k[A]}{\left[\frac{m}{s}\right] 10V} = 50s \rightarrow \begin{cases} C = 470\mu F \\ R \approx 100k\Omega \end{cases}$$

e.g. not exactly 50μF (hd.)

↳ We have to solve the problem of the ramp due to the  $+200\text{mV}$  (FINAL for distance)

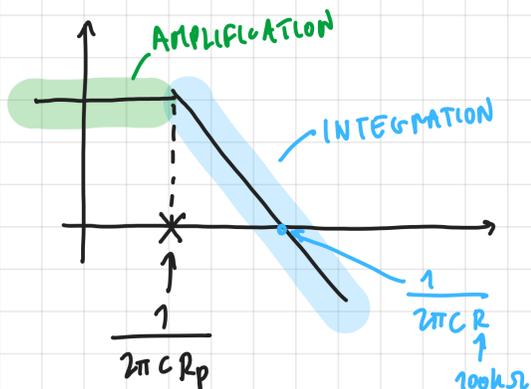


The task of  $R_2$  is to "remove" the current  $i$  and prevent it to go through the capacitor so the offset cannot be integrated into a ramp.

$$i = \frac{V_{in}}{R} = \frac{200\text{mV}}{100\text{k}\Omega} = \frac{12\text{V}}{R_2} \rightarrow R_2 = \frac{12}{0.2} \cdot 100\text{k} = 6.4\text{M}\Omega$$

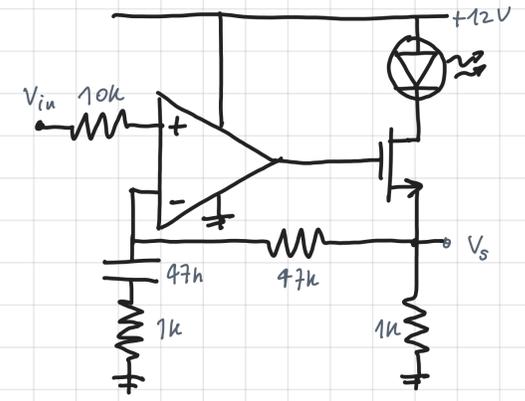
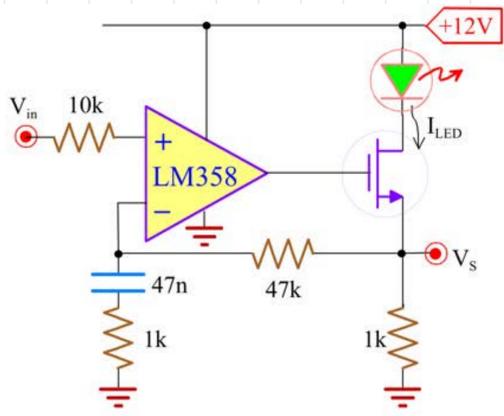
and the integrator will still produce a ramp out

Note: It's not that possible to have such precise value for  $R_2$ , so will still have this mismatch for the DC offset  $200\text{mV}$ . Since for an ideal integrator the DC gain  $\rightarrow \infty$  we can also limit it with a real integrator (with  $R_p$ )



With this we saturate the DC gain

3



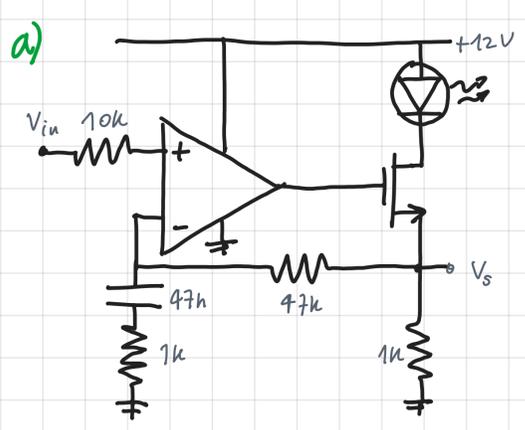
MOSFET:  $V_T = 0.5V$   $k = 1/2 \cdot \mu \cdot C_{ox} \cdot W/L = 12mA/V^2$

$V_{IN}(t)$  has 5V DC plus a  $\pm 100mV$  sinusoid  
DC VALUE

a) Compute  $I_{LED}/V_{in}$  relationship at DC and  $I_{LED}$  for  $V_{in} = 5V$

b) Plot the Bode diagram of  $i_{LED}(f)/V_{in}(f)$

$V \rightarrow I$  (TRANCONDUCTANCE)



at DC:  
[0 Hz]  
 $\hookrightarrow$  DC value: 5V  
 $\hookrightarrow$  C OPEN

MOSFET:

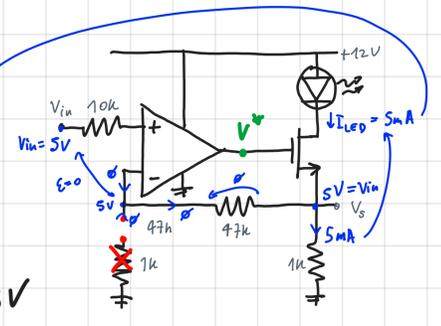
$$I_{LED} = k V_{GS}^2 = 12 \frac{mA}{V^2} (V_{GS} - V_T)^2 = 5mA$$

$$V_{GS} = V_T + \sqrt{\frac{5mA}{12 \frac{mA}{V^2}}} = 1.95V$$

$$V_S = 5V$$

$$V^* = V_{GS} + V_S = 6.95V$$

(can supply it with a 12V P.S.)



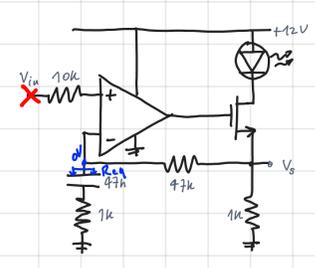
$$G_{DC} = \frac{i_{LED}}{V_{in}} = \frac{5mA}{5V} = 1 \frac{mA}{V}$$

zeros and poles

zero (from Bode plot)

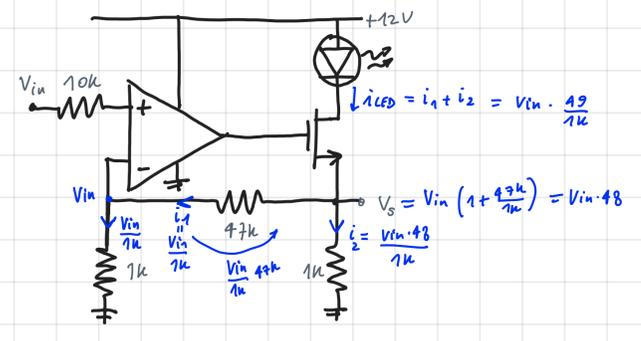
pole  $\tau_{eq} = 1\mu s$

$$pole = \frac{1}{2\pi C \cdot 1k} = 3.4 kHz$$

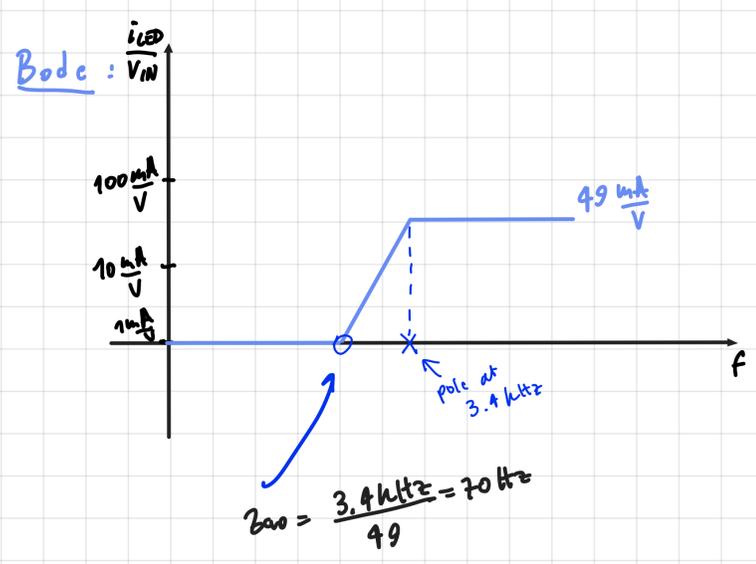


b) at AC  
[ $\infty$  Hz]

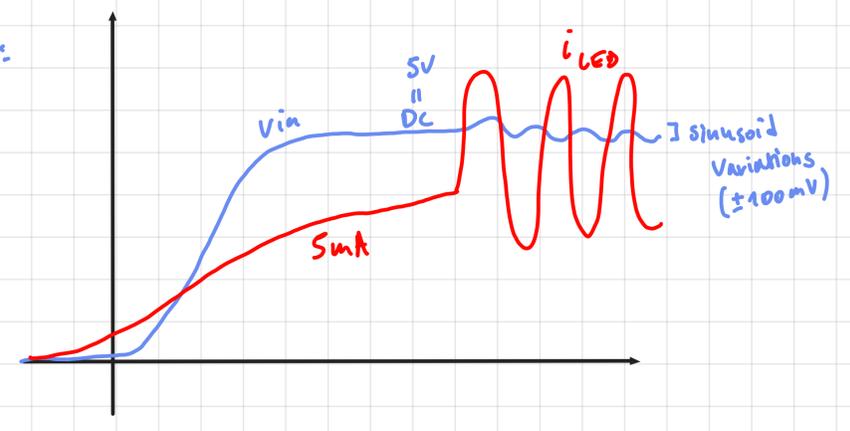
C close



$$G_{AC} = \frac{i_{LED}}{V_{in}} = \frac{49}{1k} = 49 \frac{mA}{V}$$

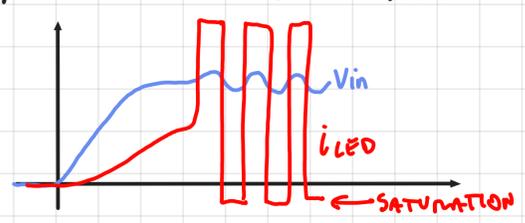


In time domain:



$\hookrightarrow$  Small signal analysis, from the text we know we have a small signal with DC value 5V  $\pm 100mV$  sinusoid

$\hookrightarrow$  Now suppose we have  $\pm 1V$  sinusoid variations  $\rightarrow$  It means the output will be saturated due to the P.S. (12V)



$\rightarrow$  Indeed for  $\pm 1V$   $G_{AC} = 49 \frac{mA}{V} \Rightarrow i_{LED} = \pm 49mA + DC \text{ value of } 5mA = 54mA$

So  $V_S = 54V \rightarrow$  so  $V_G > V_S$  but  $V_G$  is limited to +12V (P.S.)

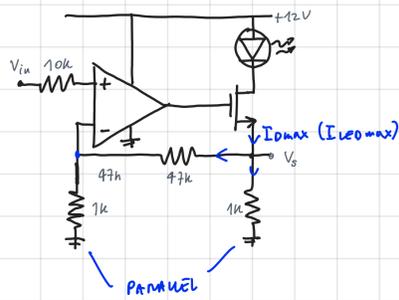
→ We can actually compute the maximum current for the max.  $V_G = 12V$

↳  $V_G|_{\max} = 12V$

$$I_D|_{\max} = 12 \frac{\text{mA}}{V^2} (V_G - V_S - V_T)^2 = 12 \frac{\text{mA}}{V^2} (12V - V_S - 0.5)^2$$

$V_S|_{\max} = I_{D\max} (1k \parallel 48k) \approx I_{D\max} \cdot 1k$

$$\hookrightarrow I_{D\max} = 12 \frac{\text{mA}}{V^2} \frac{I_{D\max}^2 (1k)^2 - 2 \cdot 11.5 \cdot I_{D\max} \cdot 1k + 11.5^2}{(11.5V - I_{D\max} \cdot 1000)^2}$$



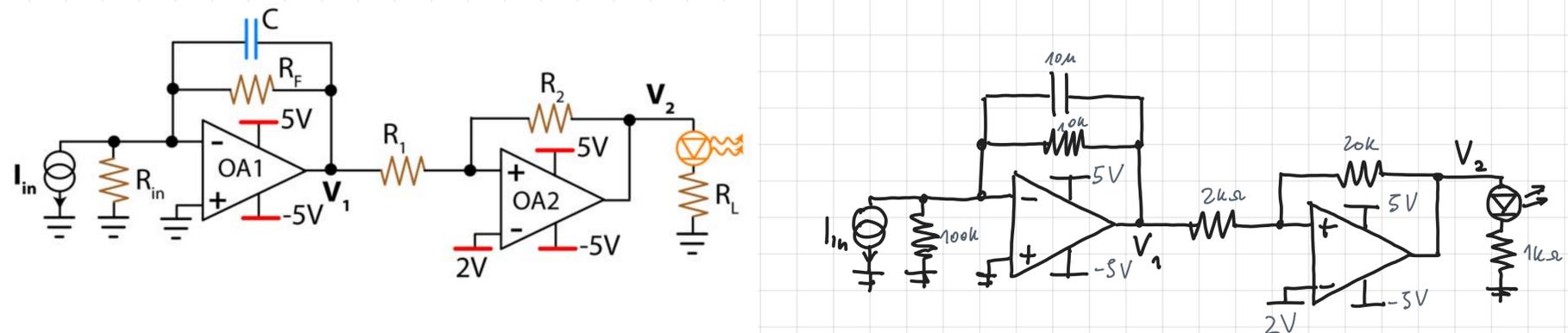
$$\hookrightarrow I_{D\max}^2 \underbrace{(1k)^2 \cdot \frac{12 \text{mA}}{V^2}}_{12k} - \underbrace{\left(1 + \frac{12 \text{mA}}{V} \cdot 2 \cdot 11.5 \cdot 1k\right)}_{2.77} I_{D\max} + \underbrace{\frac{12 \text{mA}}{V^2} \cdot 11.5^2}_{1.587} = 0$$

$I_{D\max} = 12,5 \text{ mA} \leftarrow \text{MAX one}$   
 $I_{D\max} = 10,6 \text{ mA}$

$$\frac{2.77 \pm \sqrt{2.77^2 - 4 \cdot 12k \cdot 1.587}}{24k} =$$

$\sqrt{533} \approx 23,156$

4

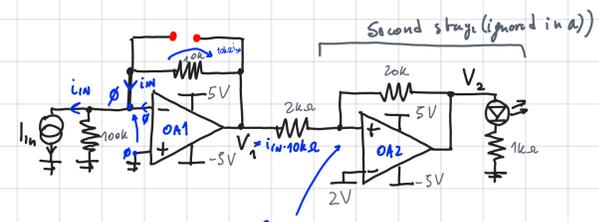


$R_{in}=100k\Omega$   $R_F=10k\Omega$   $C=10\mu F$   $R_1=2k\Omega$   $R_2=20k\Omega$   $R_L=1k\Omega$

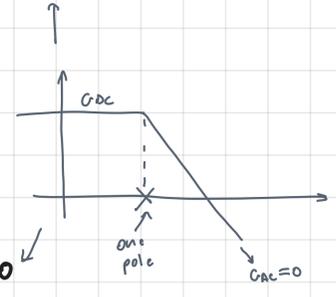
- a) Compute the effect of  $I_B=100nA$  and  $V_{OS}=3mV$  of OA1 on  $V_1$
- b) Plot the static curve  $V_1$  vs.  $V_2$
- c) Compute the minimum amplitude  $I_{in}$  (20Hz sinusoidal) to switch on the LED

a)  $\rightarrow \left| \frac{V_1}{I_{in}} \right| ?$

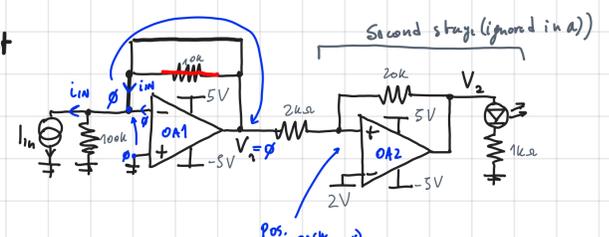
• at DC:  $C \rightarrow OPEN$   
[0 Hz]



$\Rightarrow G_{DC} = \frac{V_1}{I_{in}} = 10k\Omega$



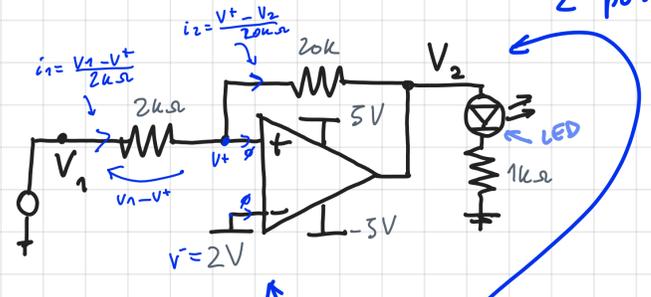
• at AC:  $C \rightarrow short$   
[∞ Hz]



$\Rightarrow G_{AC} = \frac{V_1}{I_{in}}(\infty) = 0$

• pole =  $\frac{1}{2\pi \cdot 10\mu F \cdot 10k\Omega} = 1.6 Hz$   
 $R_{eq} = R_F = 10k\Omega$

b)  $\rightarrow \frac{V_2}{V_1}$  SECOND STAGE:



2 possible outputs  $\begin{cases} +5 \\ -5 \end{cases}$

→ current KCL:

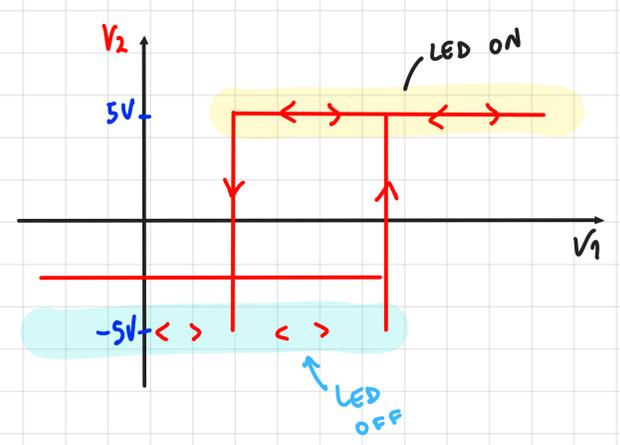
$i_{L1} = i_{L2} \rightarrow \frac{V_1 - V^+}{R_1} = \frac{V_1 - V_2}{R_2}$  Eq.1

→ We analyze when  $\epsilon = 0 \rightarrow V^+ = V^- = 2V$  Eq.2

$\hookrightarrow \begin{cases} \text{Eq.1} \\ \text{Eq.2} \end{cases} \rightarrow \frac{V_1 - 2}{2k\Omega} = \frac{2 - V_2}{20k\Omega}$

• for output  $V_2 = +5V \rightarrow V_1 = V_{TL} = \frac{(2 - 5V) \cdot 2k\Omega}{20k\Omega} + 2V = 1.7V$

• for output  $V_2 = -5V \rightarrow V_1 = V_{TH} = \frac{(2 + 5V) \cdot 2k\Omega}{20k\Omega} + 2V = 2.7V$



c)  $V_2$  LED usually requires 1.5V  
LED is ON  $\rightarrow i_{LED} = 3.5mA$

threshold to TURN ON:  $V_{TH} = 2.7V = V_1$

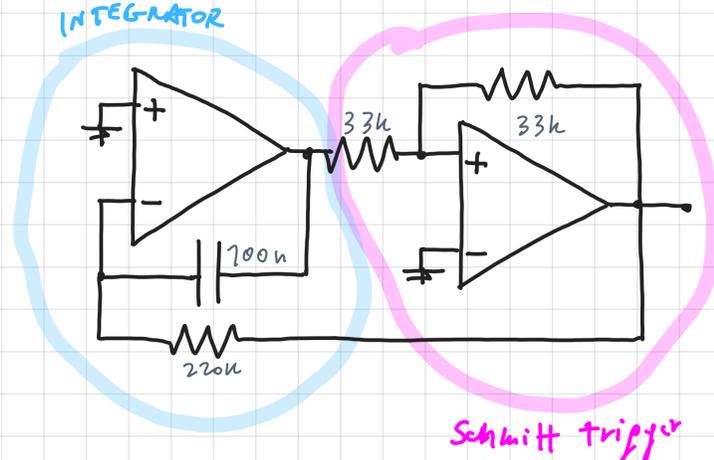
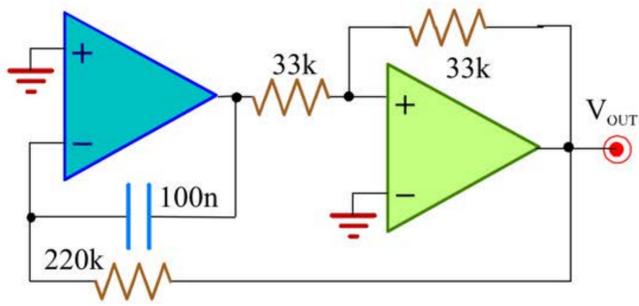
the gain to obtain  $V_1 = V_{TH}$  and turn on the LED is freq. dependent

$\hookrightarrow$  at 20 Hz  $G(20Hz)$  but from the previous Bode:  $\Rightarrow$  small gain  $\Rightarrow G(20Hz) = \frac{G_{DC}}{20} = \frac{0.8 \mu V}{1.6} \mu A$

$\hookrightarrow$  so to trigger the compensator and turn on the LED we need as input to get  $V_1 = V_{TH} = 2.7$  through an amplifier with gain

$G(20Hz)$  is:  $i_{in} = \frac{2.7V}{0.8 \frac{\mu V}{\mu A}} = 3.4 mA \rightarrow$  then the LED turns ON

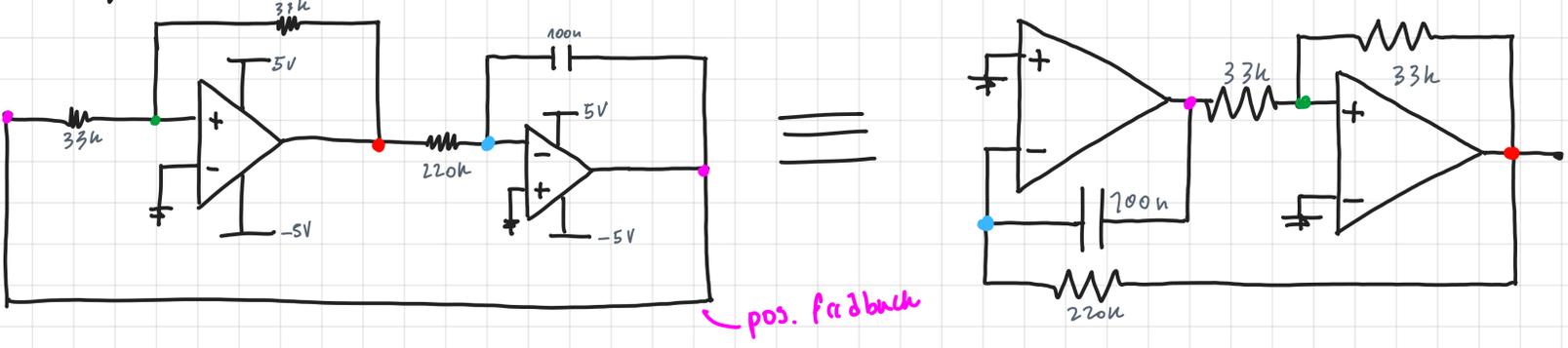
5



Rail-to-rail OpAmp biased at ±5V. At t=0s, the capacitor is discharged.

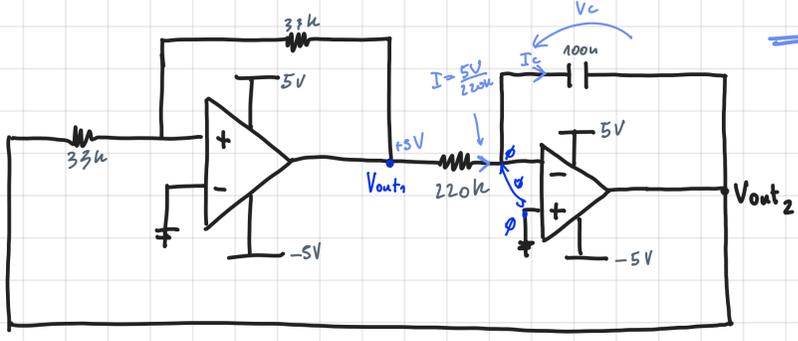
- a) Plot all waveforms
- b) Write the equation of output frequency vs. R and compute the value for R=220kΩ

Instead of the circuit in the figure we start our analysis drawing the integrator and trigger configurations in a different way:



Now, we know that the Schmitt trigger can have only two possible outputs: +5V, -5V

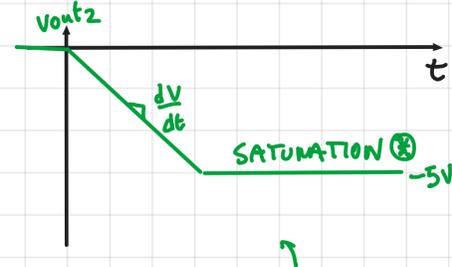
• For  $V_{out1} = +5V$



$$I = \frac{5V}{220k\Omega} = I_c = C \frac{dV_c}{dt}$$

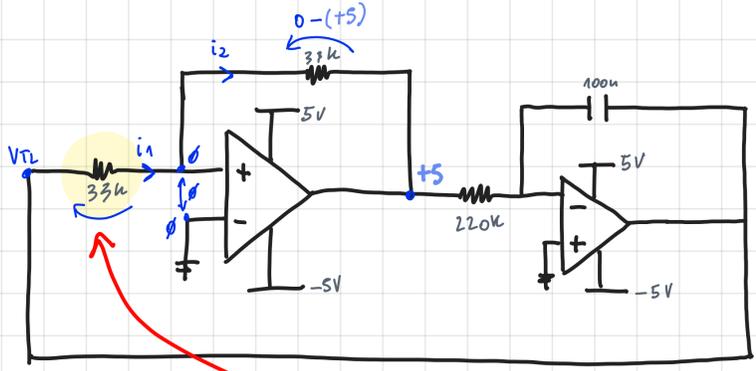
$$\hookrightarrow \frac{dV_c}{dt} = \frac{I}{C} = \frac{5V}{220k\Omega \cdot 100nF} = 227 \frac{V}{s}$$

$$V_{out2} = -V_c = -\frac{1}{C} \int_0^t I(t) dt = -\frac{1}{220k\Omega \cdot 100\mu F} \int_0^t 5V dt = -227 \frac{V}{s} \cdot t [s]$$



We actually don't know if  $V_{out2}$  will saturate to -5V before the trigger commutes (from +5V → -5V)

↳ Let's compute the threshold value for which the trigger commutes. We analyze when  $V_{in1} = V_{out2} = V_{TL}$  is such that  $i_1 = i_2$

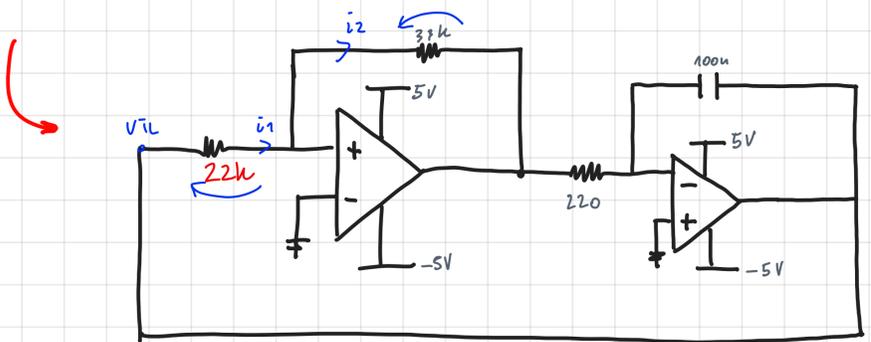


$$\hookrightarrow i_1 = \frac{V_{TL}}{33k\Omega} = i_2 = \frac{-5V}{33k\Omega}$$

$$\hookrightarrow V_{TL} = -5V \leftarrow \text{threshold on the SATURATION}$$

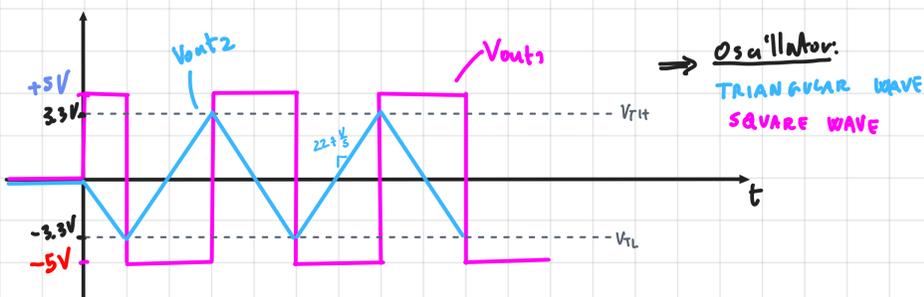
it's better to consider a different value for this resistor

↳ c.p. 22kΩ

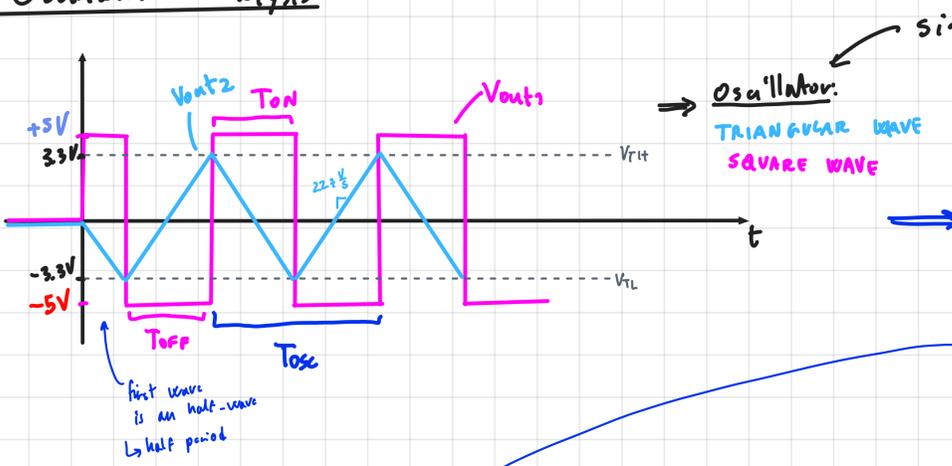


$$\hookrightarrow i_1 = \frac{V_{TH}}{22k\Omega} = i_2 = \frac{-5V}{33k\Omega} \rightarrow V_{TH} = -5 \cdot \frac{22}{33} = -3.3V$$

(Since the  $V_{out1}$  is symmetric ±5V ⇒  $V_{TH} = 3.3V$ )



# Oscillator analysis



→ simply - stable  
 Oscillator:  
 TRIANGULAR WAVE  
 SQUARE WAVE

$$T_{osc} = T_{on} + T_{off} = 58 \text{ ms} \rightarrow f_{osc} = \frac{1}{58 \text{ ms}} = 17 \text{ Hz}$$

$$T_{off} = T_{on} = \frac{2 \cdot 3.3 \text{ V}}{227 \frac{\text{V}}{\text{s}}} = 29 \text{ ms}$$

$\frac{5 \text{ V}}{220 \text{ k}\Omega \cdot 100 \text{ nF}}$   
 $\frac{5 \text{ V}}{R}$

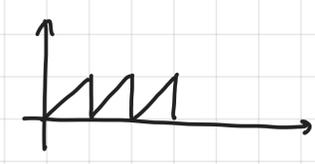
b)  $T_{osc} = \frac{2 \cdot 2 \cdot 3.3 \text{ V}}{\frac{5 \text{ V}}{R \cdot 100 \text{ nF}}} = 264 \frac{\text{ns}}{\Omega} R$

$f_{osc} \approx 3,8 \frac{\text{MHz}}{\text{s}} \cdot \frac{1}{R}$  ( $f_{osc}(220 \text{ k}\Omega) \approx 17 \text{ Hz}$ )

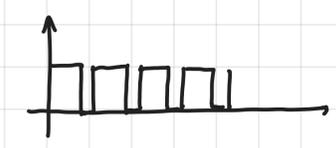
## Obs. Oscillators

We can have different trends with this oscillator configurations.

e.g. saw tooth

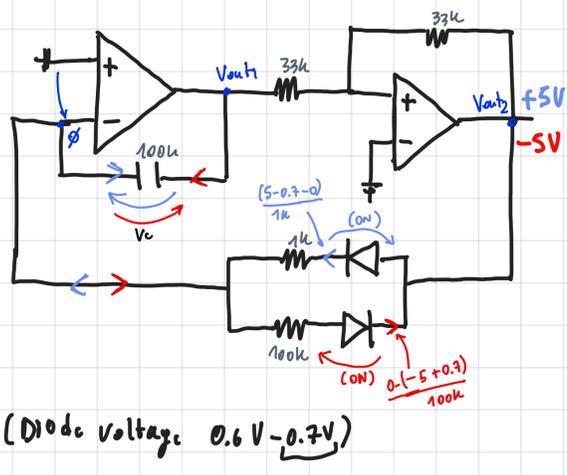


different Ton, Toff



Duty cycle change:  $\text{Duty Cycle} = \frac{T_{on}}{T_{on} + T_{off}}$

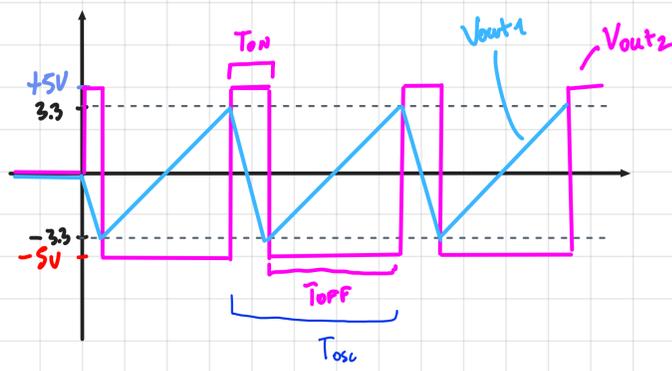
## ex.



$V_{out1} = -V_c \rightarrow \frac{dV_{out1}}{dt} = -\frac{I}{C} = -\frac{5 - 0.7 - 0}{1k \cdot 100n} = -43 \frac{\text{V}}{\text{s}}$  (much faster slope → different Ton ≠ Toff)

$V_{out1} = V_c \rightarrow \frac{dV_{out1}}{dt} = \frac{I}{C} = \frac{0 - (-5 + 0.7)}{100k \cdot 100n} = 430 \frac{\text{V}}{\text{s}}$

SQUARE WAVE with ≠ Duty Cycle  
 ≈ SAW TOOTH WAVE



$$T_{on} = \frac{2 \cdot 3.3 \text{ V}}{43 \frac{\text{V}}{\text{s}}} = 154 \mu\text{s}$$

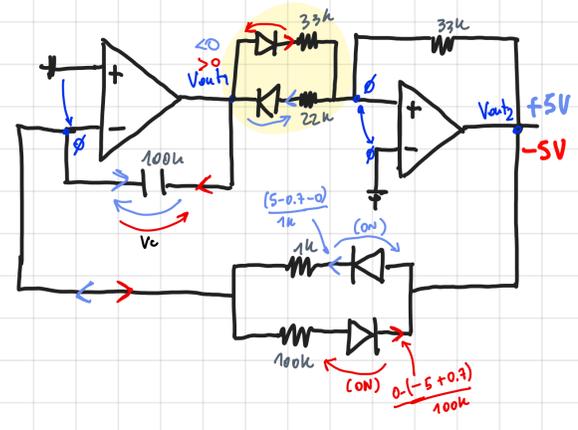
$$T_{off} = \frac{2 \cdot 3.3 \text{ V}}{430 \frac{\text{V}}{\text{s}}} = 15 \text{ ms}$$

$$T = T_{on} + T_{off} \approx T_{off} = 15 \text{ ms}$$

$$\text{Duty Cycle} = \frac{T_{on}}{T} = 0,01\%$$

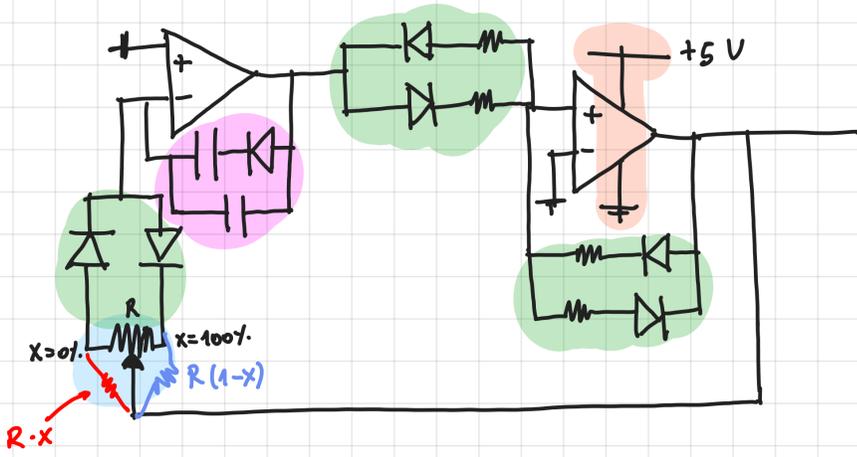
ex. Now suppose we want to change Vth and Vtl to make it for ex. non-symm.

We saw in the ex. that Vth and Vtl depend on the resistors of the Schmitt trigger, so to have different Vth, Vtl for symm. output ±5V we can implement also here the diodes that allow us to pass to specific and different resistors, based on the direction of the current, that depends on the output.

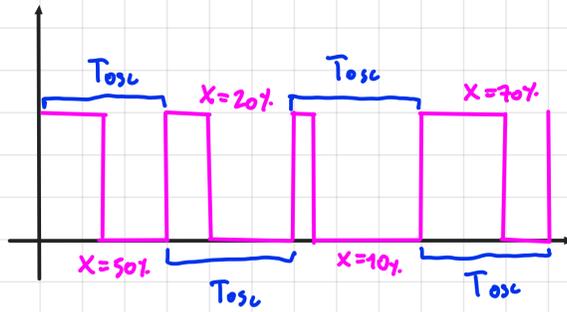


→ Different thresholds ← Otherwise we can use non symm. P.S.

ex. Playing with the different elements of a circuit can allow us to have a lot of design variability



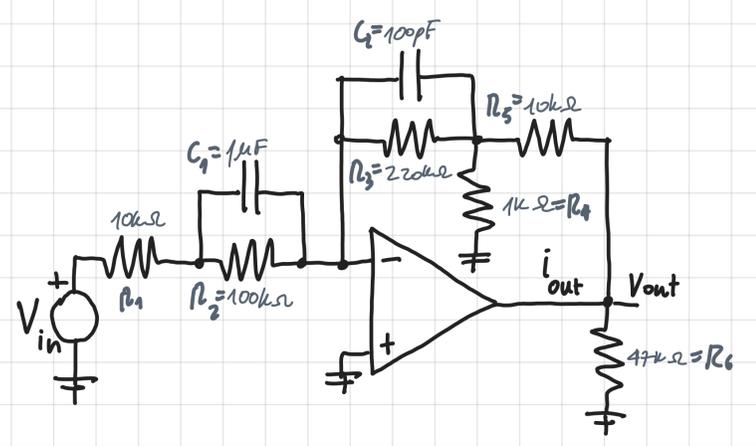
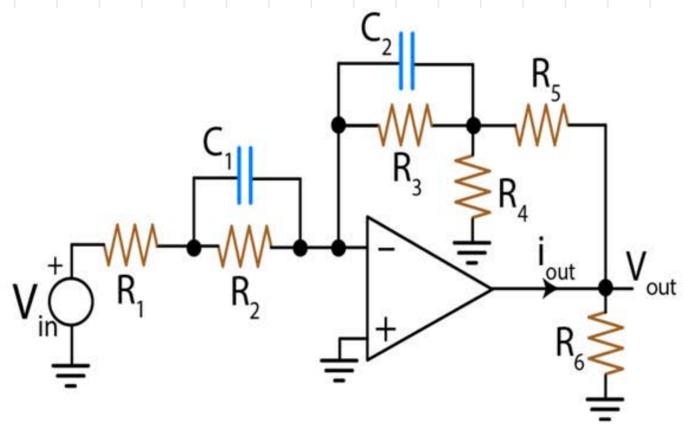
- to allow the choice bw different resistors
- choice bw sum of C or just one C ⇒ diff. slopes (parallel of C)
- potentiometer to have diff. resistors
- non symm. P.S.



We can change the duty cycle but the  $f_{osc}$  will remain const. (period stays const.)

↑  
Because  $T_{osc} \propto R_{on} + R_{off} = R$   
↑  
the sum is always the same

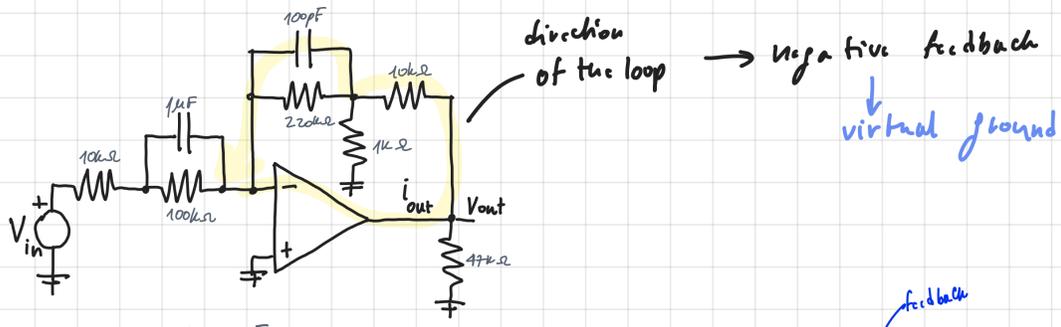
①



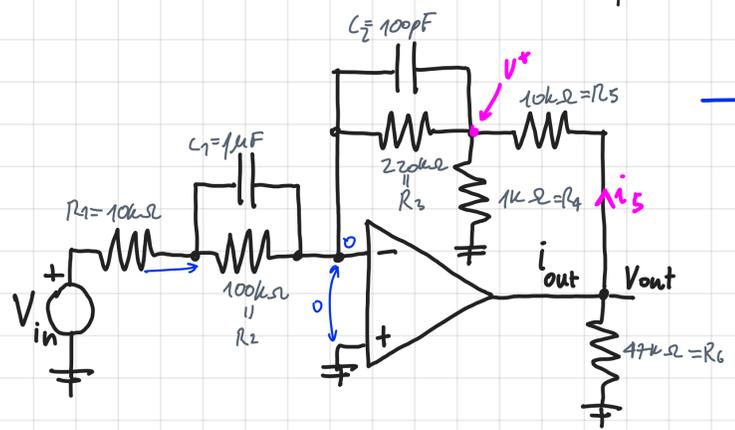
$R_1=10k\Omega, R_2=100k\Omega, R_3=220k\Omega, R_4=1k\Omega, R_5=10k\Omega, R_6=47k\Omega, C_1=1\mu F, C_2=100pF.$

- a) Plot the ideal  $|v_{out}(f)/v_{in}(f)|$  gain.
- b) Compute  $i_{out}$  when  $V_{in}=-100mV.$

↳ Negative or positive feedback

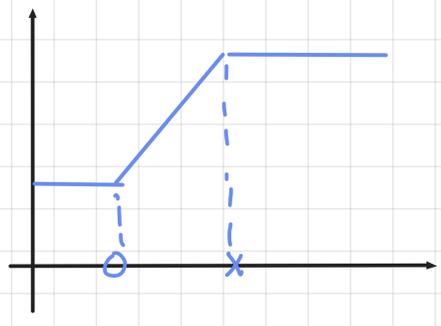


a)



1  $G(0) = -\frac{R_F}{R_1+R_2}$

2  $G(\infty) = -\frac{R_F}{R_1}$



$i_{in}(f) = \frac{V_{in}}{R_1 + Z_1} = \begin{cases} \text{DC } [0 \text{ Hz}] & \frac{V_{in}}{R_1 + R_2} \\ \text{AC } [\infty \text{ Hz}] & \frac{V_{in}}{R_1} \end{cases}$

$V^*(0) = -\frac{V_{in}}{R_1+R_2} \cdot R_3$

$i_s(0) = \frac{V^*}{R_3} + \frac{V^*}{R_4}$

$\Rightarrow V_{out}(0) = V^* + R_5 \cdot i_s = V^* + R_5 V^* \left( \frac{1}{R_3} + \frac{1}{R_4} \right) = -\frac{V_{in}}{R_1+R_2} \cdot R_3 \left[ 1 + R_5 \left( \frac{1}{R_3} + \frac{1}{R_4} \right) \right]$

$R_3 || R_4$  causes an increase in the gain

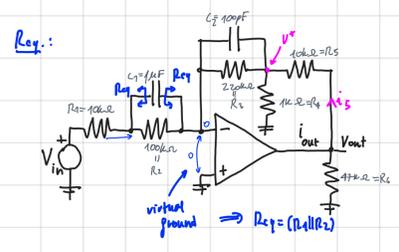
1  $G(0) = \frac{V_{out}(0)}{V_{in}} = -\frac{R_3}{R_1+R_2} \left[ 1 + \frac{R_5}{R_3 || R_4} \right] = -\frac{220k}{110k} \left[ 1 + \frac{10k}{1k} \right] = -22$

2  $G(\infty) = \frac{V_{out}(\infty)}{V_{in}} = -\frac{R_5}{R_1} = -1$

• Poles and Zeros:

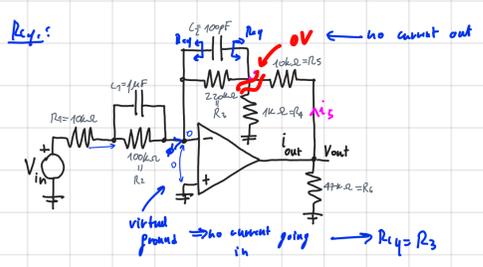
for  $C_1$   $\bullet$  pole 1 =  $\frac{1}{2\pi C_1 (R_2 || R_4)} = \frac{1}{2\pi \cdot 1\mu (10k || 100k)} = 18 \text{ Hz}$

$\bullet$  zero 1 =  $\frac{1}{2\pi C_1 R_2} = 1.6 \text{ Hz}$

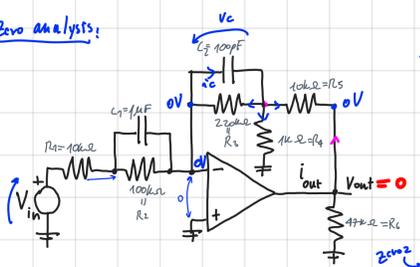


for  $C_2$   $\bullet$  pole 2 =  $\frac{1}{2\pi C_2 R_3} = 7.2 \text{ kHz}$

$\bullet$  zero 2 =  $\frac{1}{2\pi C_2 (R_3 || R_4 || R_5)} = 1.8 \text{ MHz}$

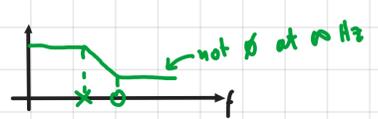


Zero analysis:

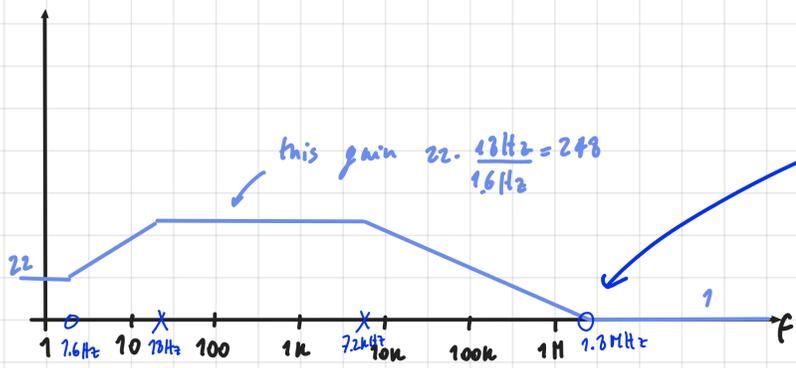


$i_c = \frac{V_c}{s} = -\frac{V_c}{R_3} - \frac{V_c}{R_4} - \frac{V_c}{R_5} = -\frac{V_c (R_4 R_5 + R_3 R_5 + R_3 R_4)}{R_3 R_4 R_5}$   
 This eq. holds for:  
 $s = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$   
 $= \frac{1}{C_2 (R_3 || R_4 || R_5)}$

↳ the value at  $\infty$  is not 0  $\rightarrow$  we have a zero too



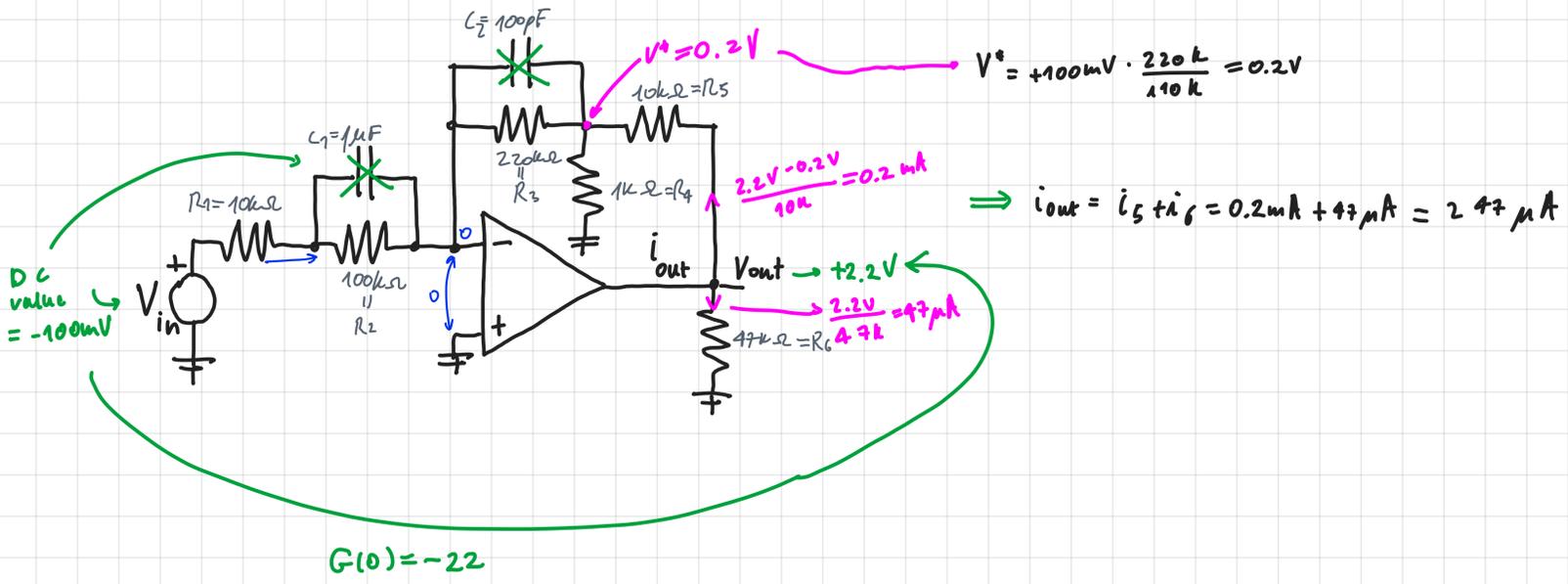
Bode diagram:



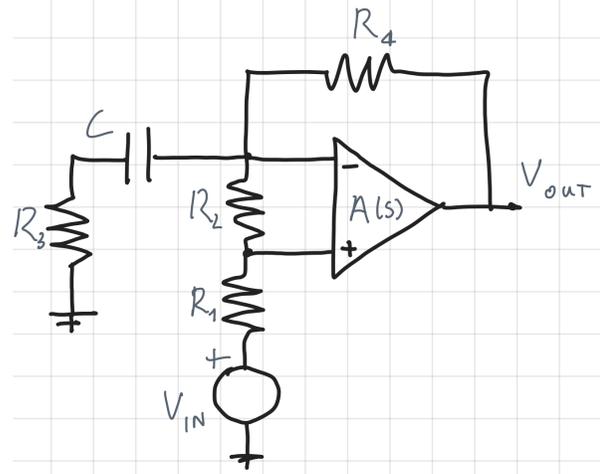
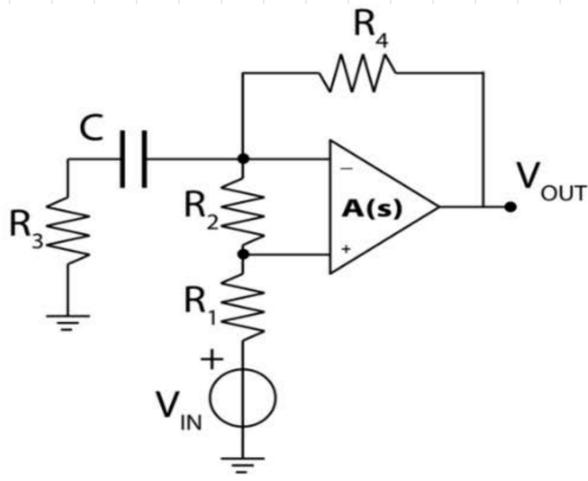
We could have also compute the zero like:

$$\hookrightarrow z_{c02} = \frac{248 \cdot 7.2 \text{ kHz}}{1} = 1.8 \text{ MHz} \quad \checkmark$$

b)



2

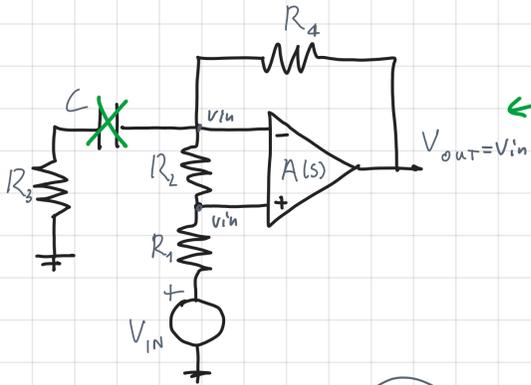


$A_0=100\text{dB}$ ,  $\text{GBWP}=100\text{MHz}$ .

$R_1=22\text{k}\Omega$ ,  $R_2=1\text{k}\Omega$ ,  $R_3=1\text{k}\Omega$ ,  $R_4=47\text{k}\Omega$ ,  $C=1\text{nF}$ .

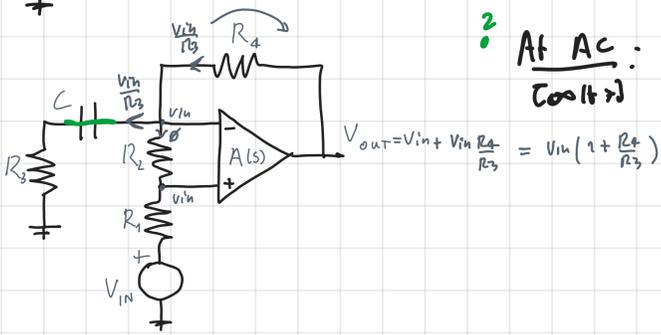
- a) Plot the Bode diagram of the **ideal** and **real**  $v_{\text{OUT}}(f)/v_{\text{IN}}(f)$ .
- b) Compute the range of GBWP values that guarantees stability with a P.M. better than  $90^\circ$ .

a)



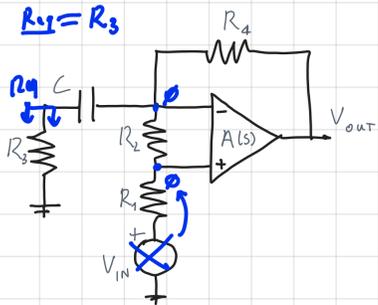
1. At DC :  $G(0) = \frac{V_{\text{out}}(0)}{v_{\text{in}}} = +1$

So there's a pole



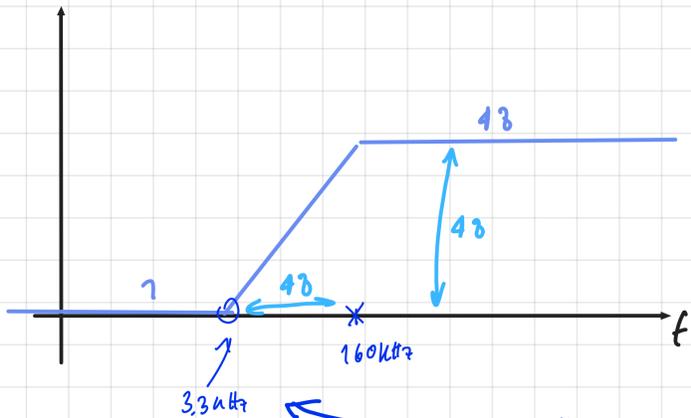
2. At AC :  $G(\infty) = \frac{V_{\text{out}}(\infty)}{v_{\text{in}}} = \left(1 + \frac{R_4}{R_2}\right) = +48$

Pole and zero:



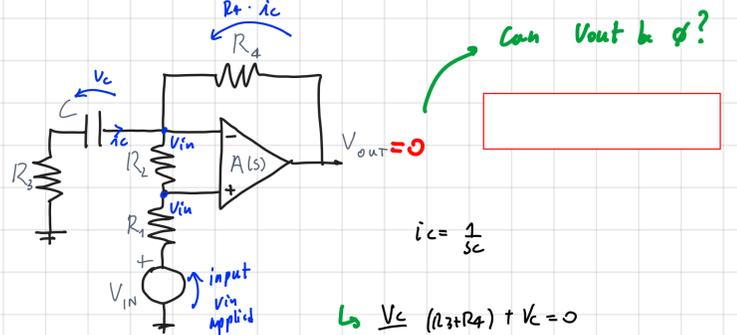
• pole =  $\frac{1}{2\pi C R_3} = \frac{1}{2\pi} = 160\text{kHz}$

Bode diagram:



We computed the zero physically but also from circuit analysis

• zero =  $\frac{1}{2\pi C (R_3 + R_4)} = 3.3\text{kHz}$



Can Vout be 0?

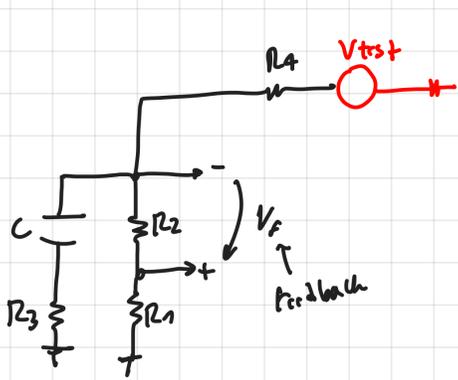
$i_c = \frac{1}{sC}$

$\hookrightarrow \frac{V_c}{sC} (R_3 + R_4) + V_c = 0$

$\hookrightarrow sC (R_3 + R_4) + 1 = 0$

$\hookrightarrow s = \frac{1}{C (R_3 + R_4)}$

b) Loop analysis:



• DC  $\rightarrow$  C open

$$\beta(0) = \frac{V_f}{V_{test}}(0) = -\frac{R_2}{R_2 + R_1 + R_4} = -\frac{1k}{22k + 47k} = -\frac{1}{69}$$

$$\hookrightarrow \frac{1}{\beta}(0) = -69$$

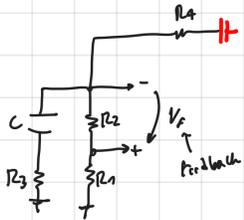
• AC  $\rightarrow$  C close

$$\beta(\infty) = \frac{V_f}{V_{test}}(\infty) = -\frac{R_2 \parallel (R_1 + R_2)}{[R_3 \parallel (R_1 + R_2)] + R_4} \cdot \frac{R_2}{R_1 + R_2} = -\frac{1k \parallel 23k}{1k \parallel 23k + 47k} \cdot \frac{1}{23} = -870 \cdot 10^{-6}$$

$$\hookrightarrow \frac{1}{\beta}(\infty) = -1151$$

• Pole and zero

$\hookrightarrow$   $V_{test}$  grounded

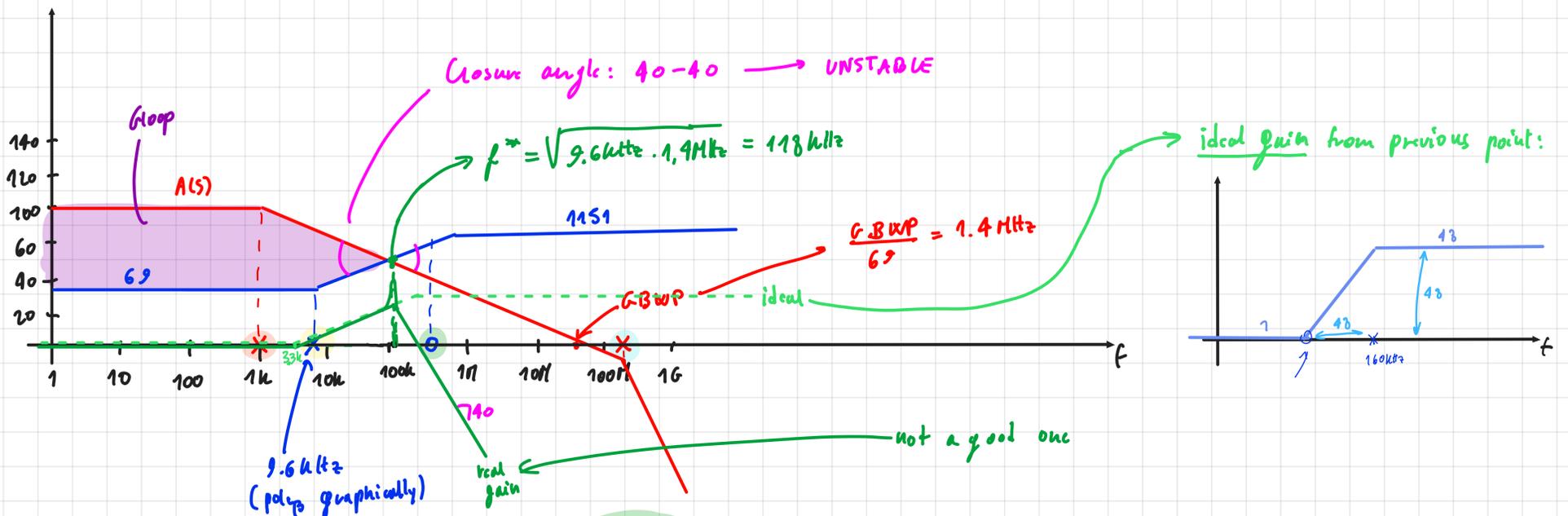


• pole  $\beta = \frac{1}{2\pi C [R_3 + R_4 \parallel (R_1 + R_2)]} = \dots$

• zero  $\beta = \frac{1}{2\pi C R_3} = 160 \text{ kHz}$

$\uparrow$  much easier to compute zero (than pole graphically)

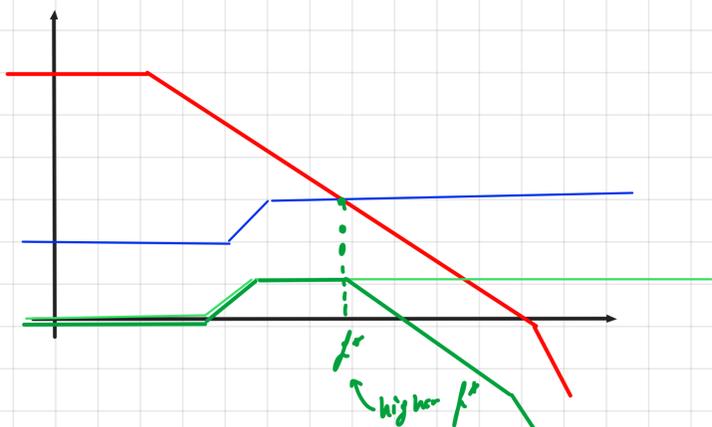
Bode diagram:



• Phase margin:  $PM = 180^\circ - 90^\circ - 90^\circ + \arctan\left(\frac{f^*}{160k}\right) - \phi = 0 + 36^\circ = 36^\circ$

$\uparrow$  negligible  
try to increase PM (but it's too late)

$\hookrightarrow$  A possible solution could be to buy another Op Amp  $\rightarrow$  different A(s)

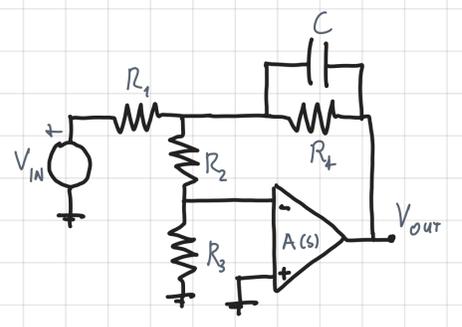
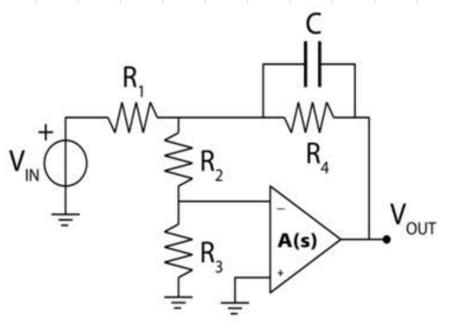


BUT NOT THAT feasible

$\hookrightarrow$  Or we can change  $\frac{1}{\beta}$   $\rightarrow$  reduce the value by removing some resistors in order to match the ideal gain



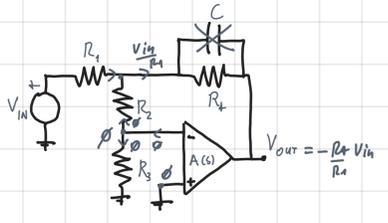
3



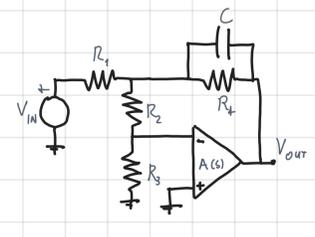
Compensated OpAmp:  $A_0=120\text{dB}$ ,  $\text{GBWP}=10\text{MHz}$ ,  $I_B=10\text{nA}$ ,  $V_{OS}=5\text{mV}$ .  $R_1=47\text{k}\Omega$ ,  $R_2=33\text{k}\Omega$ ,  $R_3=22\text{k}\Omega$ ,  $R_4=680\text{k}\Omega$ ,  $C=330\text{pF}$ .

- a) Plot the real  $v_{out}(f)/v_{in}(f)$  gain and comment stability.
- b) Compute the output static errors due to the OpAmp.
- c) Let the OpAmp be a Norton Amplifier instead, with  $A_1=5$ , compute the  $v_{out}(f)/v_{in}(f)$  gain.

a)



1 DC  $G(0) = -\frac{R_F}{R_1} = -\frac{R_4}{R_1} = -14.5$



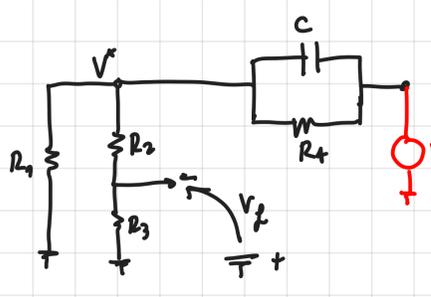
2 AC  $G(\infty) = 0$

Pole:  $\text{pole} = \frac{1}{2\pi C R_F} = 710\text{Hz}$



B computation:

DC  $\rightarrow$  Copan



$\beta(0) = -\frac{R_1 \parallel (R_2 + R_3)}{R_1 \parallel (R_2 + R_3) + R_4} \cdot \frac{R_3}{R_3 + R_2} = -0.014 \rightarrow \frac{1}{\beta}(0) = -71$

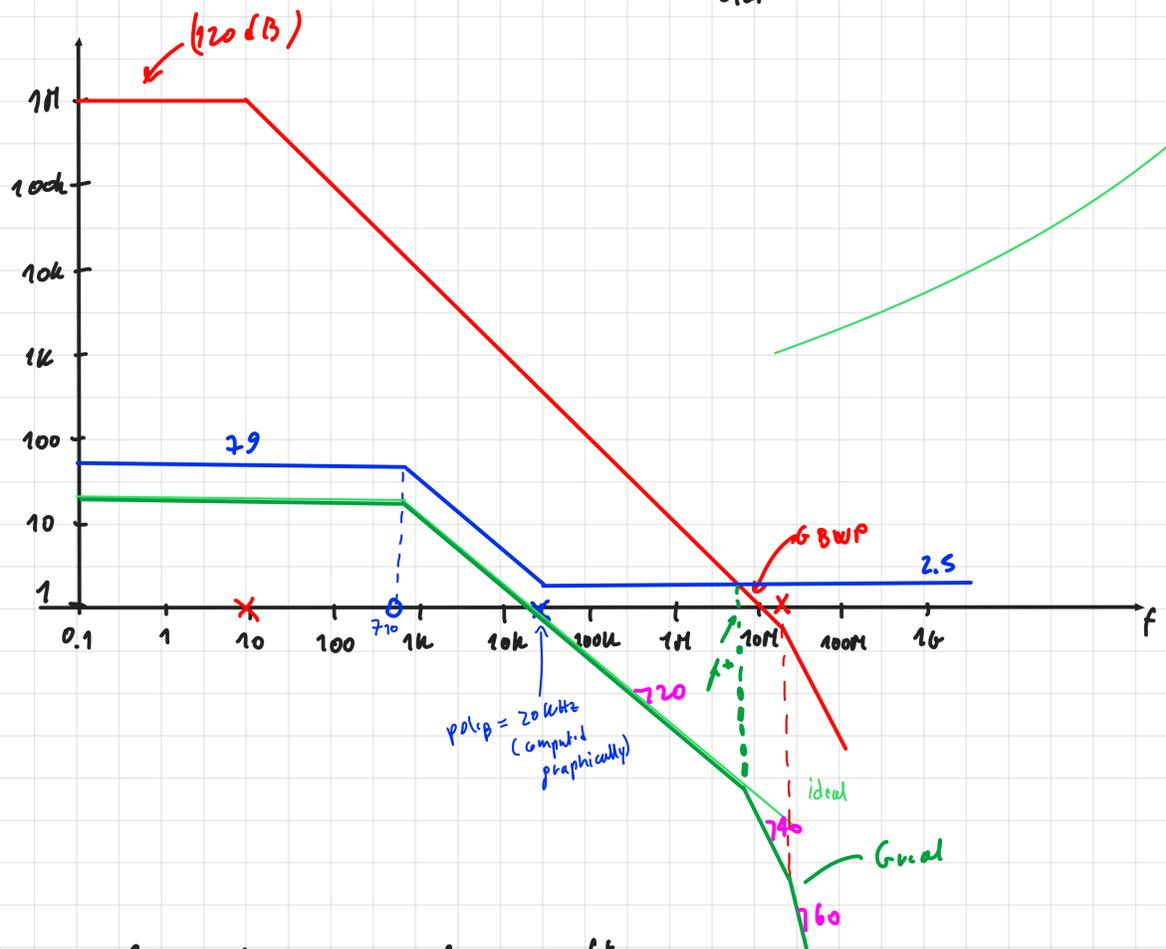
AC  $\rightarrow$  C close

$\beta(\infty) = -\frac{R_3}{R_3 + R_2} = -0.4$

$\rightarrow \frac{1}{\beta}(\infty) = 2.5$

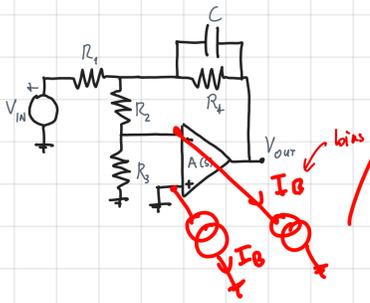
- Pole and zero:  $\text{pole}_\beta = \frac{1}{2\pi C \{R_1 \parallel [(R_2 + R_3)]\}} = \dots$
- $\text{zero}_\beta = \frac{1}{2\pi C R_1} = 710\text{Hz} \leftarrow \text{earlier}$

Bode diagram:



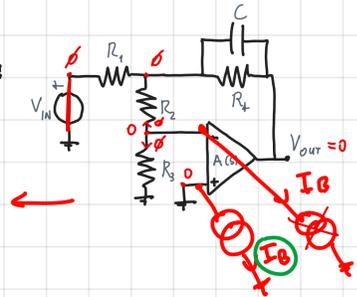
$\text{PM} = 180^\circ - 90^\circ + 90^\circ - 90^\circ - \arctan\left(\frac{f}{f_2}\right) = 90^\circ - 22^\circ = 68^\circ \rightarrow \text{STABLE}$

b) (From the part we skipped at lesson)



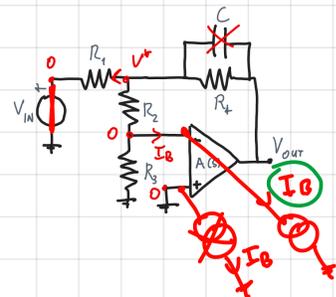
Although these current generators the input impedance are still  $\infty$   
 but there will be a leakage in the Op Amp  
 bias current  $\rightarrow$  cause errors

Effect of  $I_{B+}$ :



No effect

Effect of  $I_{B-}$ :



$$U^* = I_{B-} R_2$$

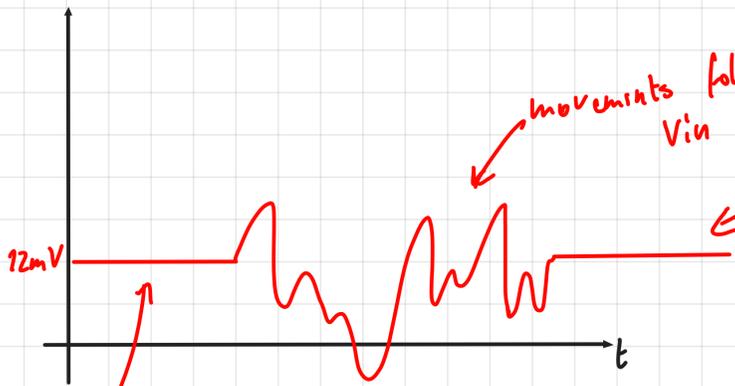
$$I_1 = \frac{U^*}{R_1} = I_{B-} \cdot \frac{R_2}{R_1}$$

$$I_2 = I_{B-}$$

$$I_4 = I_1 + I_2 = I_{B-} \left(1 + \frac{R_2}{R_1}\right)$$

$$V_{out} = R_4 \cdot I_{B-} \left(1 + \frac{R_2}{R_1}\right) + I_{B-} \cdot R_2 = 12mV \oplus$$

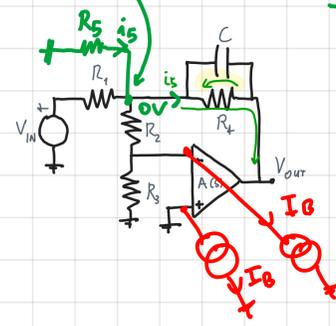
So if we look at the output of the Op Amp in time domain:



the output is not 0 when  $V_{in}$  is 0 but is at 12mV  $\oplus$

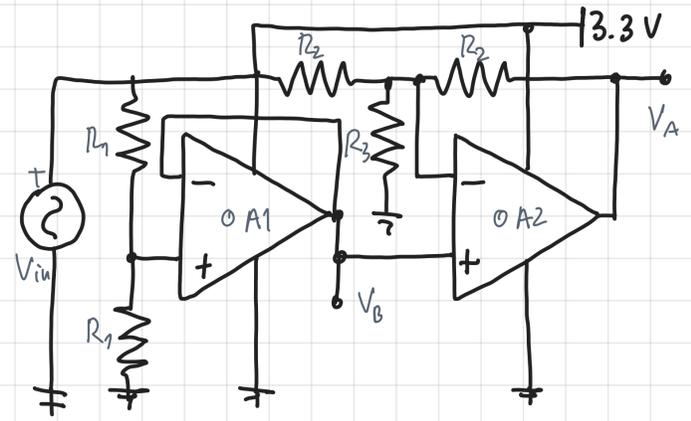
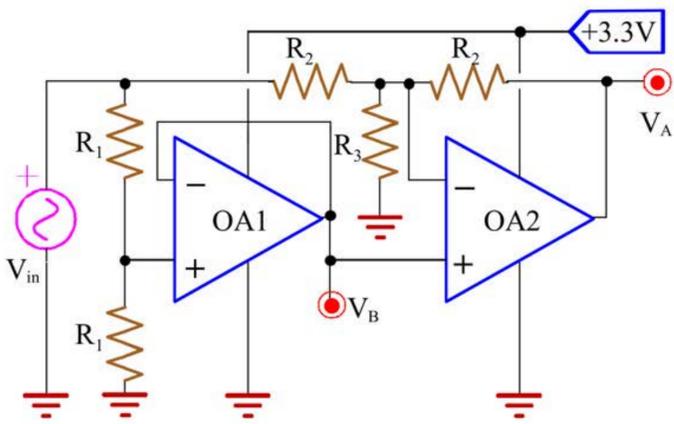
but will have this OFFSET due to bias current

to remove it we could inject some current in this node



in such a way that  $i_s$  when that node is ground, will flow on  $R_5$   
 $\rightarrow$  giving a neg. contribution to  $V_{out}$  that will counter balance that offset of 12mV given by the bias  
 $\Downarrow$   
 so that  $V_{out} = 0$  for  $V_{in} = 0$

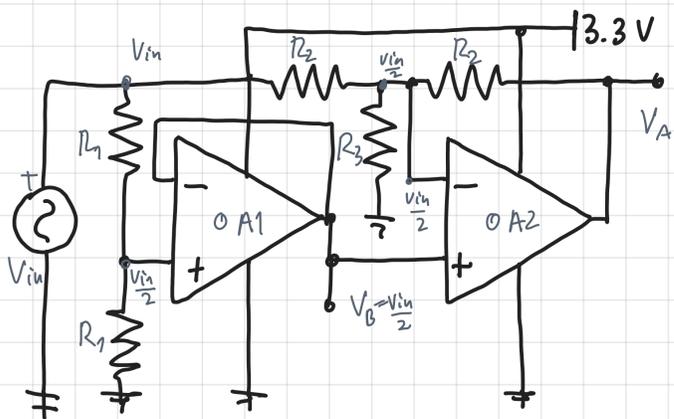
4



OpAmp with  $A_0=100\text{dB}$  and  $\text{GBWP}=100\text{MHz}$ .  $R_1=47\text{k}\Omega$ ,  $R_2=220\text{k}\Omega$ ,  $R_3=110\text{k}\Omega$ .

- a) Compute the **real**  $v_A(f)/v_{in}(f)$  and  $v_B(f)/v_{in}(f)$  gains and the input impedance.
- b) Compute the output static error on  $V_A$ , due to  $I_B=10\text{nA}$  and  $V_{OS}=5\text{mV}$  of both OpAmps.

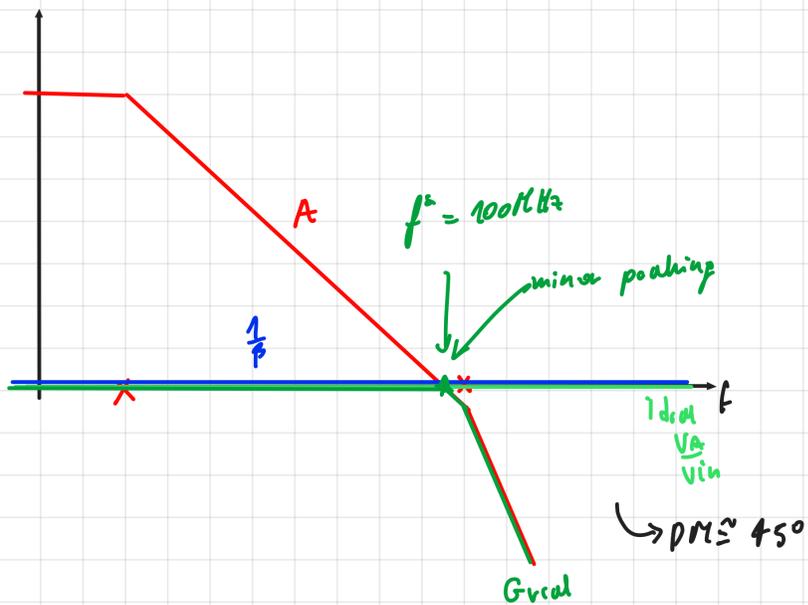
a)



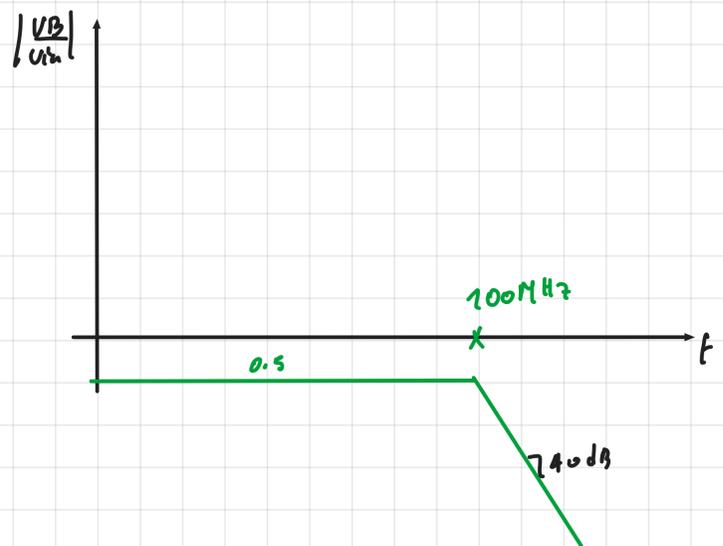
$$V_A \rightarrow \left( \frac{v_{in}}{2} + \frac{v_{in} - V_{in}}{R_2} \right) R_2 + \frac{v_{in}}{2} = v_{in} \left[ \frac{R_2}{2R_3} - \frac{R_2}{R_2} + 1 \right]$$

$$\hookrightarrow \text{Gain}_{V_A} = \frac{V_A}{V_{in}} = \frac{1}{2} \left( 1 + \frac{R_2}{R_3} - 1 \right) = \frac{1}{2} \frac{R_2}{R_3} = 1 \quad (\text{BUFFER ACTION})$$

Bode diagram:

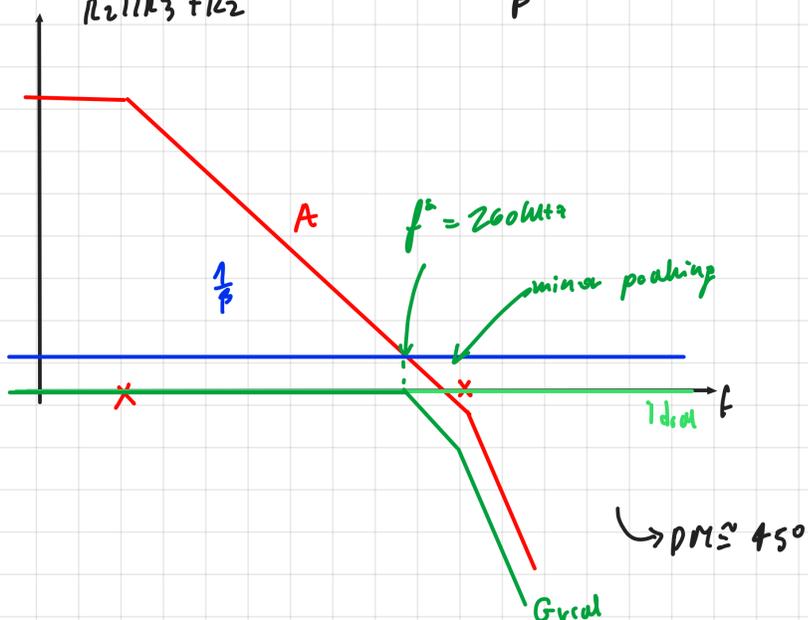


$$\text{Gain}_{V_B} = \frac{V_B}{V_{in}} = \frac{1}{2} = 0.5$$

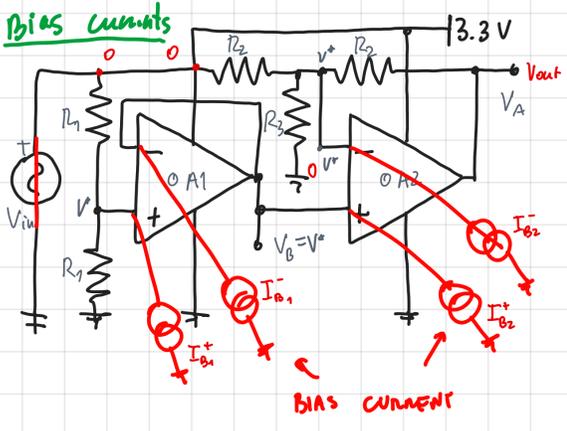


b) computation:

$$\beta = \frac{R_2 || R_3}{R_2 || R_3 + R_2} = 0.26 \quad \rightarrow \quad \frac{1}{\beta} = 3.9$$



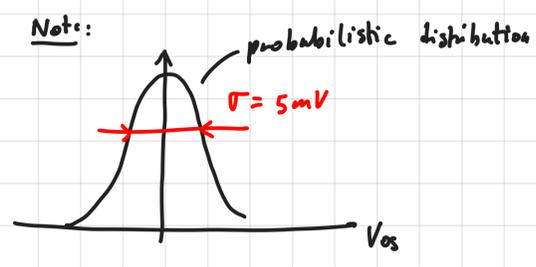
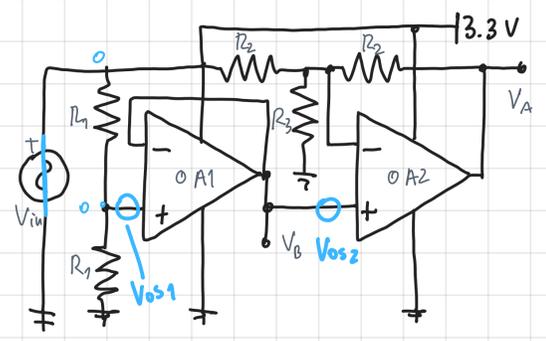
b) Bias currents



$I_{B1+} \rightarrow V^+ = I_{B1+} (R_1 || R_2) \cdot \left( \frac{R_2 || R_3 + R_4}{R_2 || R_3} \right) \left( 1 + \frac{R_2}{R_2 || R_3} \right) = 0.9 \text{ mV}$   
 $I_{B1-} \rightarrow I_{B1-} \text{ (no effect)}$   
 $I_{B2+} \rightarrow I_{B2+} \text{ (no effect)}$   
 $I_{B2-} \rightarrow I_{B2-} \cdot 220k = 2.2 \mu\text{V}$

$\rightarrow V_{out \xi} \approx 0.9 \text{ mV}$   
 ↑ error

• Offset → due to mismatch of components (e.g. diff. b/w transistors that make the op Amp → see slides skipped for ref.)

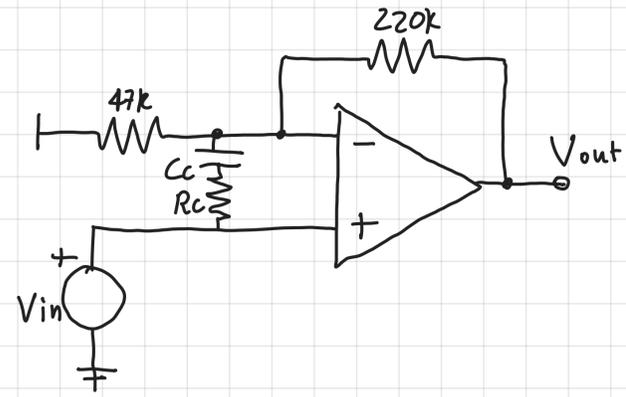
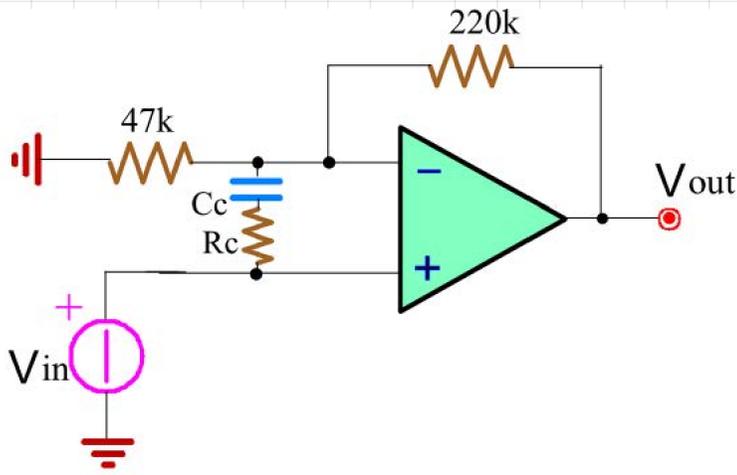


↓  
 We simply model this mismatch like a voltage offset

$V_{os1}$  (first contribution)  
 $V_{out \xi} = V_{os1} \left( 1 + \frac{220k}{220k || 110k} \right) = V_{os1} \cdot 3.9 = \pm 18 \text{ mV}$   
 $V_{os2}$  (second contrib.)  
 $V_{out \xi} = V_{os2} \left( 1 + \frac{220k}{220k || 110k} \right) = \dots = \pm 18 \text{ mV}$

$\rightarrow V_{out} = \pm 18 \text{ mV} \pm 18 \text{ mV} = \pm 36 \text{ mV}$   
 we have to consider the whole span  
 only by chance they will cancel out

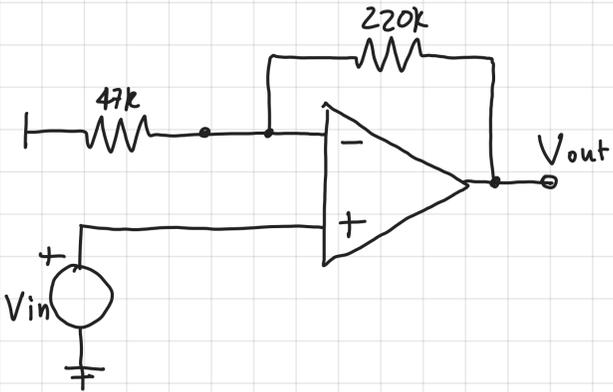
5



Uncompensated OpAmps with  $A_0=120\text{dB}$ ,  $f_0=5\text{kHz}$  and  $f_1=50\text{MHz}$ ,  $I_B=1\text{nA}$  and  $V_{OS}=3\text{mV}$ .

- a) Without  $C_C$  and  $R_C$ , compute stability and PM.
- b) Properly size  $C_C$  and  $R_C$  to attain  $\text{PM}=90^\circ$ .

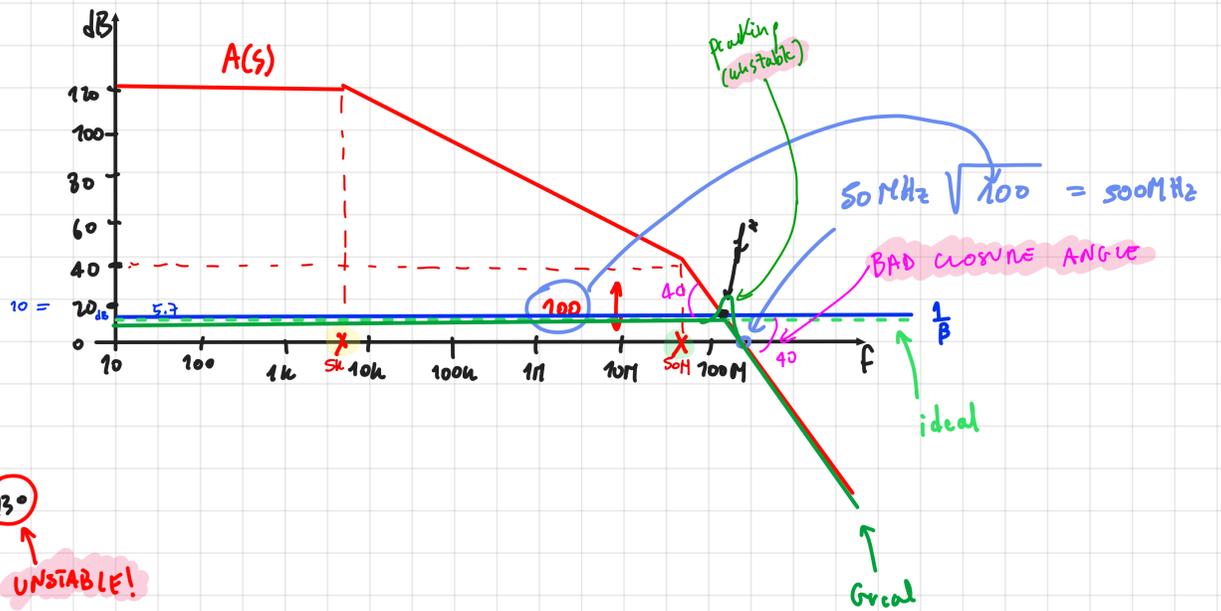
a)



→ Non-inverting

$$G_{\text{ideal}} = 1 + \frac{R_2}{R_1} = 1 + \frac{220\text{k}}{47\text{k}} = 5.7$$

Bode:



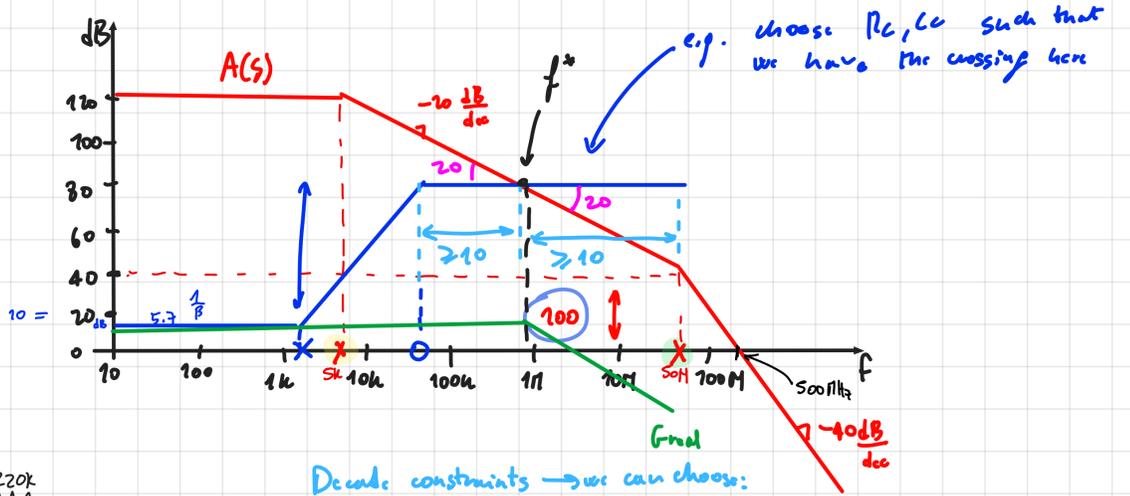
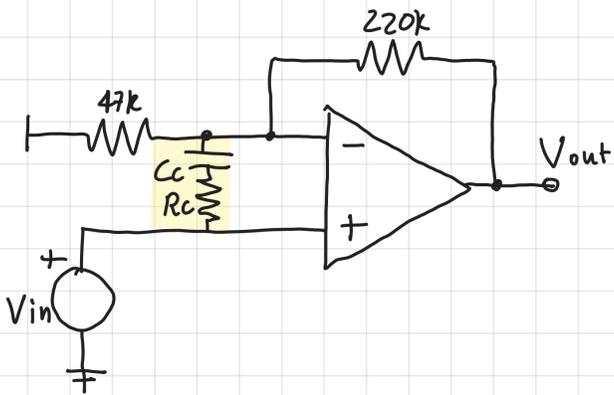
$$f^* = 50\text{MHz} \cdot \sqrt{\frac{100}{5.7}} = 20\text{MHz}$$

Phase margin:

$$\text{PM} = 180^\circ - 90^\circ - \arctan\left(\frac{f^*}{50\text{MHz}}\right) = 90^\circ - \arctan\left(\frac{20\text{MHz}}{50\text{MHz}}\right) = 90^\circ - 77^\circ = 13^\circ$$

b)

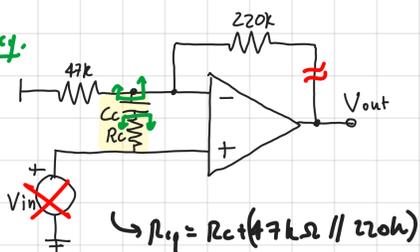
Consider now:



$$z_{\text{zero } \beta} = \frac{1}{2\pi R_C C_C}$$

$$p_{\text{pole } \beta} = \frac{1}{2\pi C_C (R_C \parallel 47\text{k} \parallel 220\text{k})}$$

→ Ref.



$$R_{\text{eq}} = R_C \parallel 47\text{k} \parallel 220\text{k}$$

$$f^* = \frac{50\text{MHz}}{10} = 5\text{MHz}$$

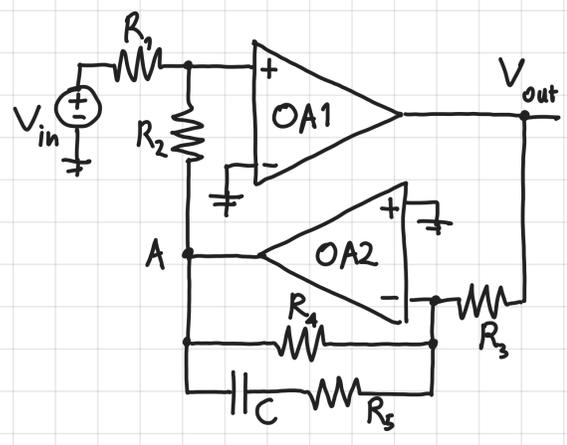
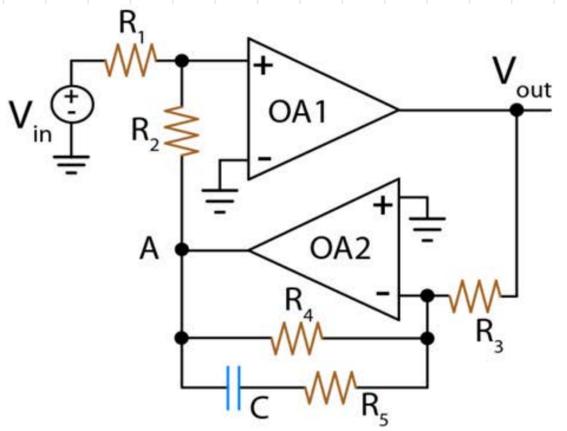
$$\text{e.g. } \frac{1}{\beta}(\infty) = 10 \cdot A_{\text{min}} = 10 \approx 1 + \frac{220\text{k}}{R_C}$$

$$R_C = \frac{220\text{k}}{99} \approx 220 \Omega$$

we want the zero to be at least one decade before  $f^*$

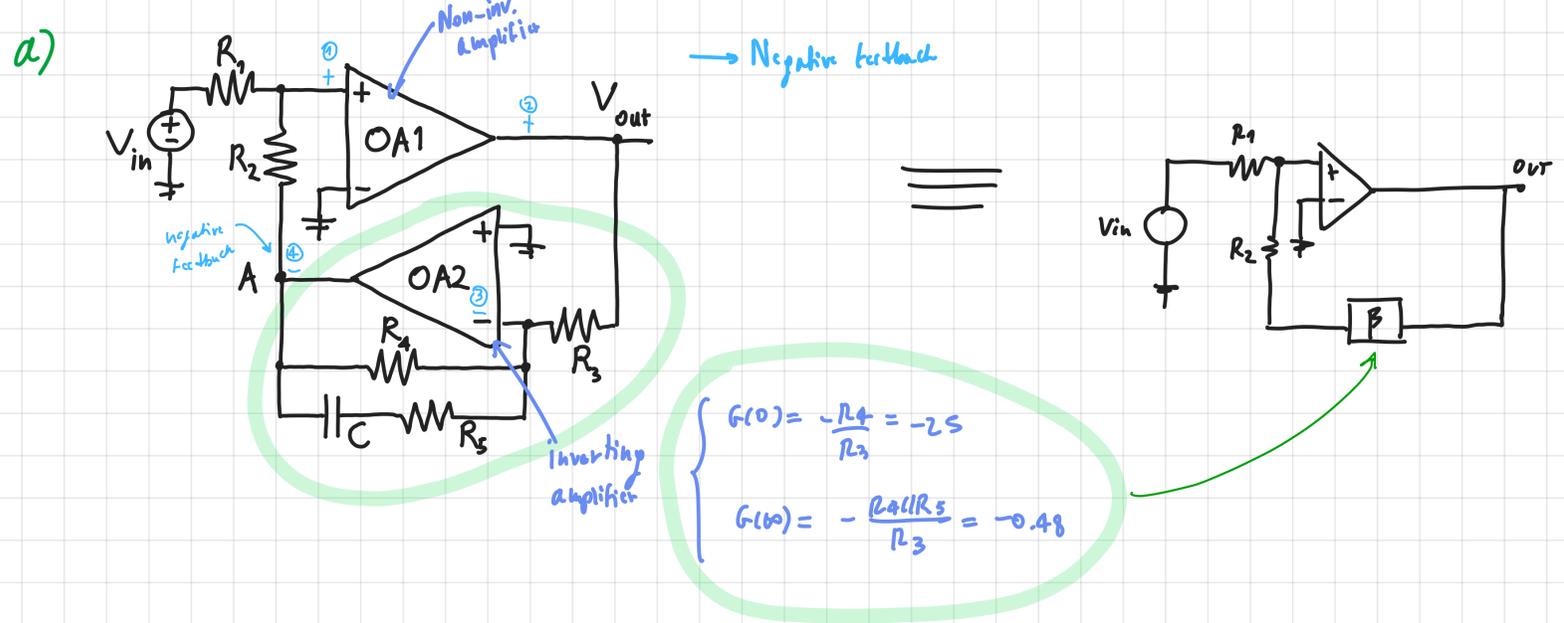
$$z_{\text{zero } \beta} = \frac{1}{2\pi C_C R_C} = \frac{f^*}{10} = 500\text{kHz}$$

$$C_C = \frac{1}{2\pi \cdot 500\text{kHz} \cdot 220\Omega} \approx 14\text{nF}$$

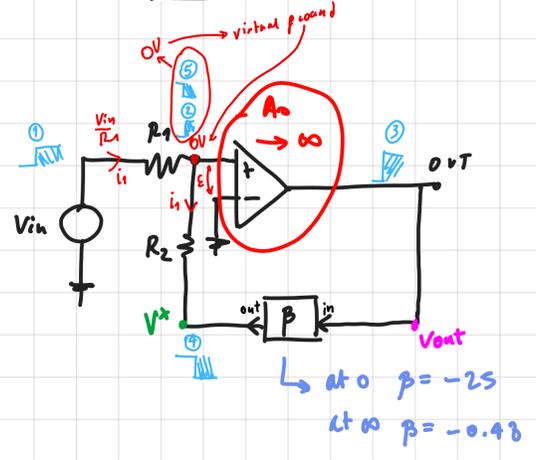


OpAmp with  $A_0=120\text{dB}$  and  $\text{GBWP}=20\text{MHz}$ , with  $I_B=10\text{nA}$  and  $V_{OS}=5\text{mV}$ .  
 $R_1=1\text{k}\Omega$ ,  $R_2=50\text{k}\Omega$ ,  $R_3=2\text{k}\Omega$ ,  $R_4=50\text{k}\Omega$ ,  $R_5=1\text{k}\Omega$ ,  $C=10\text{nF}$ .

- a) Plot the Bode diagram of the  $v_{out}(f)/v_{in}(f)$  real gain, when OA2 is still ideal.
- b) Discuss circuit stability when also OA2 is real.
- c) Compute the output error due to bias currents and offset voltages of both OpAmps.



Gain Computation:



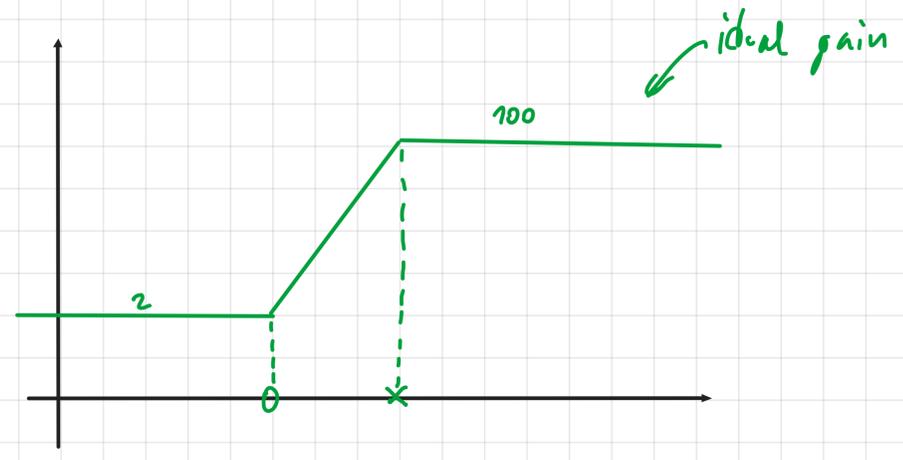
$$V^* = -V_{in} \cdot \frac{R_2}{R_1} = -V_{in} \cdot \frac{50\text{k}}{1\text{k}} = -V_{in} \cdot 50$$

$$V^* = -\beta V_{out} \Rightarrow \begin{cases} V_{out}(0) = -\frac{V^*}{25}(0) \\ V_{out}(\infty) = -\frac{V^*}{0.48}(0) \end{cases}$$

at 0  $\beta = -25$   
 at  $\infty$   $\beta = -0.48$

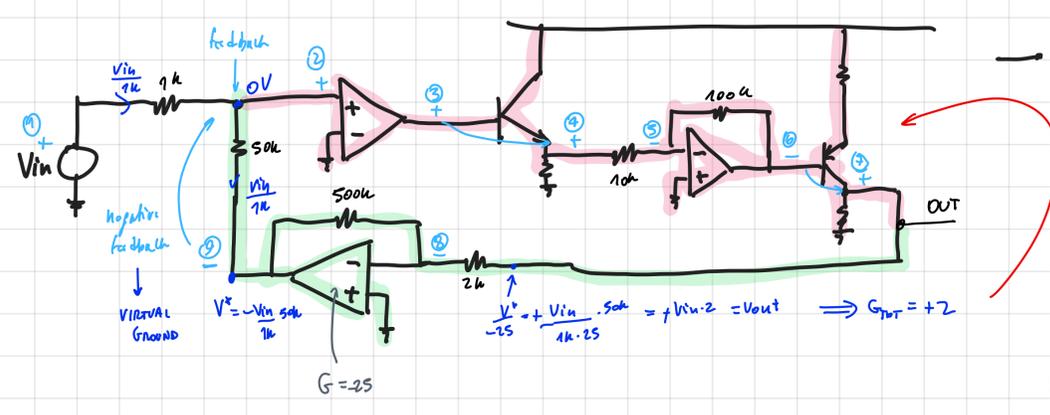
$$G_{DC} = \frac{V_{out}(0)}{V_{in}} = \frac{-50}{-25} = +2$$

$$G_{AC} = \frac{V_{out}(\infty)}{V_{in}} = \frac{-50}{-0.48} \approx +100$$



ex.

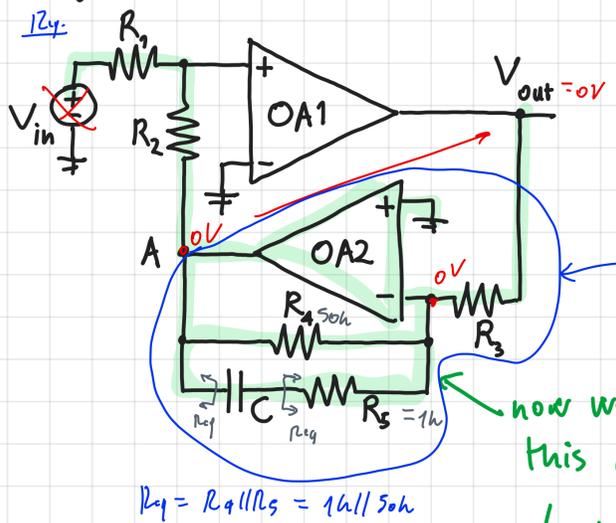
Consider a different circuit to understand how to analyze the negative feedback



we don't care about this direction thanks to the feedback we can derive to  $V_{out}$  without passing through that network

What matters is the  $\beta$  network from the output  $\rightarrow$  input

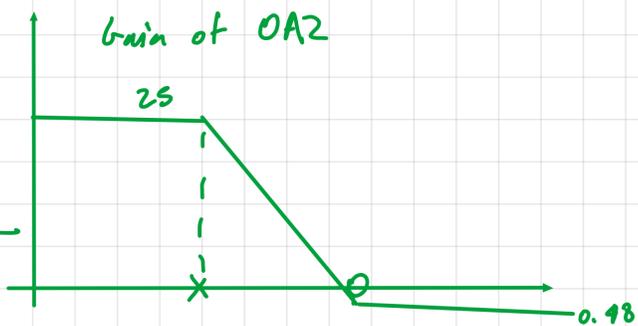
b) Going back to our circuit



$$\rightarrow \text{pole} = \frac{1}{2\pi C (50k + 1k)} = 312 \text{ Hz}$$

$$\rightarrow \text{zero} = 312 \cdot \frac{25}{0.48} = 16 \text{ kHz}$$

Gain of OA2

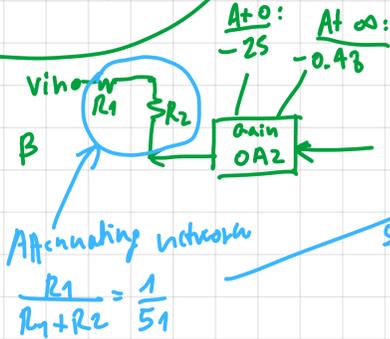


now we consider this as  $\beta$

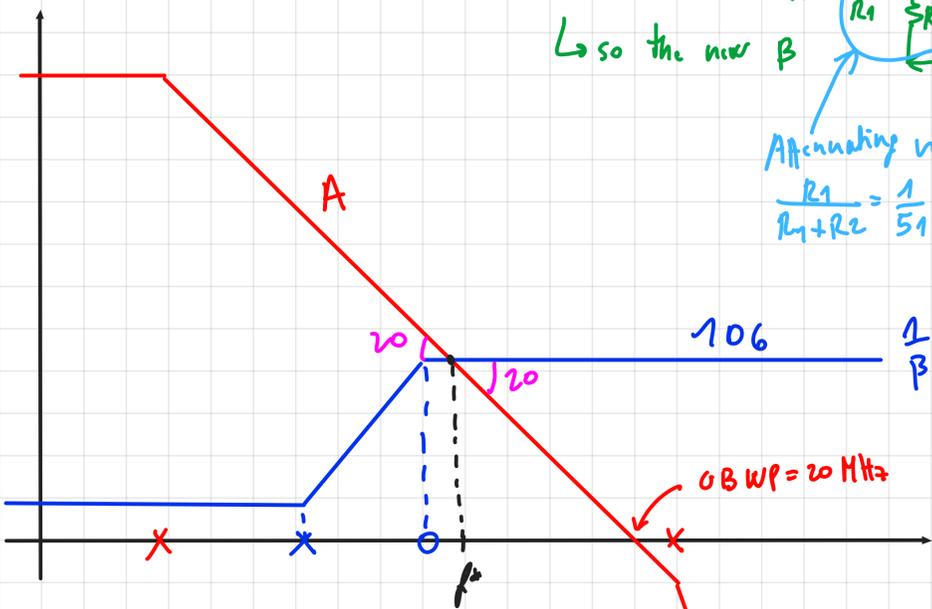
↳ before we saw that this

has this characteristic

↳ so the new  $\beta$



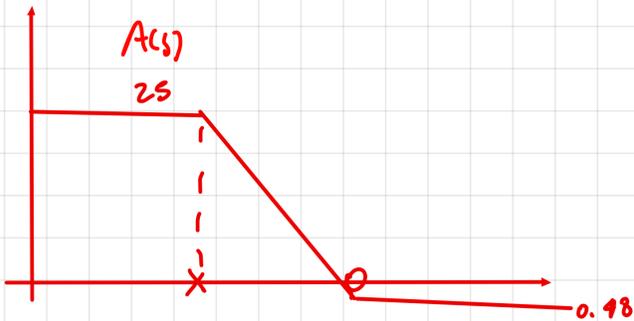
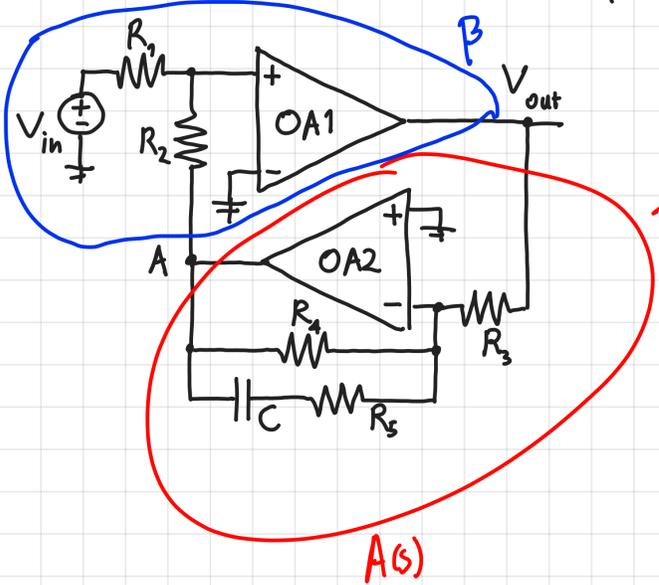
Bode



$$\rightarrow f^* = \frac{\text{GBWP}}{106} = 189 \text{ kHz}$$

Note:

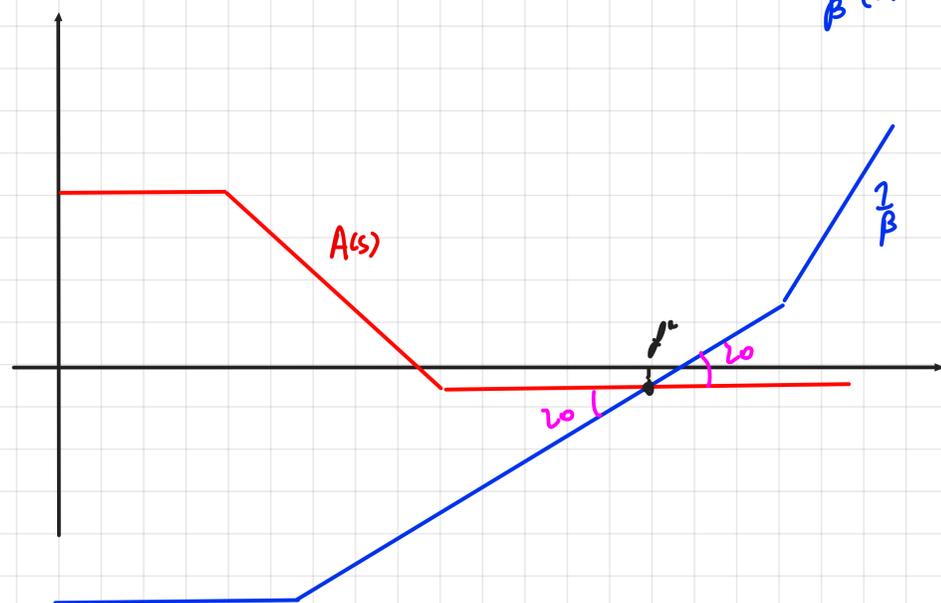
↳ Suppose now we choose different  $\beta$  and  $A(s)$



$$\beta(s) = \frac{R_1}{R_1 + R_2} \cdot A(s) = \begin{cases} \text{DC} & \frac{1}{51} \cdot 1000000 = 19 \cdot 10^3 \\ \text{AC} & \frac{1}{51} \end{cases}$$

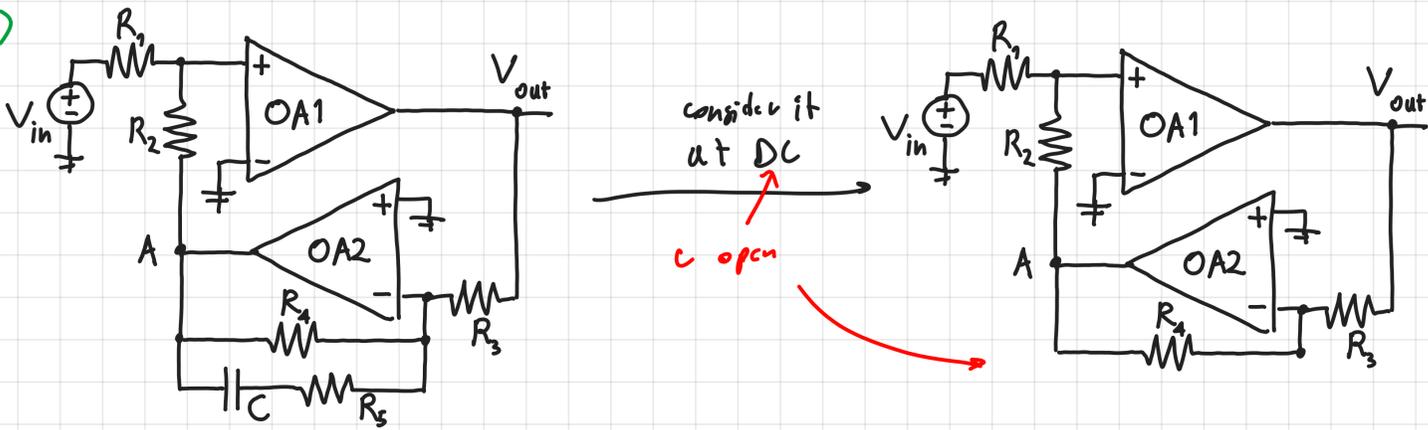
$$\rightarrow \frac{1}{\beta}(s) = \begin{cases} \text{DC} & 51 \cdot 10^{-6} \\ \text{AC} & 51 \end{cases}$$

Bode

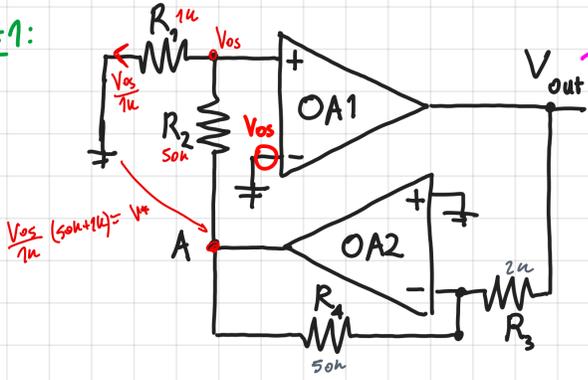


→ if we compute it  $f^*$  will be the same computed before

c)



o Offset 1:

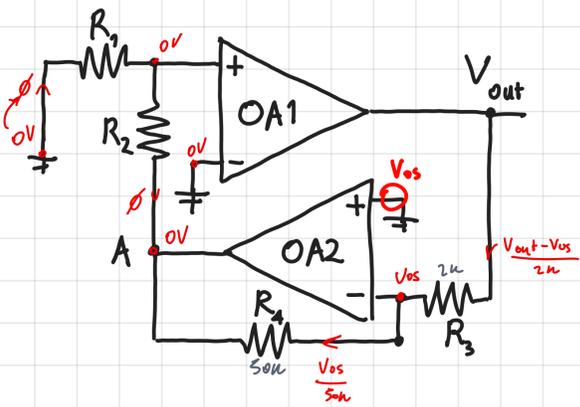


$$V_{out} = \frac{V_{os}}{1\mu} \cdot \frac{51k}{25}$$

Offset 1 contribution on the output:

$$\hookrightarrow V_{out_{os1}} \approx V_{os} \cdot 2$$

o Offset 2:



$$\frac{V_{out}}{50k} = \frac{V_{out} - V_{os}}{2k}$$

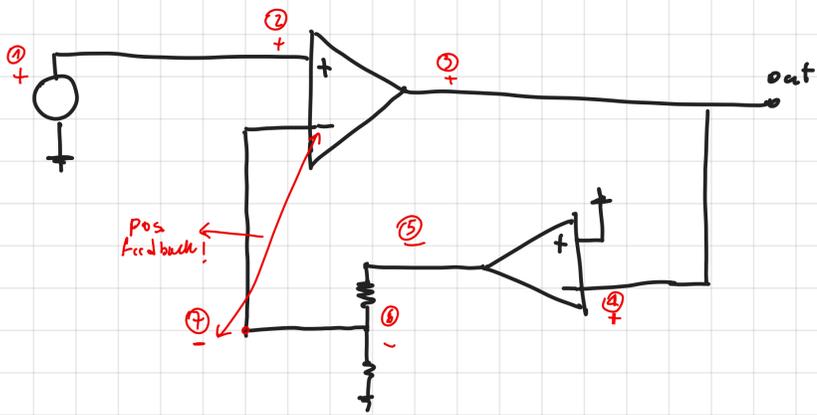
$$V_{out} \cdot \frac{50k}{50k+2k} = V_{os}$$

Offset 2 contribution on the output:

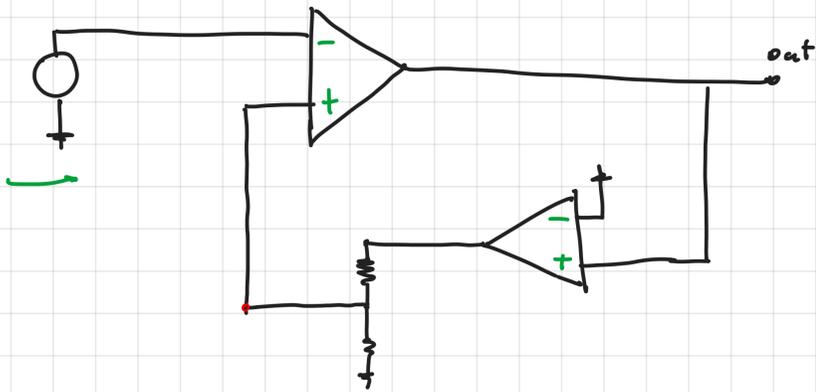
$$V_{out_{os2}} = V_{os} \cdot \frac{52}{50}$$

ex. Feedback effects

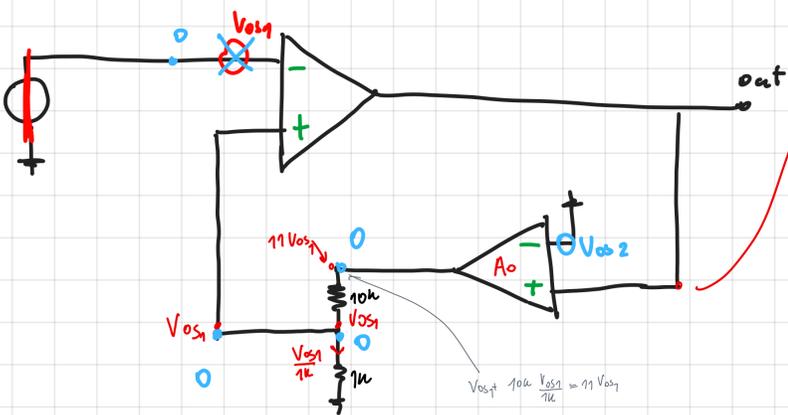
Consider the following circuit:



to make it  
neg. feedback  
we can invert  
the signs in the  
op amp



↳ If we have an offset



$$V_{out} = \frac{11 V_{os1}}{A_o} \approx 0V$$

↑  
high

$$\leftrightarrow V_{out} = V_{os2}$$

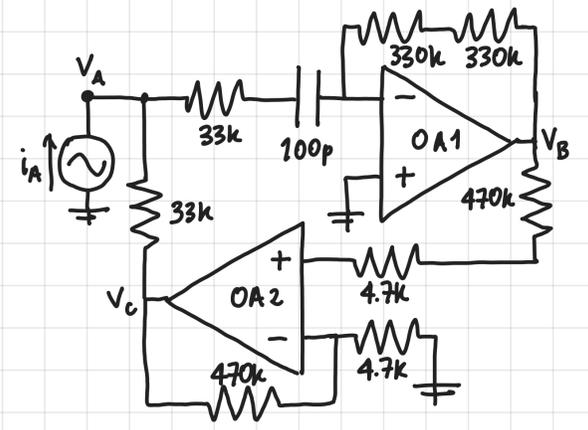
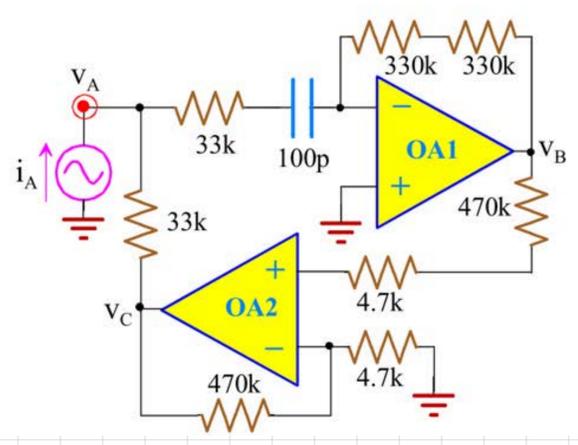
E01 (Beretta)

①

**Ex. 1**

Note that current (not voltage) input signal source. Consider both OpAmps ideal.

- a) Compute the value of the  $v_A/i_A$  ratio at DC and AC (i.e. at very high frequency) in two ways, as a gain (output/input=...) and as an input impedance ( $Z_{in}$ =...).
- b) Compute poles and zeros and plot the Bode diagram of  $v_A(f)/i_A(f)$ , when both OpAmps are still ideal.



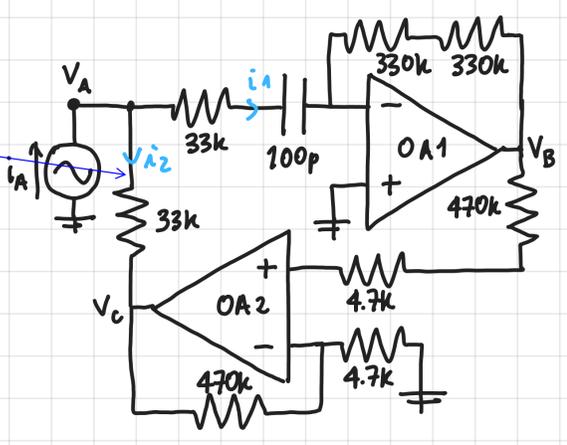
a)  $\frac{v_A}{i_A}$  (I)  $\frac{\text{output}}{\text{input}}$  (II)  $Z_{in}$

@ DC  $\rightarrow C_{open}$

$$i_A = \frac{v_A}{33k} \Rightarrow \left. \frac{v_A}{i_A} \right|_{DC} = 33k$$

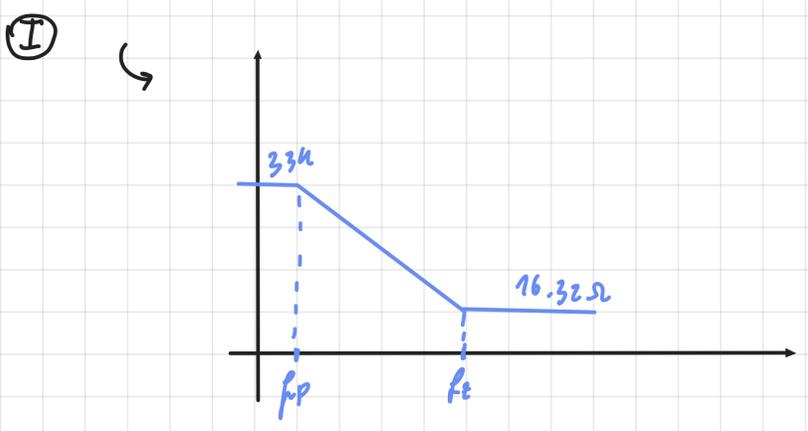
@ HF  $\rightarrow C_{short}$

$$i_A = \frac{v_A}{33k} + \frac{v_A - v_C}{33k}$$



$$\left. \begin{aligned} v_B &= v_A \left( -\frac{660k}{33k} \right) \\ v_C &= v_A \left( -\frac{660k}{33k} \right) \left( 1 + \frac{470k}{4.7k} \right) \\ v_C &= v_B \left( 1 + \frac{470k}{4.7k} \right) \end{aligned} \right\}$$

$$i_A = \frac{v_A}{33k} + \frac{v_A - v_A \left( -\frac{660k}{33k} \right) \left( 1 + \frac{470k}{4.7k} \right)}{33k}$$



$$\left. \frac{v_A}{i_A} \right|_{HF} = \frac{33k}{2022} = 16.32 \Omega$$

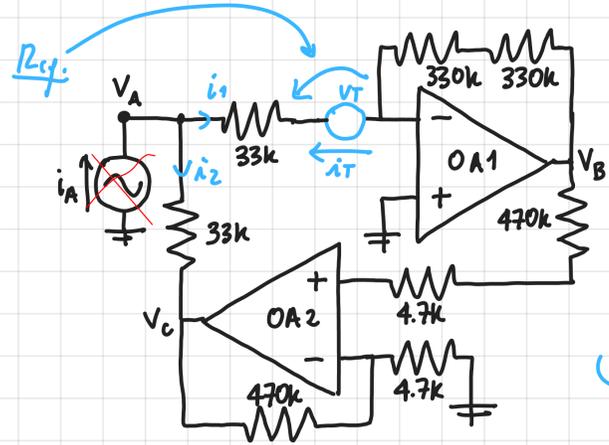
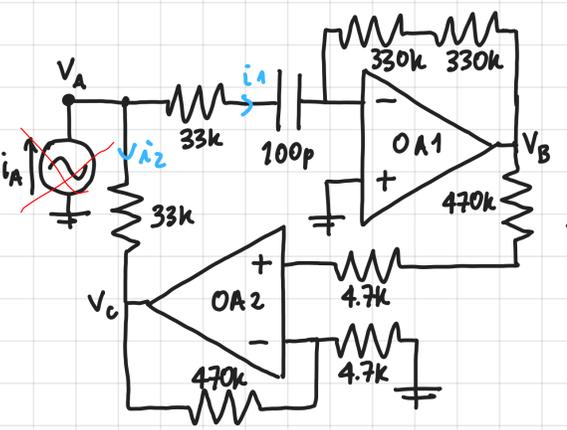
(I)  $Z_{in} = \frac{Z_{in-no feedback}}{1 - G_{loop}}$

@ DC  $\begin{cases} Z_{in,DC} = 33k \\ G_{loop}(0) = 0 \end{cases}$

@ HF  $\begin{cases} Z_{in,HF} = 33k // 33k \\ G_{loop}(\infty) = \left( 1 + \frac{470k}{4.7k} \right) \left( -\frac{660k}{66k} \right) = -1010 \end{cases}$

$\hookrightarrow Z_{in,HF} = \frac{33k // 33k}{1 - (-1010)} = 16.32 \Omega$

b)  $\frac{v_A}{i_A}(f) = ? \rightarrow$  Bode



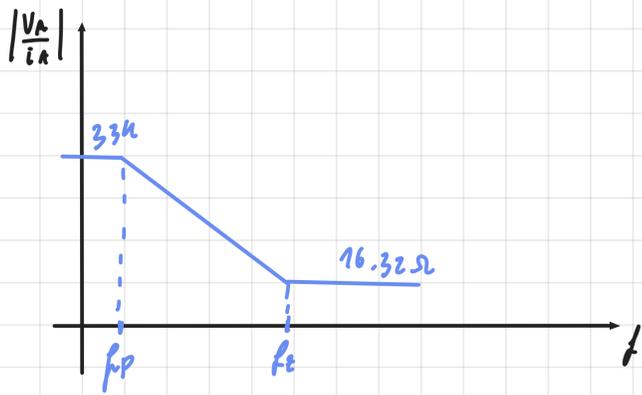
$$R_{eq} \triangleq \frac{v_T}{i_T}$$

$$v_C = v_T - i_T(66k)$$

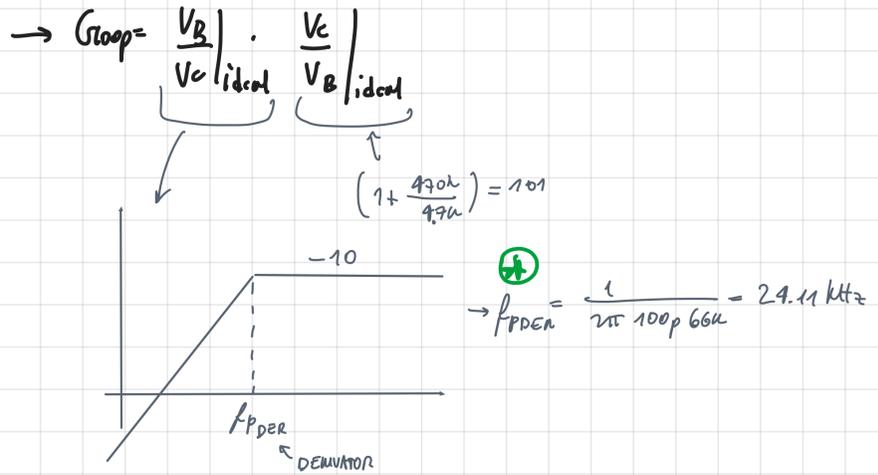
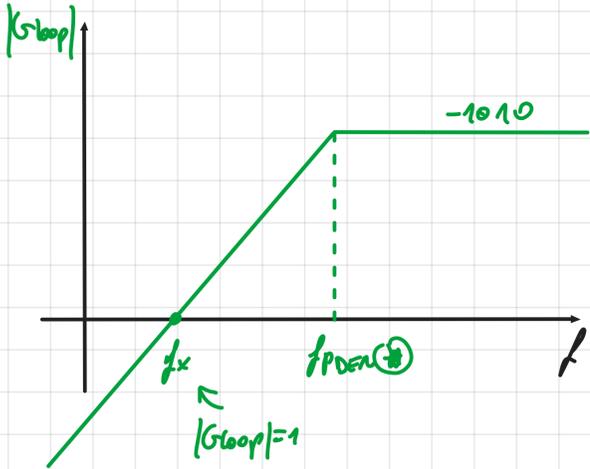
$$\begin{cases} v_C = v_B \cdot 101 \\ v_B = i_T 660k \end{cases} \Rightarrow v_C = i_T 660k \cdot 101$$

$$\hookrightarrow i_T 660k \cdot 101 = v_T - i_T(66k) \Rightarrow R_{eq} = 6672k \Omega$$

↳ Polc:  $f_p = \frac{1}{2\pi C R_{eq}} = 23.37 \text{ Hz}$   
100p



Obs. We saw that  $G_{loop}$ :



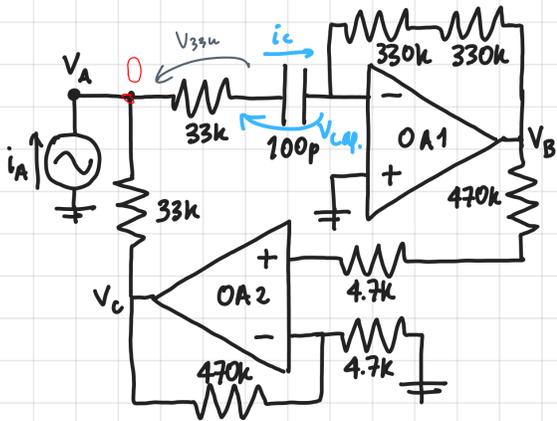
↳ We want to compute  $f_x$  in order to see when  $G_{loop}$  takes action on the circuit (i.e. when  $G_{loop} > 1$ )

→  $f_x = \frac{24.11 \text{ kHz}}{(10 \cdot 101)} = 23.87 \text{ Hz}$

↳ Methods to find  $f_z$

Ⓘ graphically  $f_z = \frac{z_{in}(0)}{z_{in}(\infty)} f_p = 48.22 \text{ kHz}$

Ⓜ compute it from the circuit



$V_{cap} + V_{33} = 0$

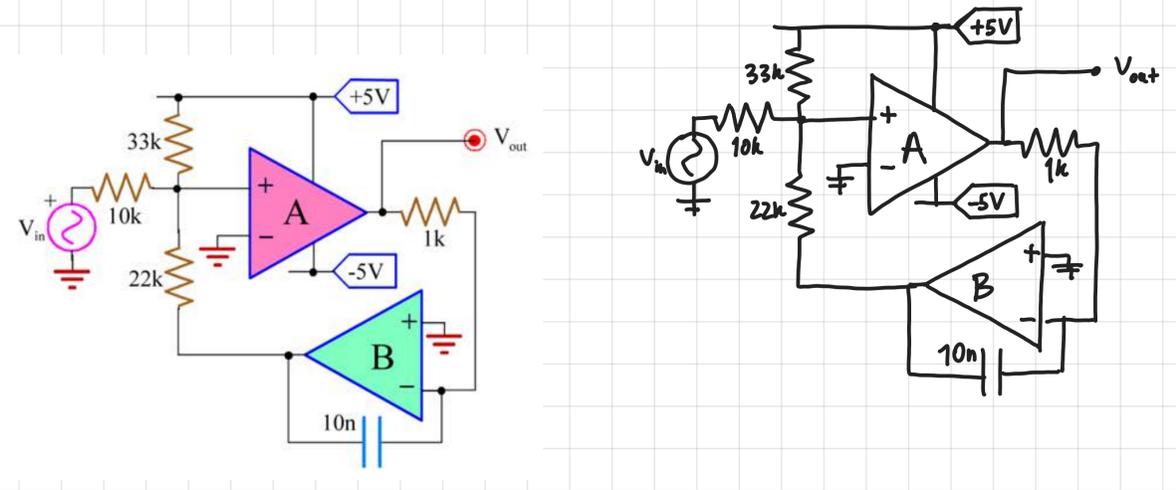
$V_{cap} + V_{cap} \cdot 33k = 0 \rightarrow s = \frac{1}{C \cdot 33k}$

↳  $f_z = \frac{1}{2\pi \cdot 100p \cdot 33k} = 48.22 \text{ kHz}$

2

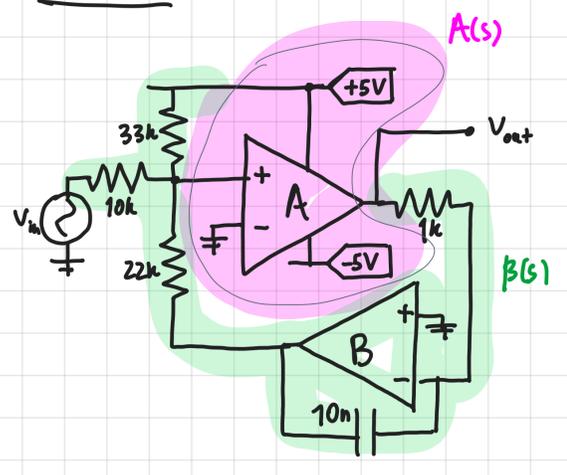
**Ex. 2**

- OpAmps with  $A_0=100\text{dB}$  and  $\text{GBWP}=50\text{MHz}$ .
- Check the stability of the stage, when both OpAmps are considered real.
  - Plot the ideal and the real  $V_{out}(f)/V_{in}(f)$  gains.
  - Propose a way to compensate the stage if needed.

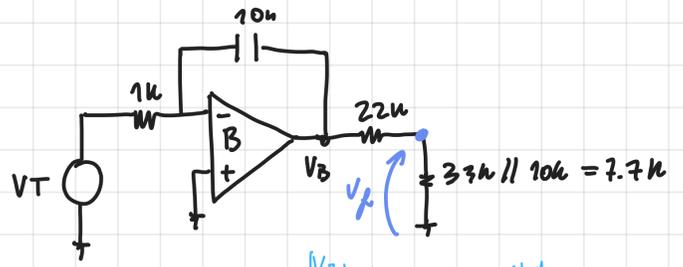


a) Stability?

Consider:

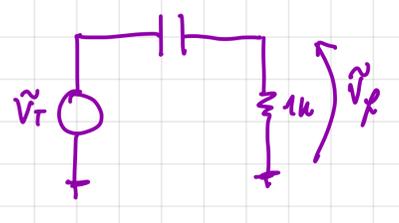
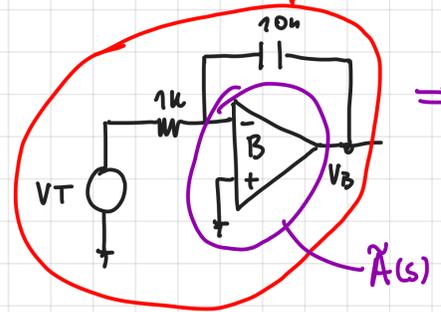
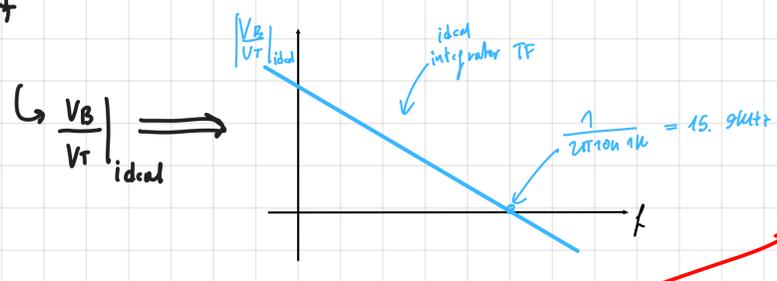


- $\beta(s)$  (switch off P.S. and all the voltage sources and apply a test volt.  $V_T$ )



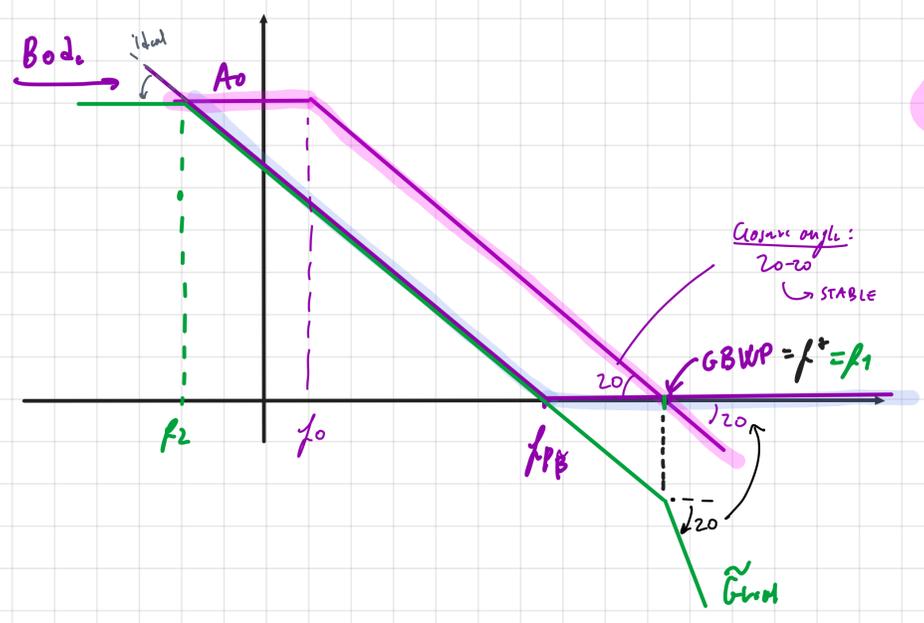
$$\beta(s) = \frac{V_B}{V_T}$$

$$\beta(s) = \frac{V_B}{V_T} \bigg|_{\text{real}} \cdot \frac{7.7\mu}{7.7\mu + 22\mu}$$



To compute  $\frac{V_B}{V_T} \bigg|_{\text{real}}$  we consider this circuit and for it we analyze again  $\tilde{A}(s)$  and  $\tilde{\beta}(s)$

$$\begin{aligned} \tilde{\beta}(0) = 0 &\longrightarrow \frac{1}{\tilde{\beta}(0)} = \infty \\ \tilde{\beta}(\infty) = 1 &\longrightarrow \frac{1}{\tilde{\beta}(\infty)} = 1 \end{aligned} \quad \left[ \right. \quad f_{p\tilde{\beta}} = \frac{1}{2\pi \cdot 10\text{n} \cdot 1\text{k}} = 15.9 \text{ kHz}$$

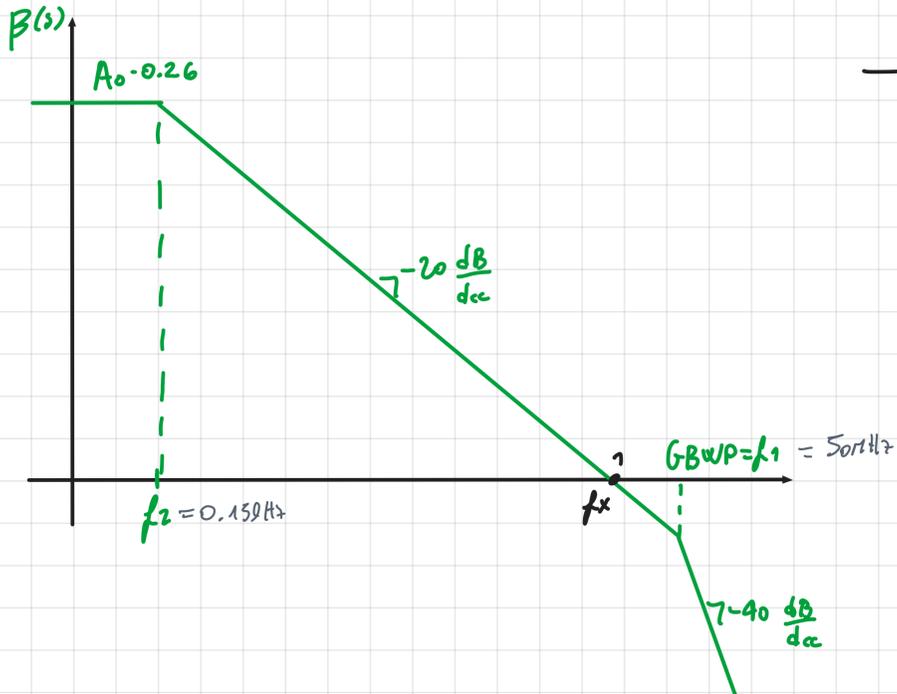


$\tilde{A}(s)$   
 $\frac{1}{\tilde{\beta}(s)}$

$$f_z = \frac{f_{p\tilde{\beta}} \cdot 1}{A_0} = 0.159 \text{ Hz}$$

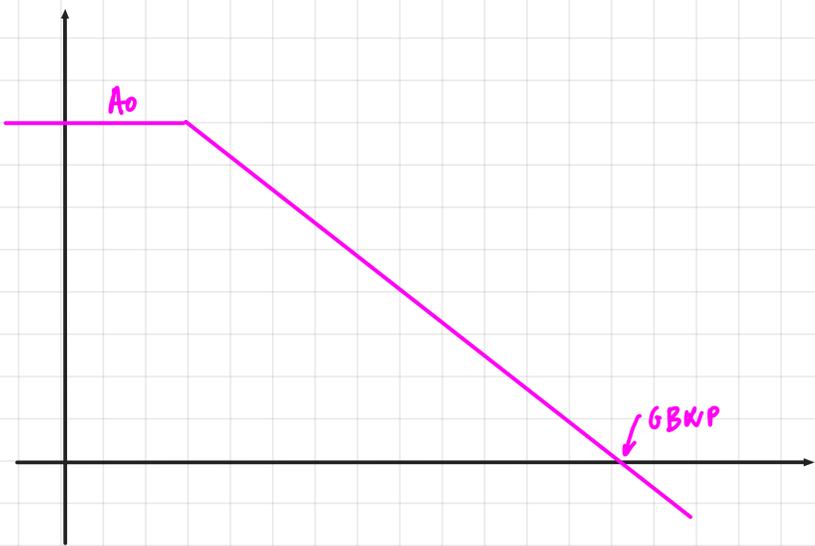
Then we can derive  $\beta(s)$  from:

$$\beta(s) = \frac{V_B}{V_T} \Big|_{\text{ideal}} \cdot \frac{7.7k}{7.7k + 22k} = 0.26$$

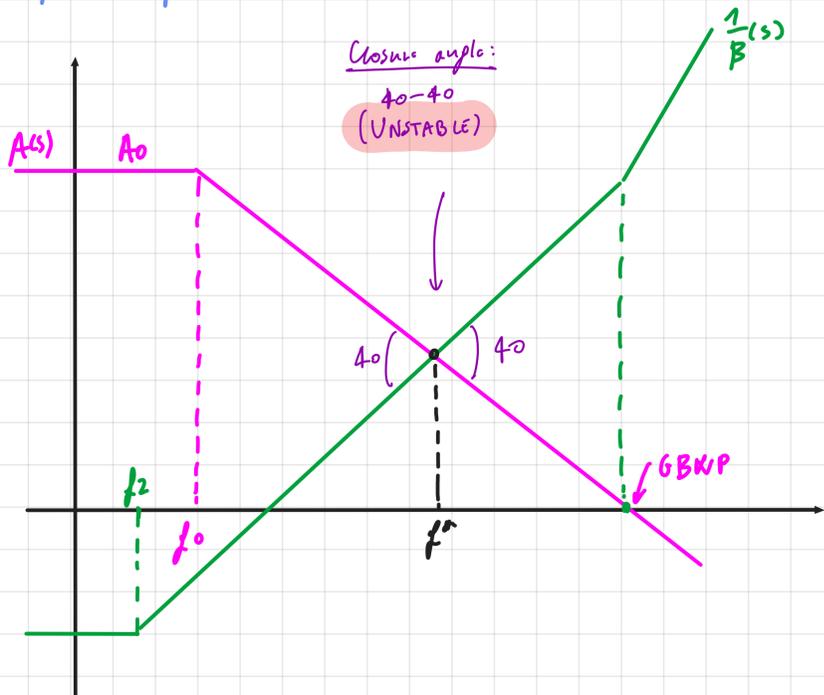


$$f_x = A_0 \cdot 0.26 \cdot f_2 = 4.1 \text{ kHz}$$

$A(s)$

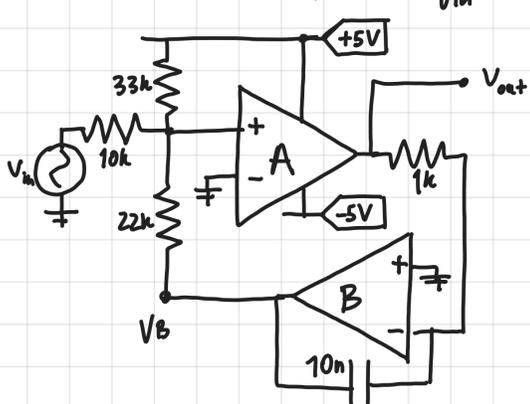


$\Rightarrow$  Bode  $G_{loop} = A(s) \cdot \beta(s)$



$$f^* = \sqrt{f_x \cdot GBWP} = 454 \text{ kHz}$$

b) Ideal and real gain  $\frac{V_{out}}{V_{in}}$



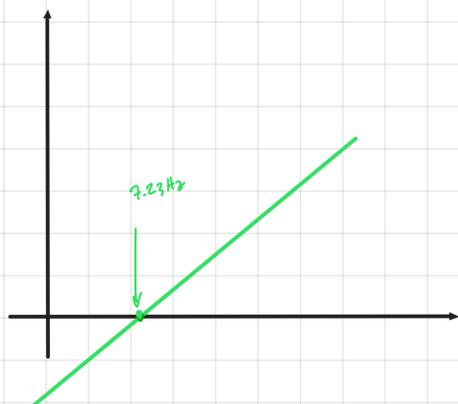
ideal

$$\frac{V_B}{V_{in}} = -\frac{22k}{10k} = -2.2$$

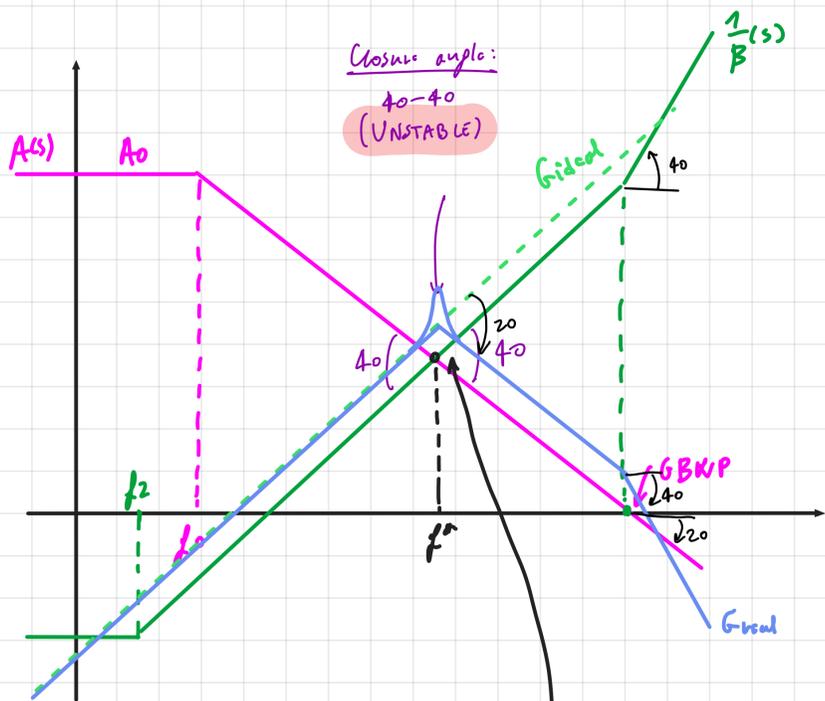
$$\frac{V_{out}}{V_B} = -5 \cdot 10n \cdot 1k$$

$$\frac{V_{out}}{V_{in}} \Big|_{\text{ideal}} = 5 \cdot 10n \cdot 2.2k \text{ (driver)}$$

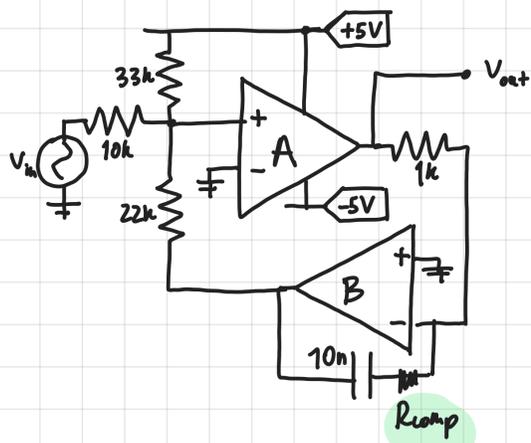
Obs.  $\frac{V_{out}}{V_{in}} \Big|_{\text{ideal}} = 1$  for  $f = \frac{1}{2\pi C \cdot 2.2k} = 7.23 \text{ kHz}$



The real gain depends on the stability of the loop:



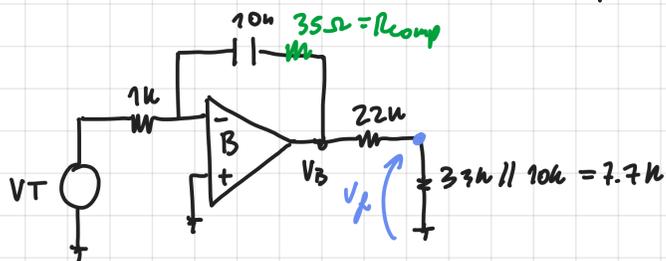
c) We would like to compensate this instability a good way could be to introduce a pole in the ideal gain in order to have a closure angle of 40-20  $\rightarrow$  marginally stable (PM=+50)



$\rightarrow R_{comp}$

$$f_{comp} = f^* = \frac{1}{2\pi R_{comp} C} \rightarrow R_{comp} = \frac{1}{2\pi C f^*} = 35 \Omega$$

We have to reperform the computations



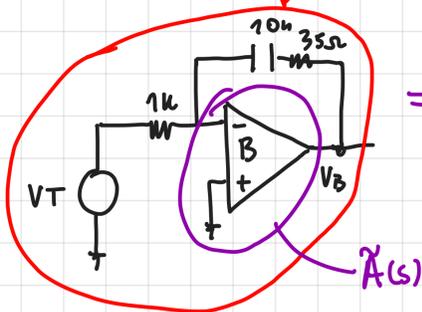
$$\beta(s) = \frac{V_B}{V_T} \Big|_{red}$$

$$0.26 \frac{7.7k}{7.7k + 22k}$$

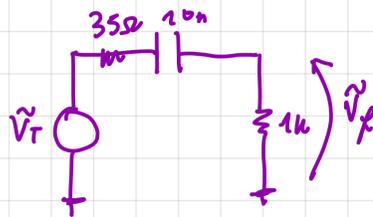
$$\tilde{\beta}(0) = 0$$

$$\tilde{\beta}(\infty) = \frac{1k}{1k + 35\Omega} = \frac{1}{1.035}$$

$$f_{P\tilde{\beta}} = \frac{1}{2\pi(1k + 35\Omega)10^{-8}} = 15.3 \text{ kHz}$$



$$\tilde{\beta}(s)$$

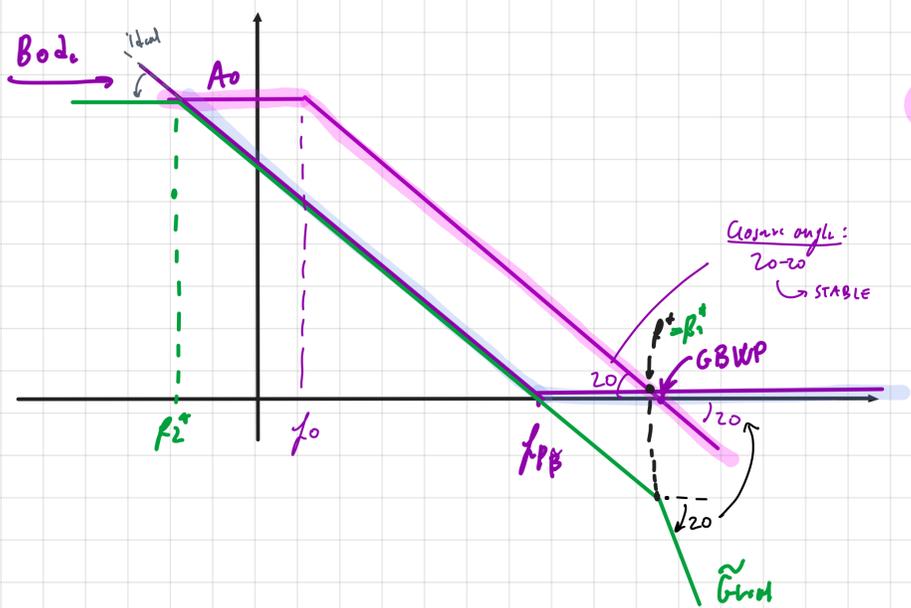


$$\tilde{A}(s)$$

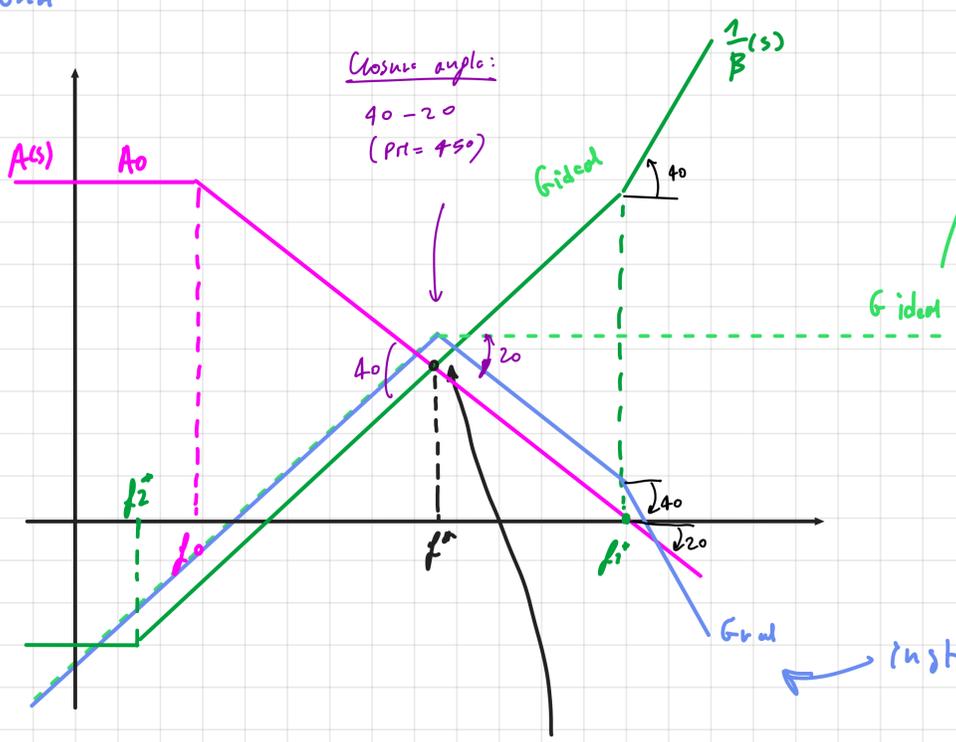
$$\frac{1}{\tilde{\beta}(s)}$$

$$f_z^* = \frac{f_{P\tilde{\beta}}}{A_0 \cdot 1.035} = 0.148 \text{ kHz}$$

$$f^* = f_1^* = \frac{GBWP}{1.035} = 48.3 \text{ MHz}$$



→  $G_{out}$



$G_{ideal} : \textcircled{HF}$

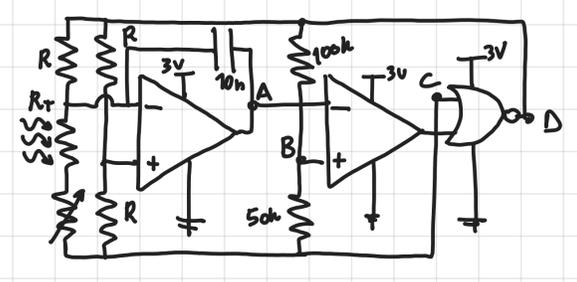
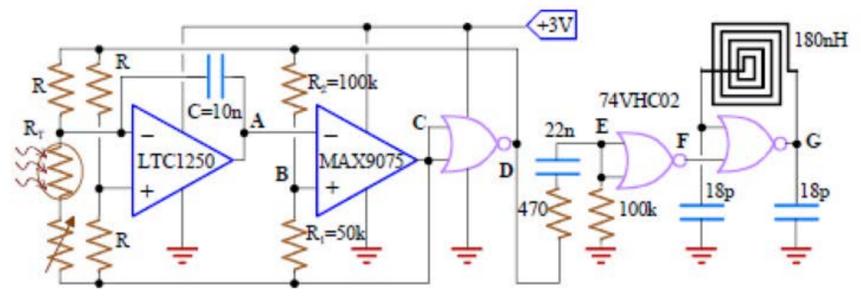
$$\frac{V_{out}}{V_{in ideal}} = 62.8$$
$$\left\{ \begin{array}{l} \frac{V_B}{V_{in}} = 2.2 \\ \frac{V_{out}}{V_B} = \frac{1k}{35} \end{array} \right.$$

→ instability has been compensated

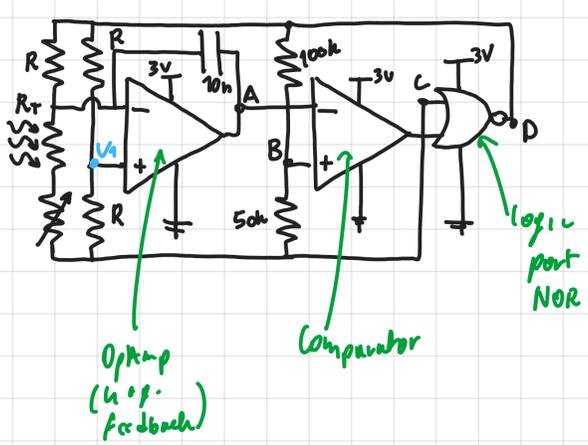
3

Ex. 3

The telemetry transmitter employs a sensor  $R_T=1k\Omega$  with a signal fluctuation  $r/R_T=0.1\%$ . The trimmer is used just to perfectly balance the bridge when no signal is applied. Hint: start when  $V_C(0)=0V$ .  
 a) Find the two thresholds of the comparator and the corresponding currents through the 10nF capacitor.  
 b) Draw the voltage waveforms at nodes A, B, C, D, and compute the oscillation period at node C as a function of the signal  $r/R_T$  and C. Say if the front end is "ratiometric" or not (i.e. the output is independent of  $V_{DD}$ ).  
 c) Draw the voltage waveforms at nodes E, F, G (hint: the last gate oscillates at 80MHz when enabled).



a)



$V_{th1}$  and  $V_{th2}$

• low-comp.  $V_C = 0V \rightarrow V_D = 3V \rightarrow V_B = \frac{3V \cdot 50k}{150k} = 1V$

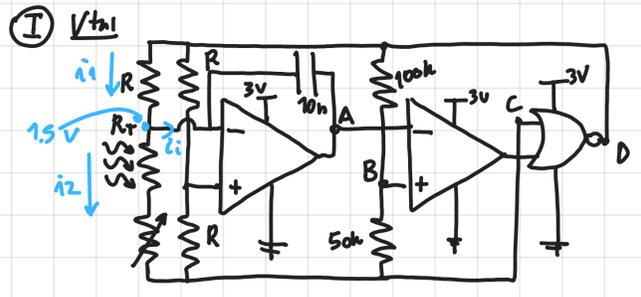
• high-comp.  $V_C = 3V \rightarrow V_D = 0V \rightarrow V_B = \frac{3V \cdot 100k}{150k} = 2V$

When  $V_A^+ = V_A^- = 3 \frac{R}{R+R} = 1.5V = V_{th1}$

( $V_{th2} = -1.5V$  symmetrical)

switching the P.S. up with the one down ( $V_D=3 \rightarrow 0$ ) ( $V_D=0 \rightarrow 3$ )

• current through capacitor



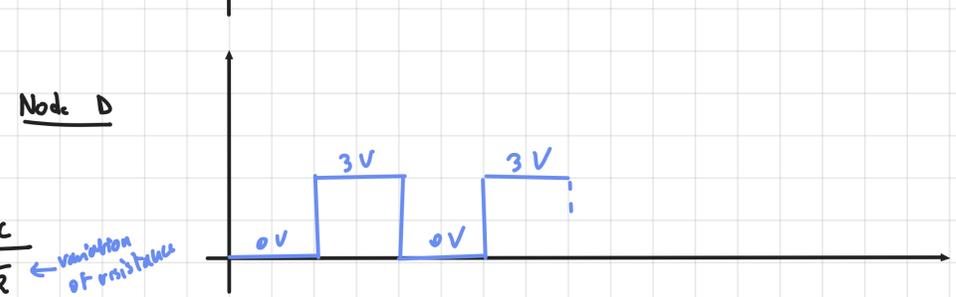
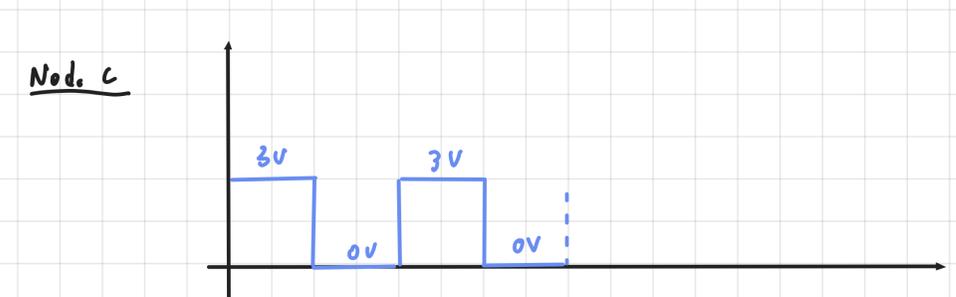
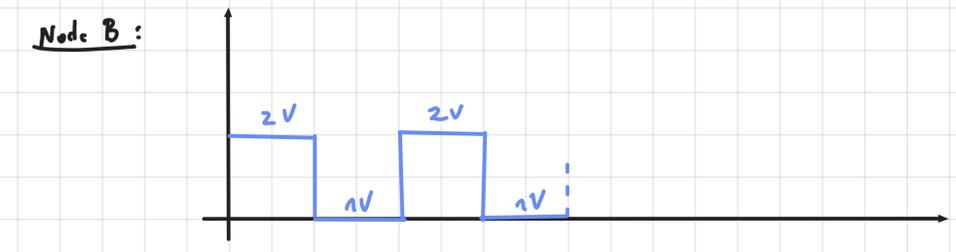
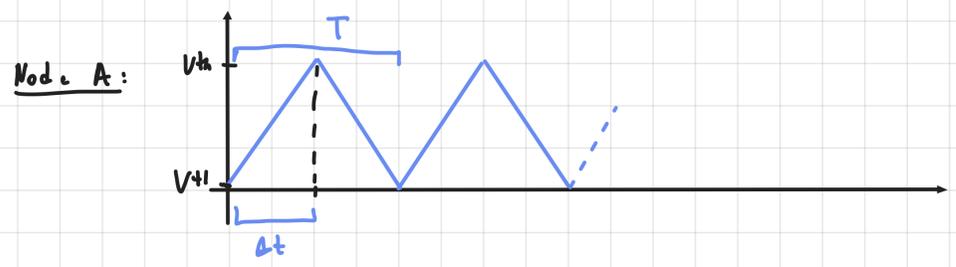
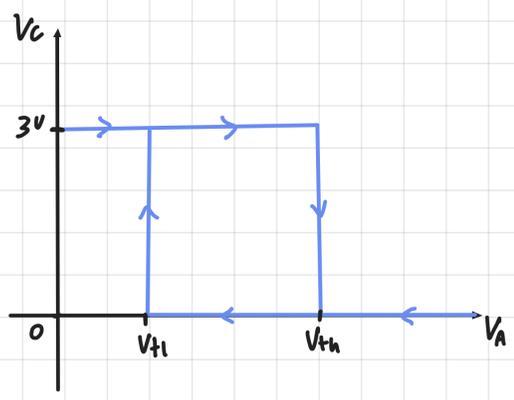
$i_1 = \frac{1.5V}{R}$        $i_2 = \frac{1.5V}{R+V}$

$i_i = i_1 - i_2 = \frac{(R+r-R) 1.5V}{R(R+r)} \approx 1.5 \mu A$

negligible  $\frac{r}{R} = 0.1\%$  ( $R=1k\Omega$ )

II  $V_{th2} \rightarrow i_1 = -\frac{1.5V}{R}$        $i_2 = -\frac{1.5V}{R+r} \rightarrow i_i = -1.5 \mu A$

b)



To compute the oscillation period:

$\frac{\Delta V_A}{\Delta t} = \frac{i_i}{C}$

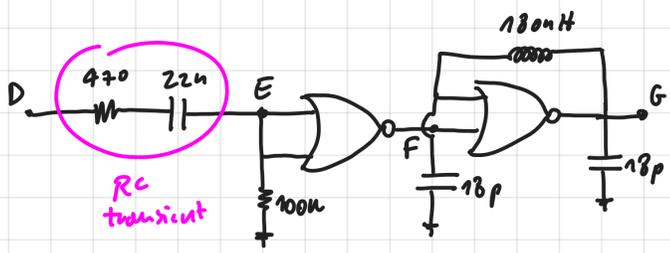
$i_i = \frac{V_{DD}}{2} \cdot \frac{1}{R} - \frac{V_{DD}}{2} \cdot \frac{1}{R+r} = \frac{V_{DD} r}{2R(R+r)} \approx \frac{V_{DD} r}{2R^2}$

$\Delta V_A = V_{th2} - V_{th1} = \frac{V_{DD} \cdot 100k}{150k} - \frac{V_{DD} \cdot 50k}{150k} = \frac{V_{DD}}{3}$

$\frac{\Delta V_A}{\Delta t} = \frac{V_{DD} r}{2R^2 C}$

$\Rightarrow \Delta t = \frac{2}{3} \frac{R^2 C}{r}$  (no P.S.  $V_{DD}$  dependence)  $\rightarrow T = 2\Delta t = \frac{4}{3} \frac{RC}{r}$  ← variation of resistance

c)

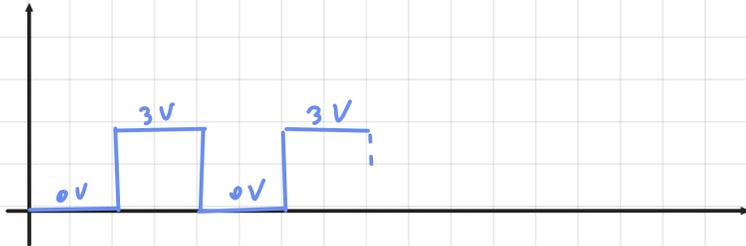


↳ We want to compute the period of the transient due to the RC

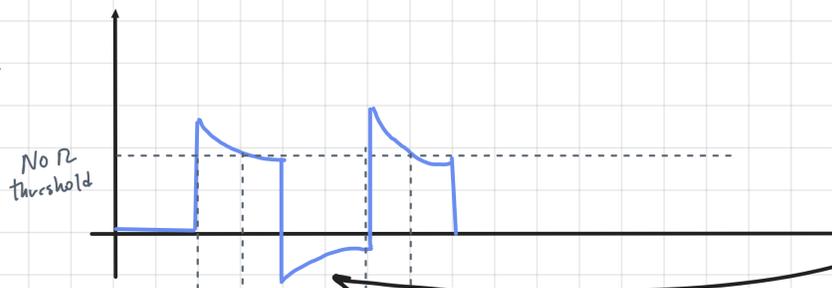
$$\tau = C(100k + 470) \approx 2.2 \text{ ns}$$

↳ Waveforms:

Node D



Node E:

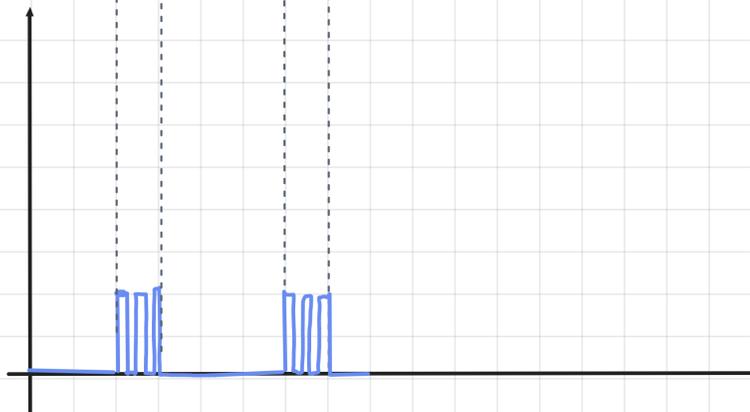


↳ We are changing the capacitance but the  $5\tau > \Delta t = \frac{T}{2}$   
 So at the end of the capacitor switch mode = 3V  
 We'll still have some residual charge.

Node F:



Node G:



↳ 80 MHz oscillation when G is enabled

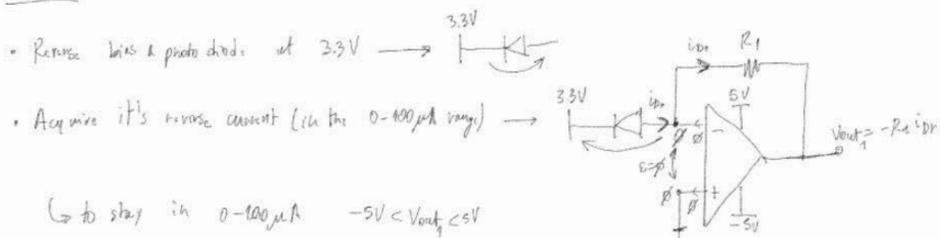
**Ex. 4**

Reverse bias a photodiode at 3.3V and acquire its reverse current, in the 0-100  $\mu$ A range, from DC to 10kHz.

- a) Every 100ms, detect the max intensity and restart.
- b) Turn on an LED when the light varies faster than  $\pm 1 \mu\text{A}/\mu\text{s}$  (both increasing and decreasing)

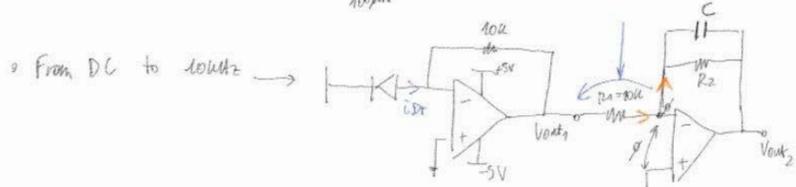
also look at prof sol. and see comments

Ex. 4 (Est Barba)



$\hookrightarrow$  choose  $R_1 < \frac{5V}{100\mu A} = 10k$

so  $V_{out} = -R_1 i_{DR} \rightarrow i_{DR} = \frac{V_{out}}{R_1} = i_{DR}$



@ DC  $V_{out2} = -\frac{R_2}{R_1} V_{out1} = -\frac{R_2}{R_1} (-R_1) i_{DR} = R_2 i_{DR}$

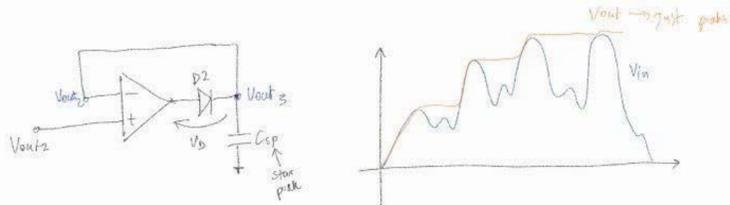
@ AC  $V_{out2} = 0 \rightarrow G_{AC} = 0$

pole  $\rightarrow \frac{1}{2\pi C R_2} = 10kHz$

e.g.  $C = 500 pF, R_2 = 32k\Omega$

a) Peak detection: every 100ms, detect the max intensity and restart

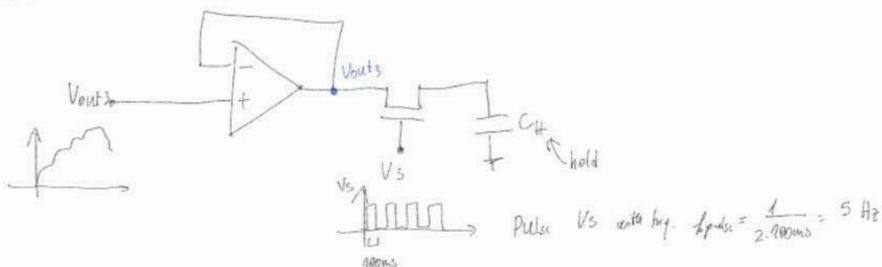
Peak detection:



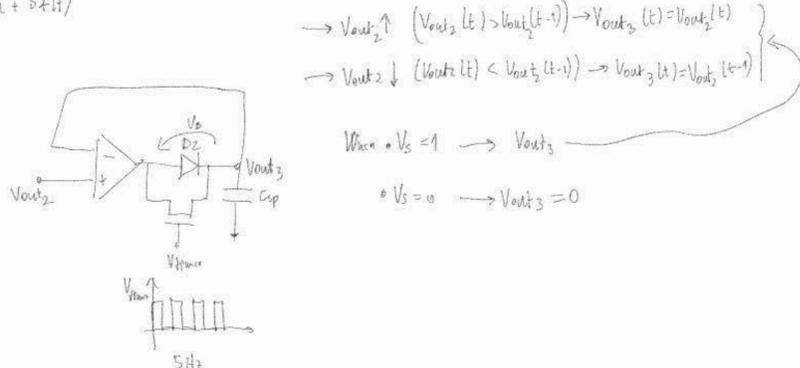
start:  $V_{out2}^0 > 0 \rightarrow D2 \text{ ON} \rightarrow$  Till  $(\phi = \pi) \rightarrow V_{out3} = V_{out2}$  stored in  $C_{sp}$

- tricks in time
- $\hookrightarrow$  If  $V_{out2} < V_{out2}^0 \rightarrow V_D < 0 \Rightarrow D2 \text{ OFF} \rightarrow C_{sp}$  stay still with the stored  $V_{out2}$
  - $\hookrightarrow$  If  $V_{out2} \uparrow (> V_{out2}^0) \rightarrow V_D > 0 \Rightarrow D2 \text{ ON} \rightarrow C_{sp}$  store new value

Sample: (SAH)

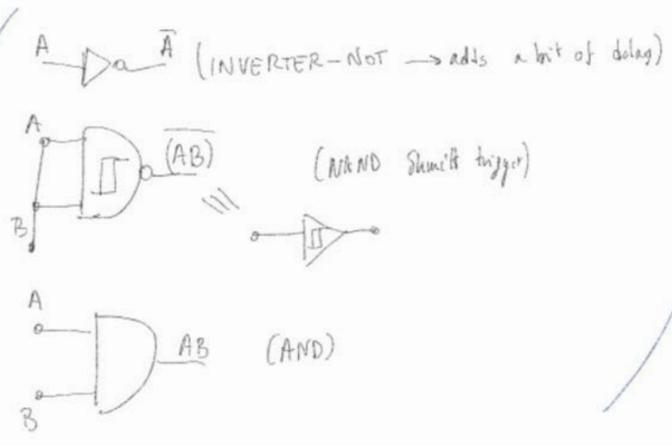
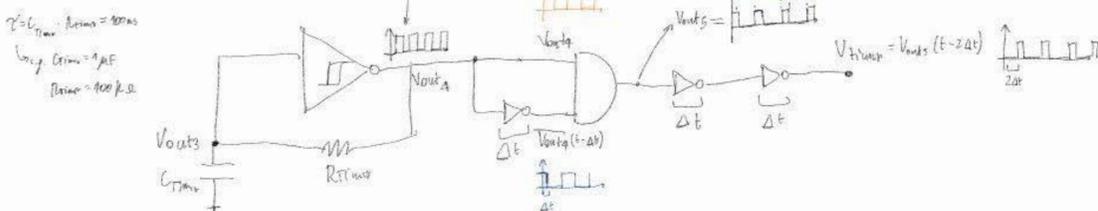


Restart (Peak + SAH)

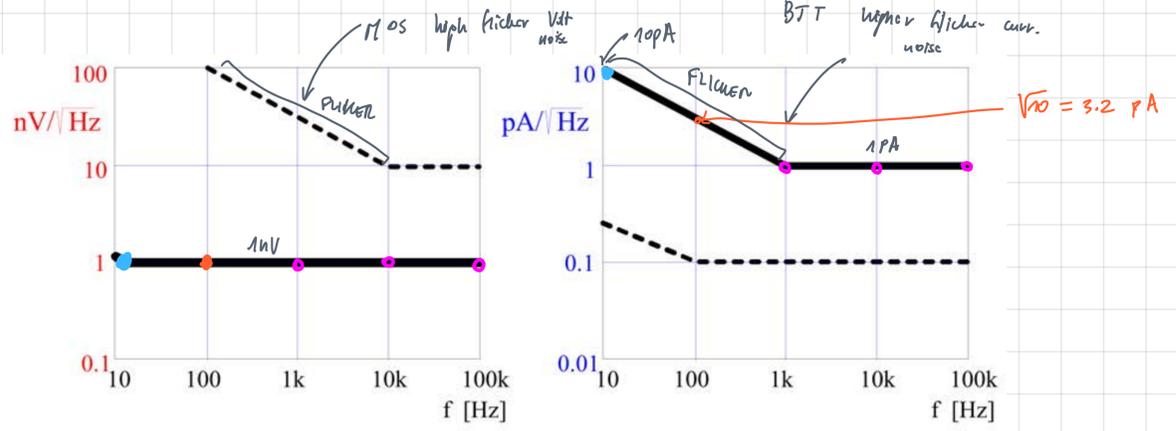


$\hookrightarrow$  It's better to wait a little bit to allow the  $C_{sp}$  capacitor to discharge before resetting with  $V_s = 0$

So we can talk:



①

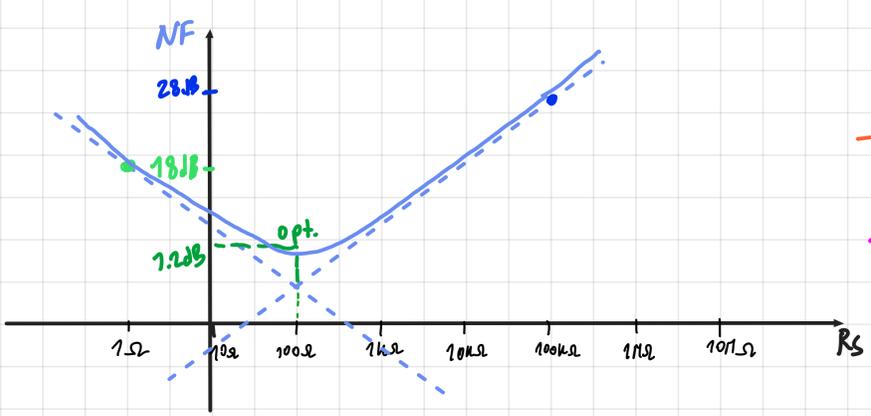


MOS (dashed lines) and BJT (solid lines) transistors at  $I_C = I_D = 1 \text{ mA}$

a) Draw the NF plots vs.  $R_S = 10 \Omega \div 10 \text{ M}\Omega$ , at 10, 100, 1k, 10k and 100kHz

b) Select the lowest noise transistor for  $R_S = 10 \text{ k}\Omega$

a) NF for BJT:



$$R_{Sopt} = \sqrt{\frac{V_{in}^2 \Delta f}{i_{in}^2 \Delta f}} = \sqrt{\frac{V_{in}^2}{i_{in}^2}} = \frac{1 \text{ nV}}{10 \text{ p}} = 100 \Omega$$

$$R_{Sopt} = \frac{1 \text{ nV}}{3.2 \text{ pA}} = 316 \Omega$$

$$R_{Sopt} = \frac{1 \text{ nV}}{1 \text{ pA}} = 1 \text{ k}\Omega$$

$$NF_{opt} = 10 \log_{10} \left( 1 + 2 \frac{V_{in} \cdot i_{in}}{4kT} \right) \frac{10^{-21}}{1.66 \cdot 10^{-20}} = 1.2 \text{ dB}$$

$$4kT = 1.66 \cdot 10^{-20} \dots$$

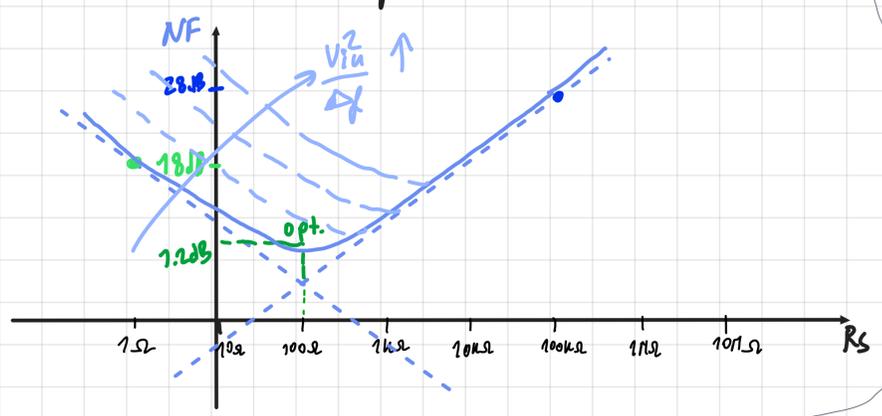
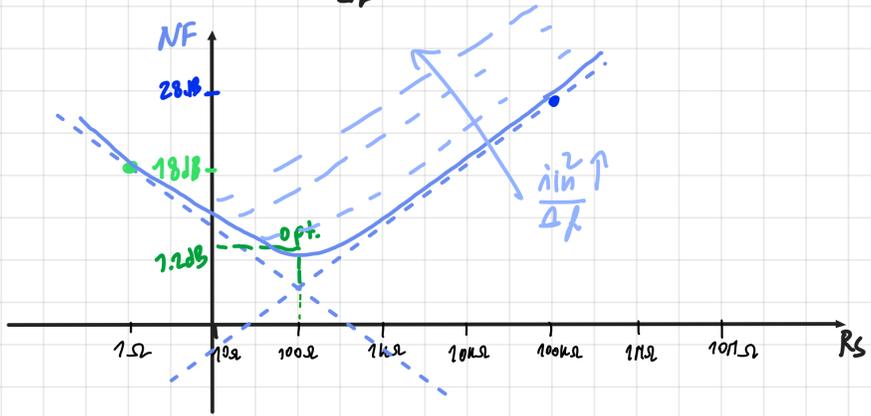
→ We compute  $NF_{high}$  and  $NF_{R_{Slow}}$  in order to understand the slopes  
like 100kΩ, 1MΩ...

$$NF_{100k\Omega} = 10 \log_{10} \left( 1 + \underbrace{\left( \frac{1 \text{ n}}{4 \text{ n}} \right)^2}_{\text{negligible}} + \underbrace{\left( \frac{10 \text{ p} \cdot 100 \text{ k}}{4 \text{ n}} \right)^2}_{\text{negligible}} \right) \approx 28 \text{ dB}$$

$$NF_{1\Omega} = 10 \log_{10} \left( 1 + \underbrace{\left( \frac{1 \text{ n}}{0.12} \right)^2}_{\text{negligible}} + \underbrace{\left( \frac{10 \text{ p} \cdot 1 \Omega}{0.12} \right)^2}_{\text{negligible}} \right) \approx 13 \text{ dB}$$

Note If we increase  $\frac{i_{in}^2}{\Delta f}$  this is the NF:

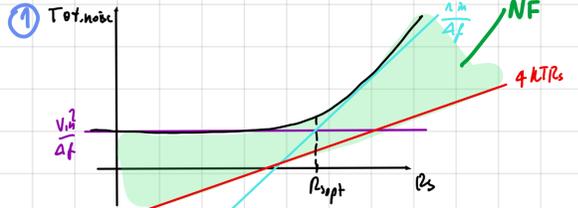
If we increase  $\frac{V_{in}^2}{\Delta f}$  this is the NF:



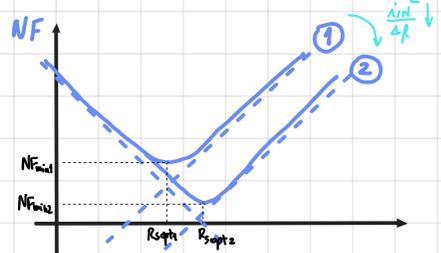
→  $R_{Sopt} \downarrow$

→  $R_{Sopt} \uparrow$

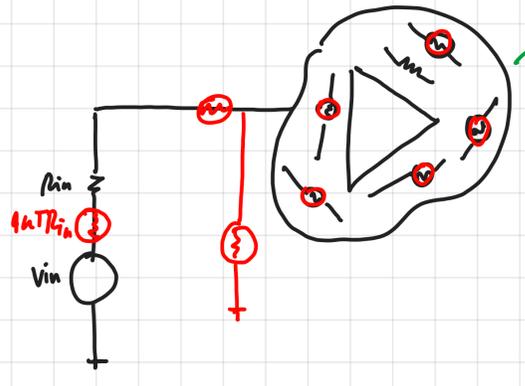
Obs. Total noise cases



(so for  $\frac{i_{in}^2}{\Delta f} \downarrow$   $R_{Sopt} \uparrow$ )  
very low  $\frac{i_{in}^2}{\Delta f}$   
→ The min NF is much lower than the previous case cause we have decreased  $\frac{i_{in}^2}{\Delta f}$  in order for the total noise to "touch" the thermal error  $4kTR_S$

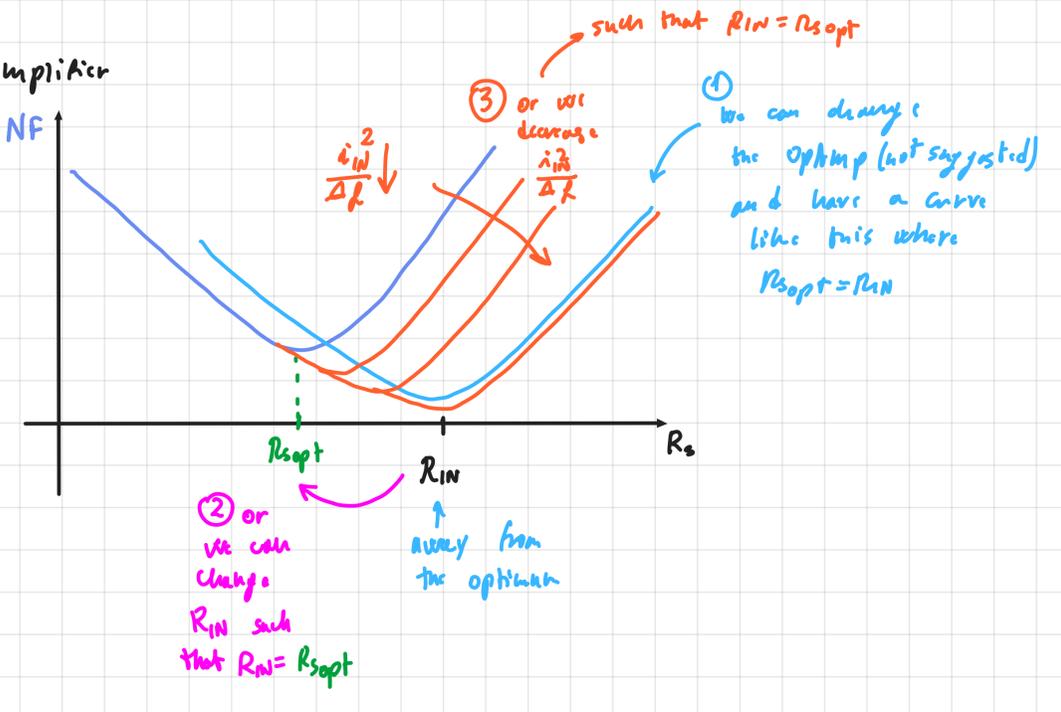


Note Suppos. we have a circuit connected to an amplifier

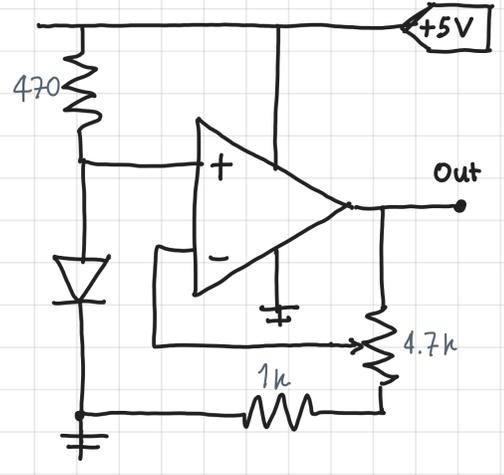
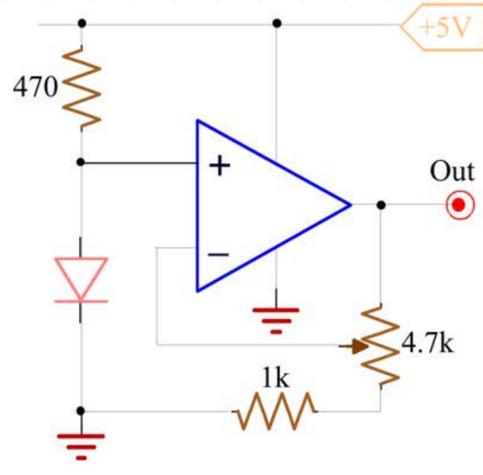


considered noiseless  
(but in reality has noise)

suppos. that the NF



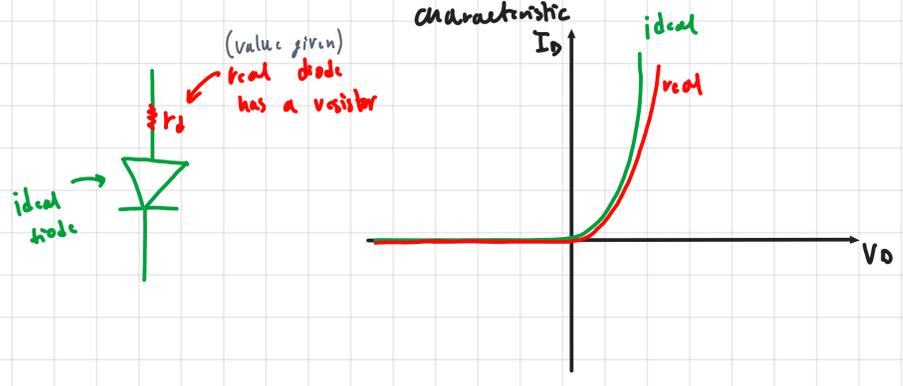
2



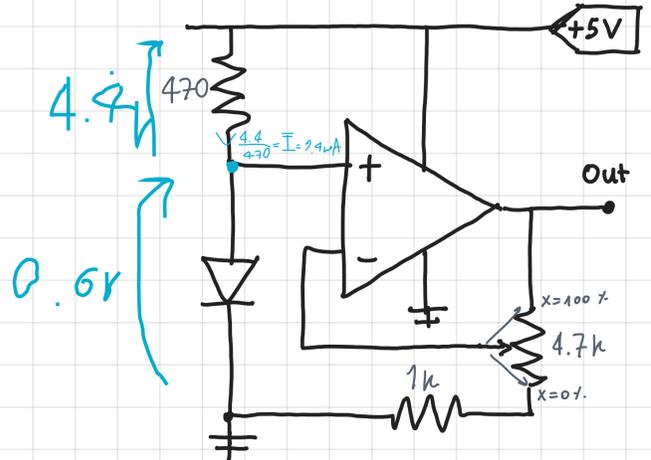
OpAmp:  $A_0=100\text{dB}$ ,  $\text{GBWP}=10\text{MHz}$ ,  $4\text{nV}/\sqrt{\text{Hz}}$  and  $5\text{pA}/\sqrt{\text{Hz}}$  noise

- a) Compute output rms noise, with trimmer's cursor at the two ends
- b) Discuss the role of the  $1\text{k}\Omega$  resistor

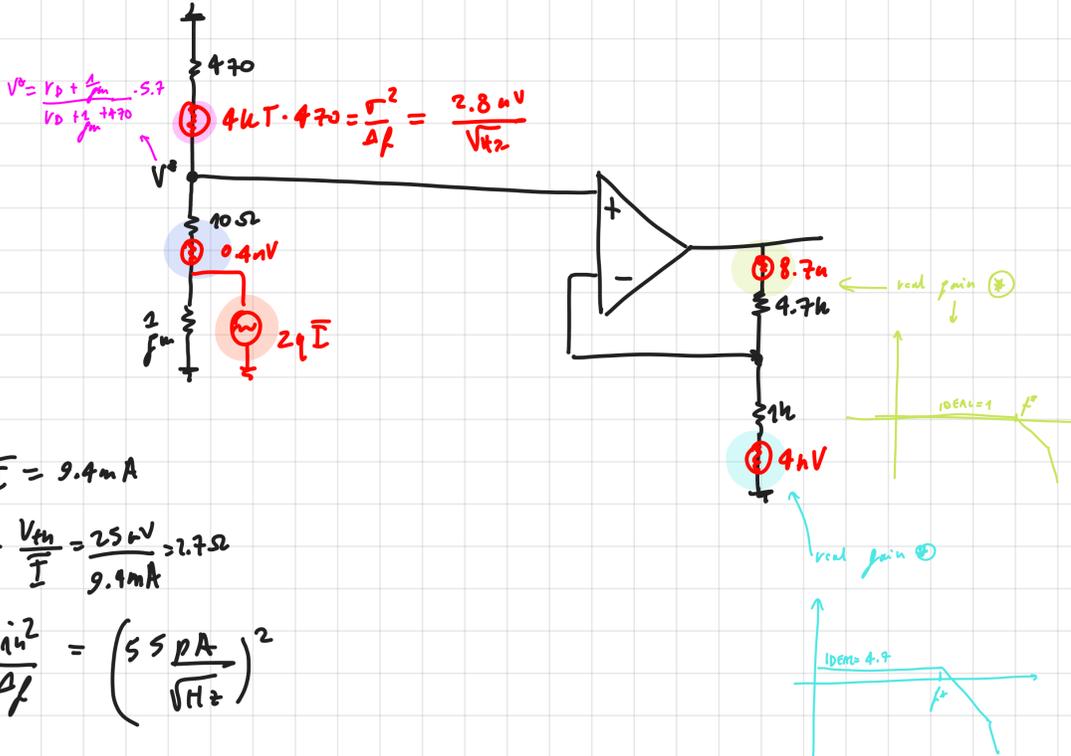
a) Remember:



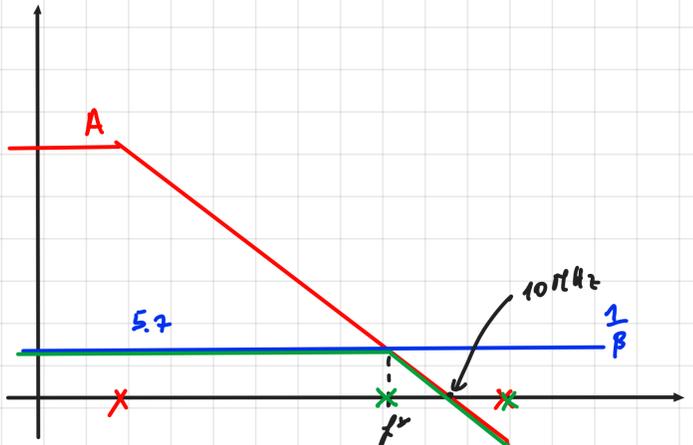
Consider the all circuit:



$x=0.1$



Bode:



$$I = 9.4\text{mA}$$

$$\frac{1}{f_m} = \frac{V_{th}}{I} = \frac{25\text{mV}}{9.4\text{mA}} = 2.7\Omega$$

$$2qI = \frac{i_n^2}{\Delta f} = \left(55 \frac{\text{pA}}{\sqrt{\text{Hz}}}\right)^2$$

we consider just 1 (dominant) pole for the real gain

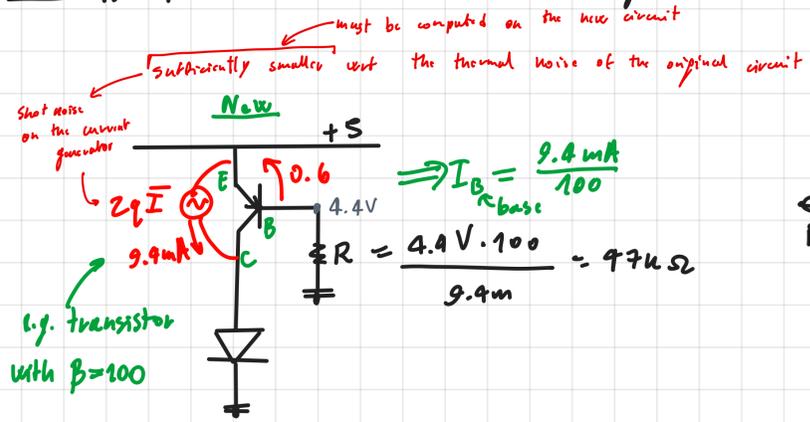
$$f_{w,pole} = \frac{I}{2} f_m$$

Noise analysis: 5 contributions

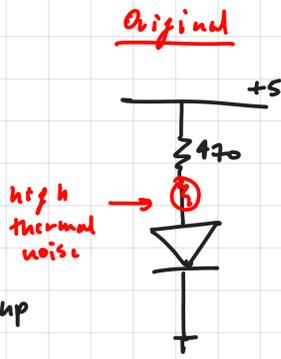
$$V_{out,noise,rms}^2 = V_{out}^2 = \left( \frac{2.8\text{nV}}{\sqrt{10}} \cdot \frac{V_D + \frac{1}{f_m}}{V_D + \frac{1}{f_m} + 470} \cdot 5.7 \right)^2 \Delta f + \left( 0.4\text{nV} \cdot \frac{470}{470 + V_D + \frac{1}{f_m}} \cdot 5.7 \right)^2 \frac{\pi}{2} \cdot 1.8\text{MHz} + \left( 55\text{pA} \cdot \frac{\frac{1}{f_m}}{1/f_m + V_D + 470} \cdot 470 \cdot 5.7 \right)^2 \frac{\pi}{2} \cdot 1.8\text{MHz} + \left( 8.7\text{nV} \cdot 1 \right)^2 \frac{\pi}{2} \cdot 1.8\text{MHz} + \left( 4\text{nV} \cdot \frac{4.7\text{k}}{1\text{k}} \right)^2 \frac{\pi}{2} \cdot 1.8\text{MHz} \approx 726\text{pV} = 0.9\text{mV}_{rms}$$

↳ So When  $X = 0\%$ . Gain = 5.7  $V_{rms} = 0.7 \text{ mV}_{rms}$

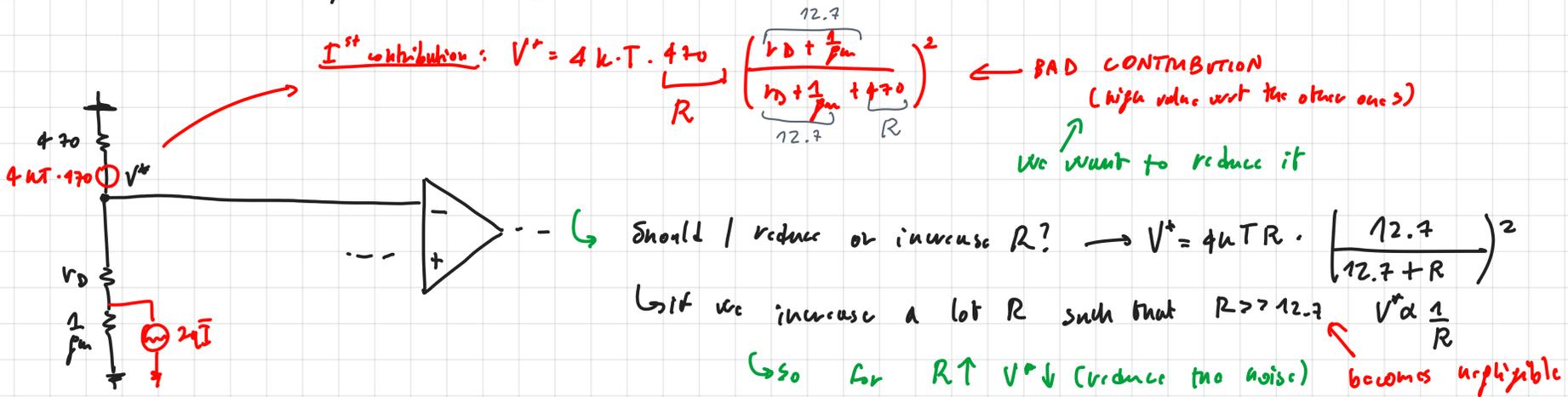
Obs. If the circuit has a too high noise we can change the circuit like this:



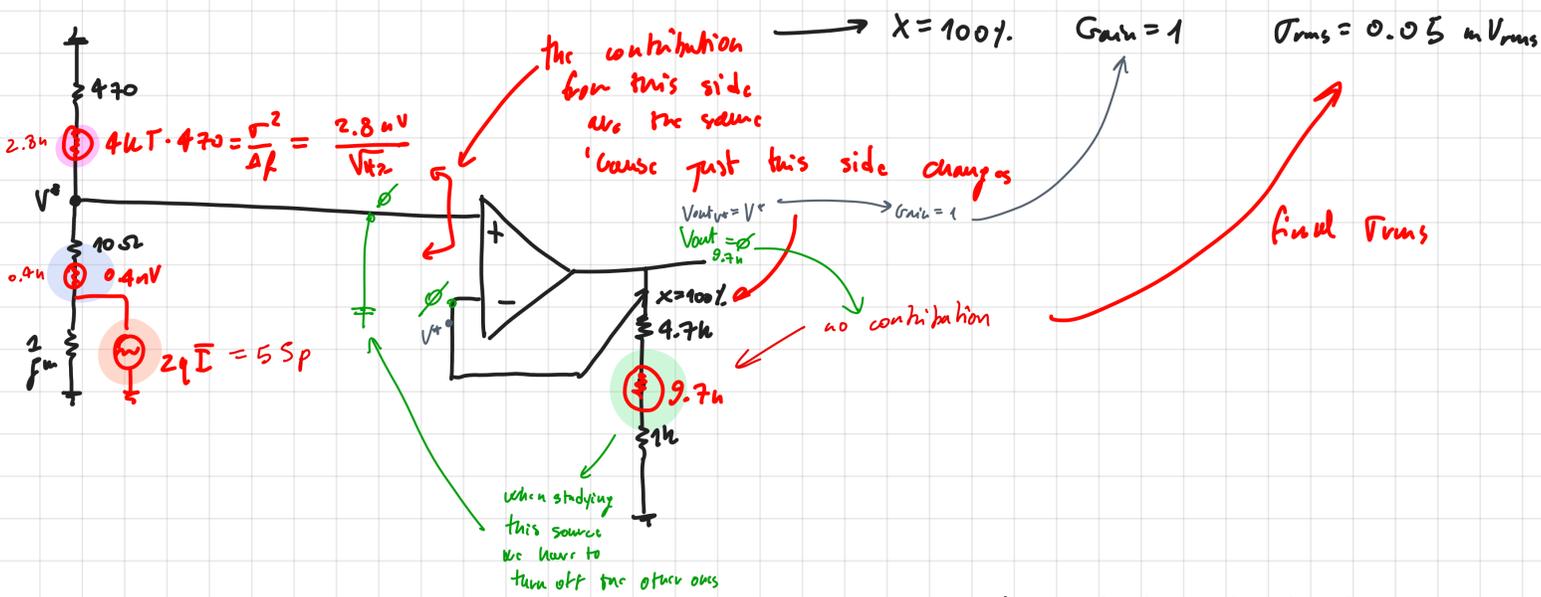
Instead of using the resistor (too high thermal noise) we can use a transistor pnp



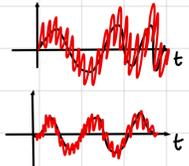
Obs. To reduce the noise, what should we do?



•  $X = 100\%$ .

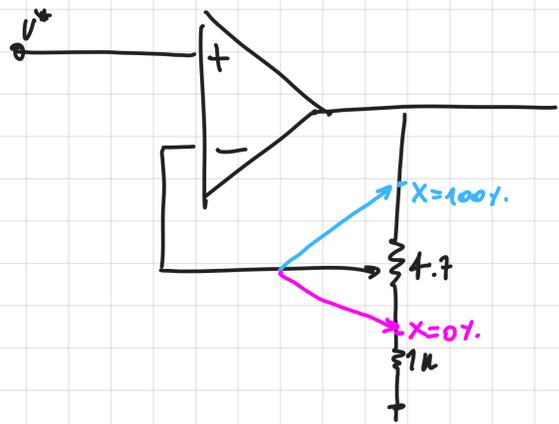


Obs.  $X = 0\%$ . Gain = 5.7  $V_{rms} = 0.7 \text{ mV}_{rms}$   
 $X = 100\%$ . Gain = 1  $V_{rms} = 0.05 \text{ mV}_{rms}$



concerning noise the best config. is  $X = 100\%$ , but we should also compare them computing the SNR

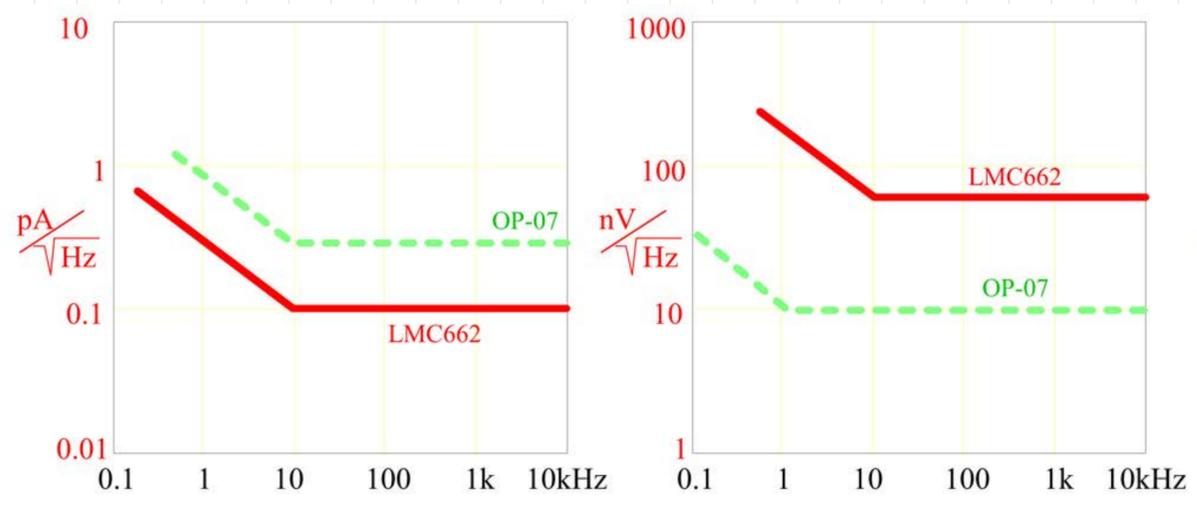
b)



•  $G = 1 + \frac{R_f}{R_i} = 1 + \frac{4.7k}{1k} = 5.7$  ← vol of the depending on  $X = 0\% / 100\%$ .

•  $G = 1 + \frac{R_f}{R_i} = 1 + \frac{4.7k}{\emptyset} = \infty$

3

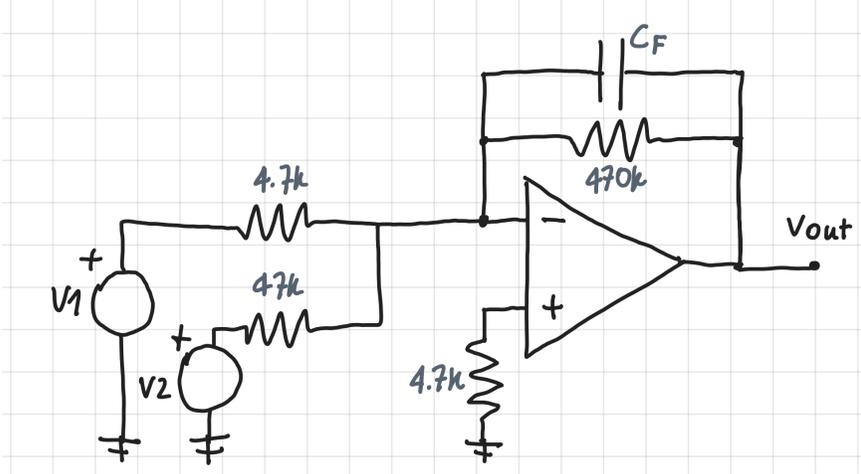
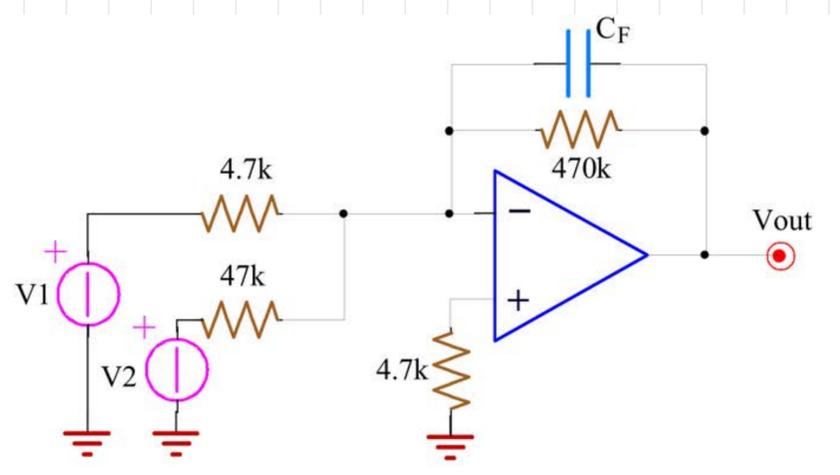


Given these two OpAmps

- a) Draw the  $NF$  plots vs.  $R_S$  in the  $1k\Omega - 1G\Omega$  range
- b) Select the best OpAmp for  $R_S=1M\Omega$  at  $1Hz$  and at  $1kHz$

(Skip. d)

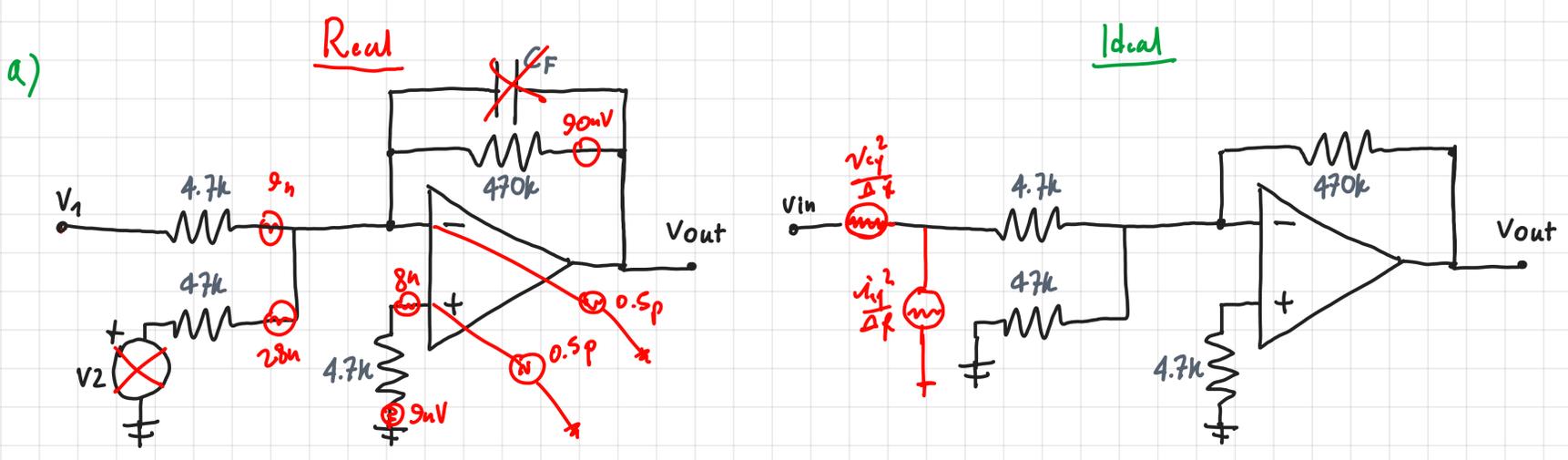
4



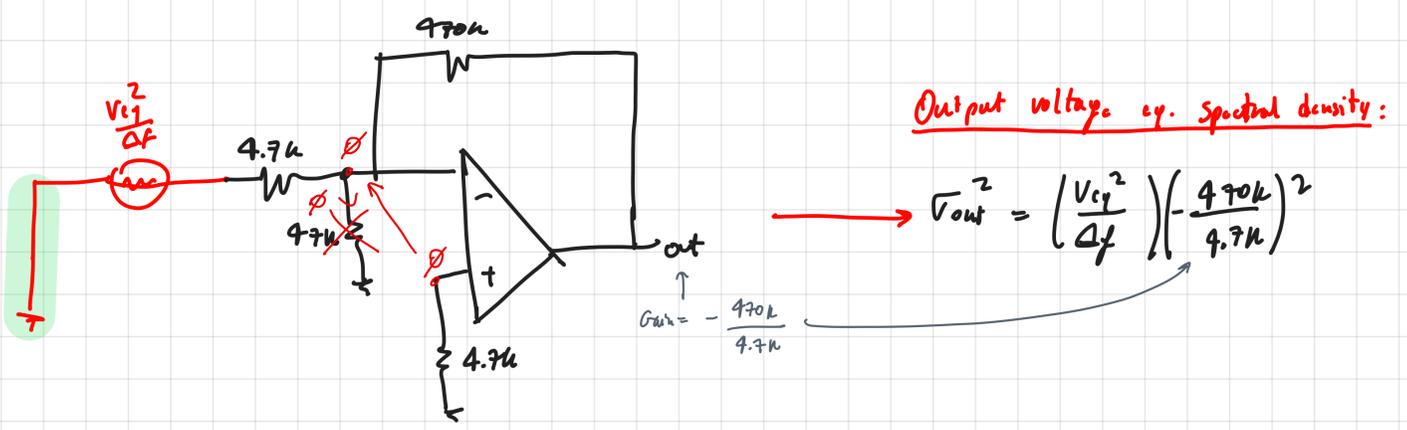
OpAmp with  $v_{in}^2/\Delta f = (8nV/\sqrt{Hz})^2$  and  $i_{in}^2/\Delta f = (0.5pA/\sqrt{Hz})^2$

a) Compute the noise equivalents for input V1

b) Compute the output rms noise

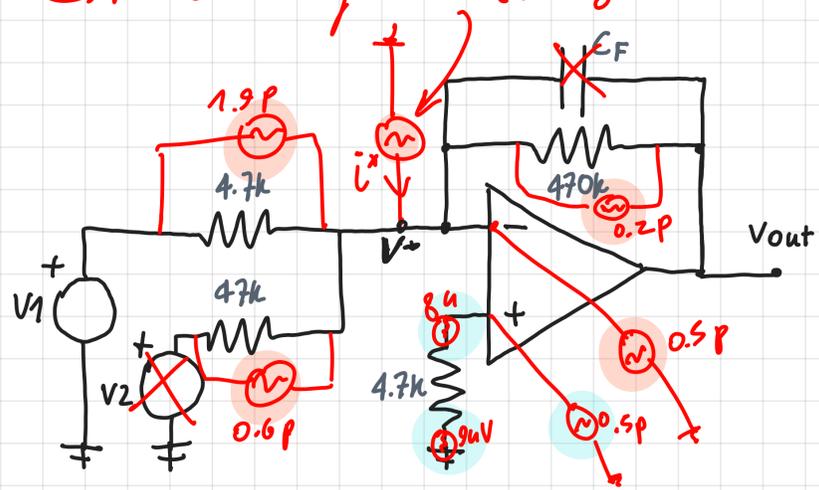


1. Compute the noise equivalent VOLTAGE generator  $\frac{V_{eq}^2}{\Delta f}$  → to do so we shortcircuit ( $R_{in}=0$ ) the input  
 ↓  
 (no current generator  $\frac{i_{eq}^2}{\Delta f}$ )



Let's study now the real circuit and compute the contributions of all the single sources

Sometimes, instead of studying the voltage eq. generator sources, it's easier to directly study their contributions for the current equivalent generator:  
 consider  
 → real current generator (taking into account all the noise contributions)



We'll study some contributions with the real curr. gen. sum:

$$\frac{i_{eq}^2}{\Delta f} = (1.9p)^2 + (0.2p)^2 + (0.6p)^2 + (0.5p)^2 = (2.1p)^2 \approx (2p)^2$$

$$\sigma_{out, i_{eq}}^2 = \left(\frac{2.1p \cdot 470k}{\sqrt{Hz}}\right)^2 \approx \left(\frac{1nV}{\sqrt{Hz}}\right)^2$$

We study the rest of the contributions with a real voltage gen. sum:

$$\frac{V_n^2}{\Delta f} = \left(\frac{9nV}{\sqrt{Hz}}\right)^2 + \left(\frac{8nV}{\sqrt{Hz}}\right)^2 + \left(\frac{0.5p \cdot 4.7k}{\sqrt{Hz}}\right)^2 = \left(\frac{12n}{\sqrt{Hz}}\right)^2$$

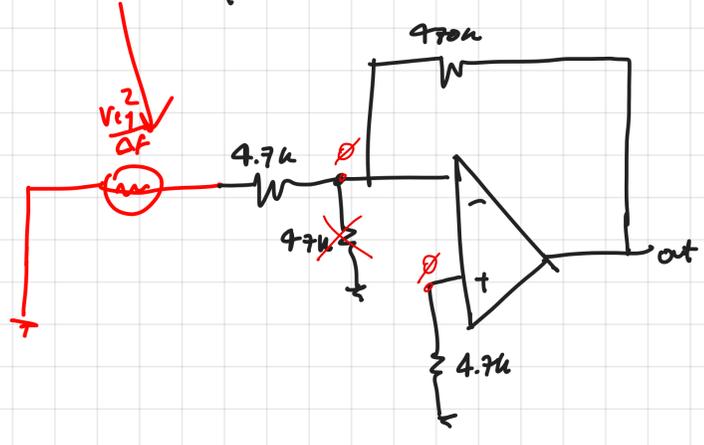
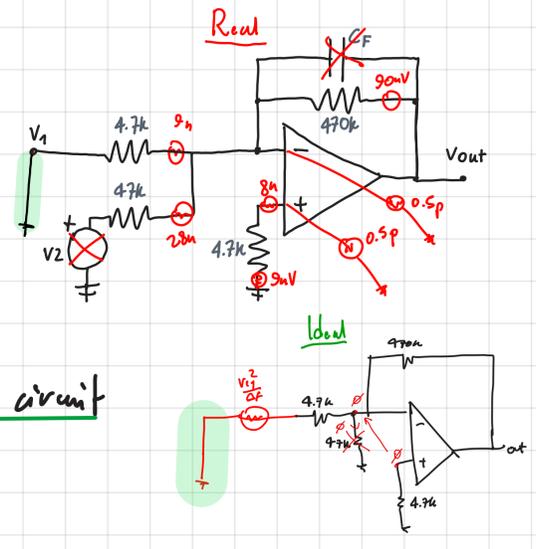
$$\sigma_{out}^2 \frac{V_n^2}{\Delta f} = (12n)^2 \left(1 + \frac{470k}{(47k||47k)}\right)^2 \approx \left(\frac{1.3\mu V}{\sqrt{Hz}}\right)^2$$

We have the sum of all the contributions from the real circuit

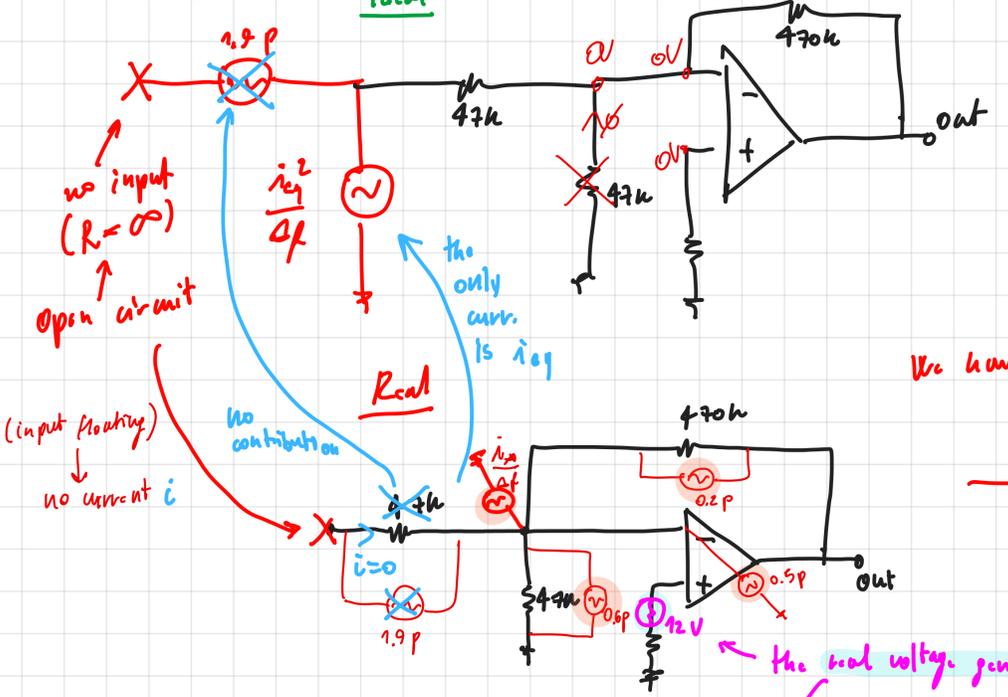
Total noise:  $\frac{V_{out}^2}{\Delta f} = \left(\frac{1\mu V}{\sqrt{Hz}}\right)^2 + \left(\frac{1.3\mu V}{\sqrt{Hz}}\right)^2 = \left(\frac{1.6\mu V}{\sqrt{Hz}}\right)^2$

We can now compute the equivalent gen. value for the ideal circuit

$$\frac{V_{eq}^2}{\Delta f} \cdot 100^2 = \left(\frac{1.6\mu V}{\sqrt{Hz}}\right)^2 \rightarrow \frac{V_{eq}^2}{\Delta f} = \left(\frac{160nV}{\sqrt{Hz}}\right)^2$$



2. Now we compute the equivalent current generator



Output current eq. spectral density:

$$\frac{I_{out}^2}{\Delta f} = \frac{i_{eq}^2}{\Delta f} \cdot 470k^2$$

We have to recompute the real curr. gen. contributions (no 1.9p):

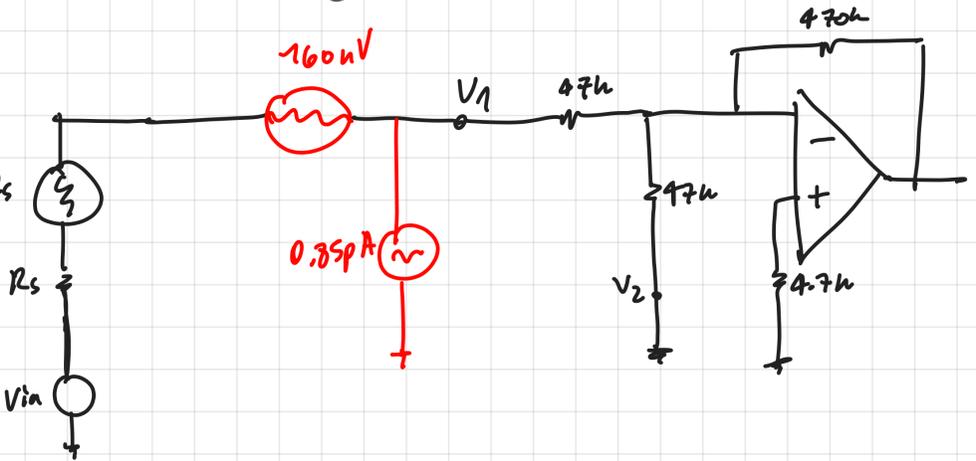
$$\frac{i_n^2}{\Delta f} = (0.2p)^2 + (0.6p)^2 + (0.5p)^2 = (0.8p)^2$$

⇒ Total noise: (in voltage form)  $\frac{V_{out}^2}{\Delta f} = \frac{i_n^2}{\Delta f} \cdot 470k^2 + (12n)^2 \left(1 + \frac{470k}{47k}\right)^2 = \underbrace{(0.8p \cdot 470k)^2}_{376n} + (132n)^2 = (400nV/\sqrt{Hz})^2$

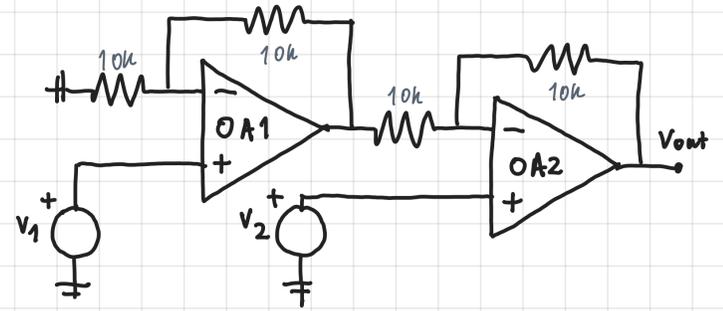
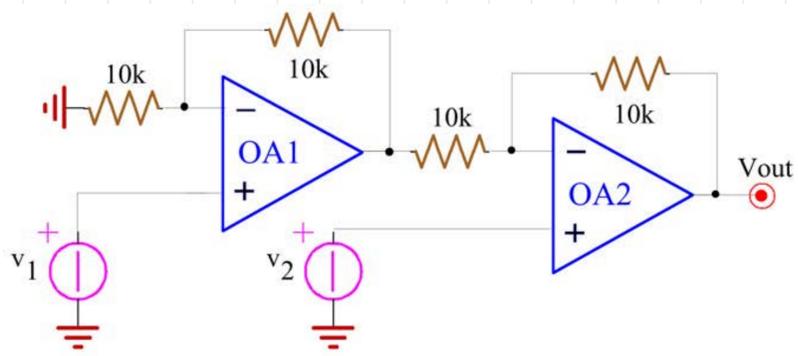
↳ The equivalent curr. noise:  $\frac{i_{eq}^2}{\Delta f} = \left(\frac{\frac{400nV}{\sqrt{Hz}}}{470k}\right)^2 = \left(0.85 \frac{pA}{\sqrt{Hz}}\right)^2$

⇒ IDEAL CIRCUIT:

When we connect something we can compare the source noise  $\frac{V_n}{\sqrt{R_s}}$  with the rest of the noise with NF, SNR...



5



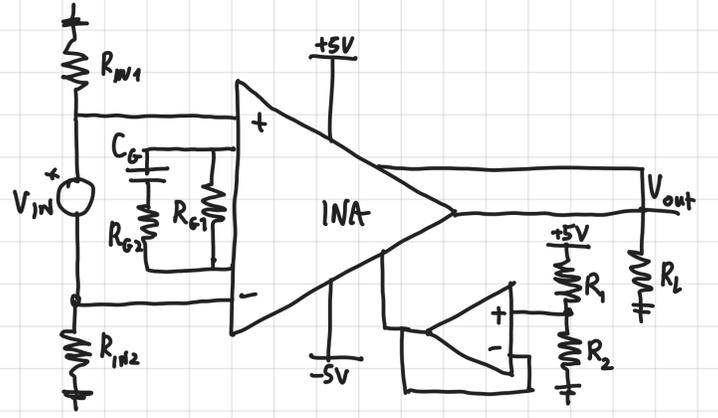
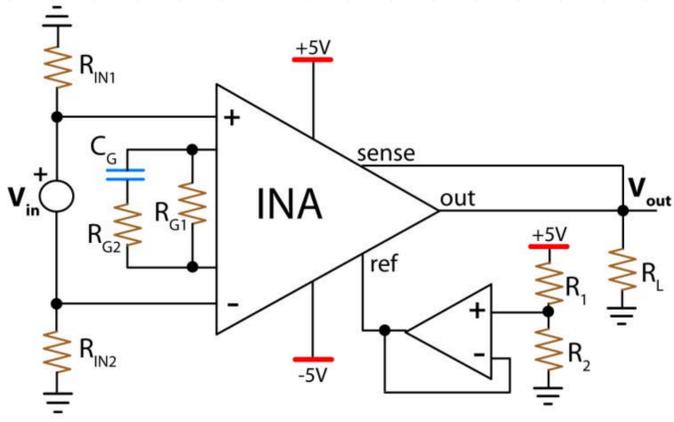
OpAmps:  $A_0=100\text{dB}$ ,  $\text{GBWP}=10\text{MHz}$ , and  $1\text{nV}/\sqrt{\text{Hz}}$   $1\text{pA}/\sqrt{\text{Hz}}$  noise

a) Compute output spectral density (neglect  $4kTR$  contributions)

b) Comment on the negligibility of resistors' contributions

(skipped)

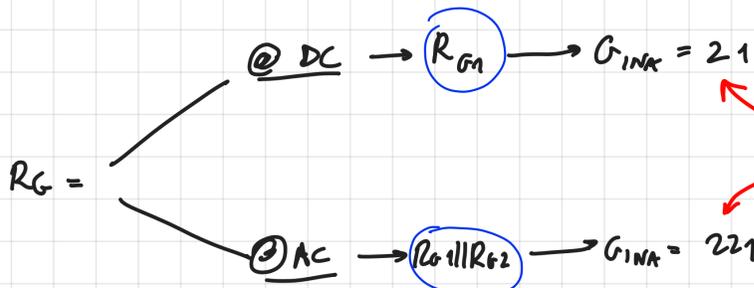
1



$R_{IN1}=100k\Omega$   $R_{IN2}=100k\Omega$   $R_{G1}=10k\Omega$   $R_{G2}=1k\Omega$   $C_G=1.6nF$   $R_1=3.3k\Omega$   
 $R_2=2.2k\Omega$   $R_L=1k\Omega$  all resistors inside INA be  $100k\Omega$

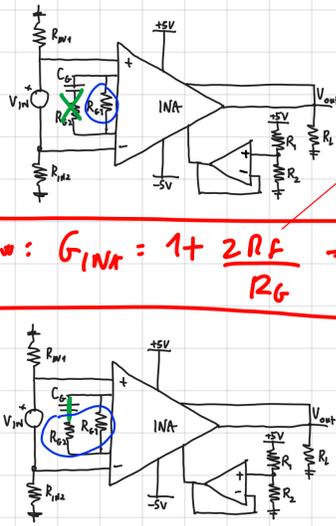
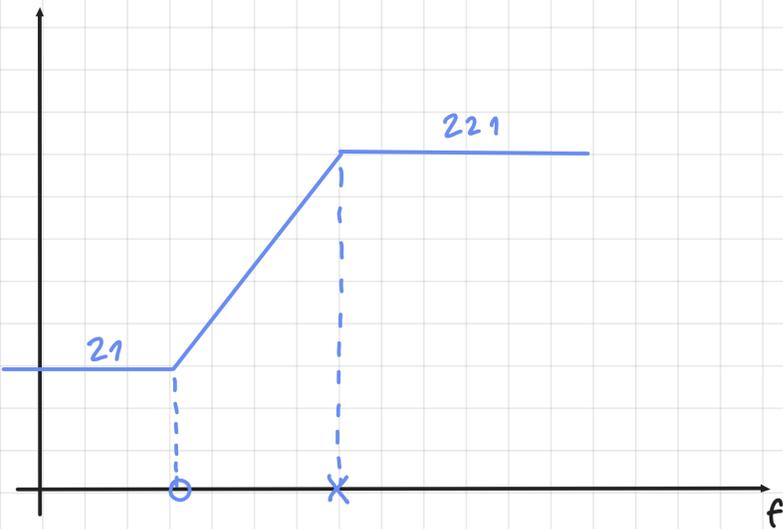
- a) Compute  $V_{out}$  for  $V_{in} = 0V$
- b) Compute the ideal gain  $v_{out}/v_{in}$  at low and high frequency
- c) Compute poles and zeros

(a) b)



We know:  $G_{INA} = 1 + \frac{2RF}{R_G} = 1 + \frac{200k}{R_G}$

Bode



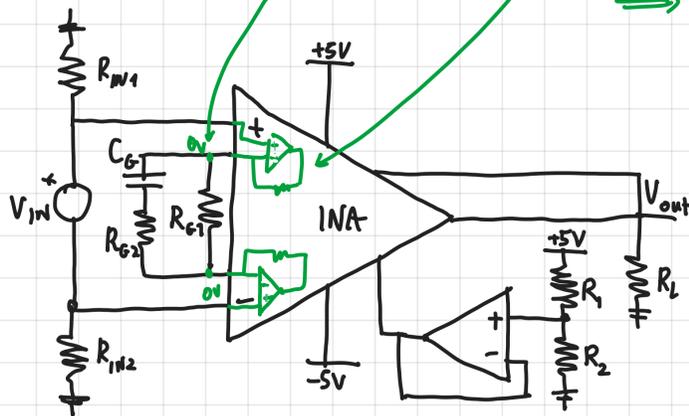
c) Poles and zeros:

$pole = \frac{1}{2\pi C_G R_{G2}}$

Remember that inside the INA we have these OpAmp  $\rightarrow$  for the computation of the pole ( $R_{eq}$ ) we have to consider the voltage  $v_{in} = 0$

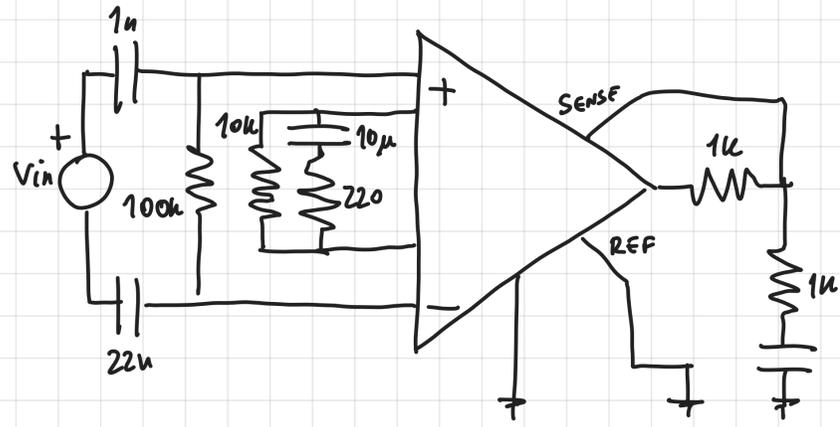
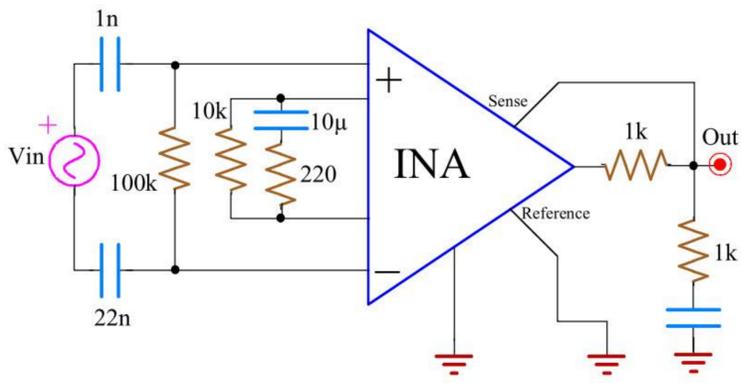
$zero = \frac{1}{2\pi C_G (R_{in1} + R_{G2})}$

also can be computed graphically



2

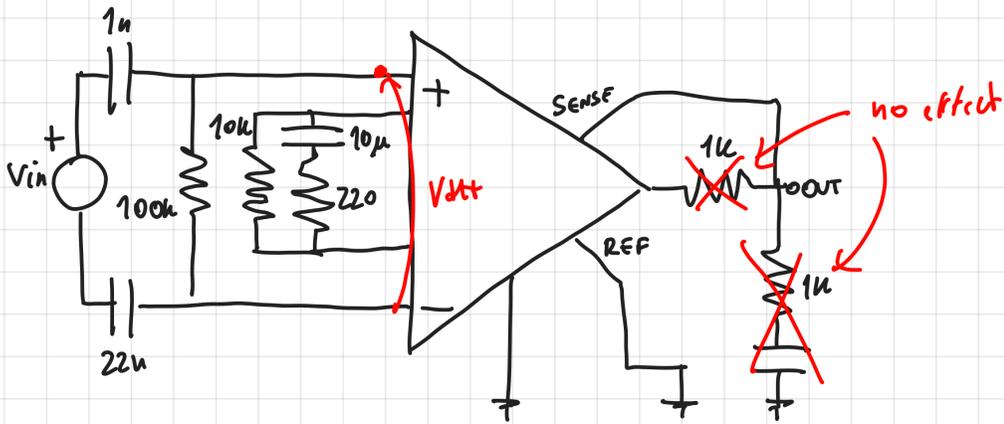
NICO



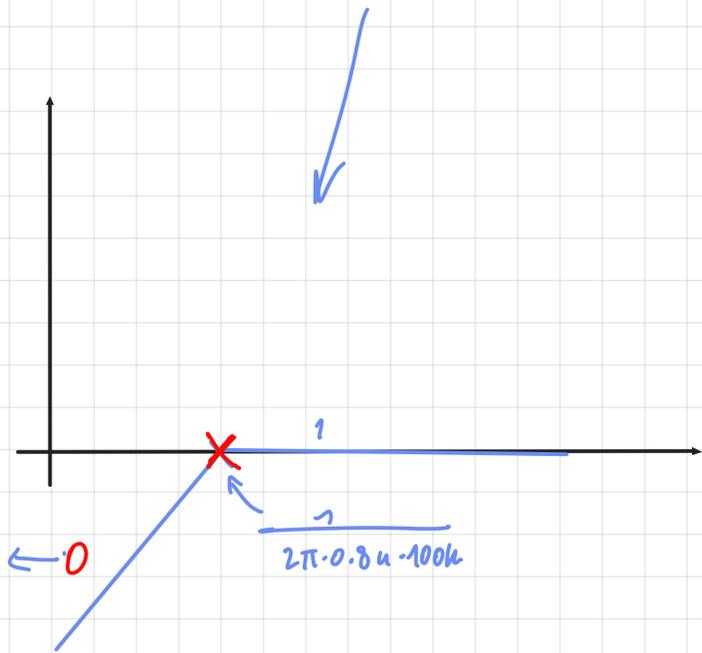
INA with  $R_f=100k\Omega$

a) Plot the Bode diagram for the  $v_{out}(f)/v_{in}(f)$  ideal gain

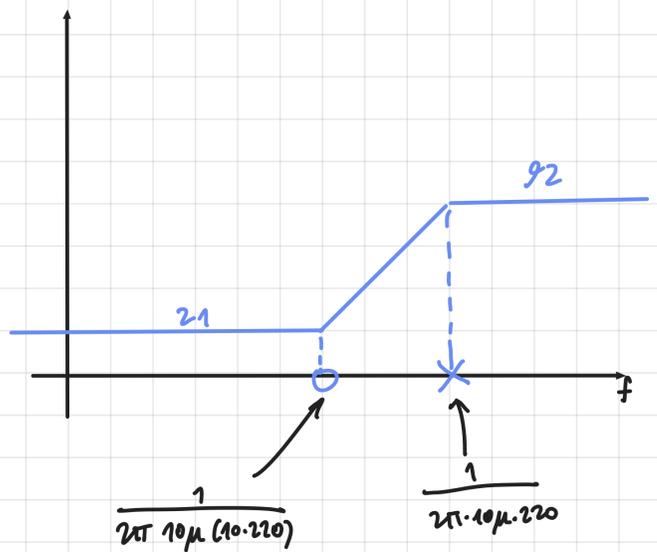
b) Modify the circuit to provide a DC gain of 1000 and a low-pass filtering action with 2 coincident poles at 100kHz



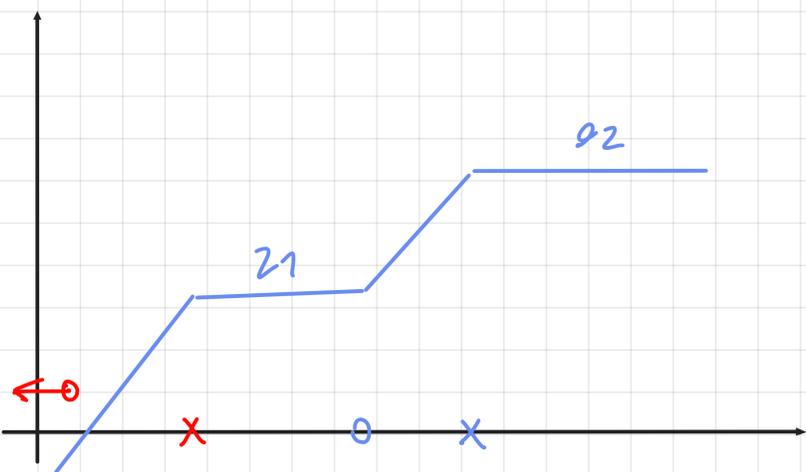
$$C_{in} = \frac{1}{\frac{1}{22n} + \frac{1}{1n}} = \frac{1n \cdot 22n}{1n + 22n} = 0.8 \mu F$$



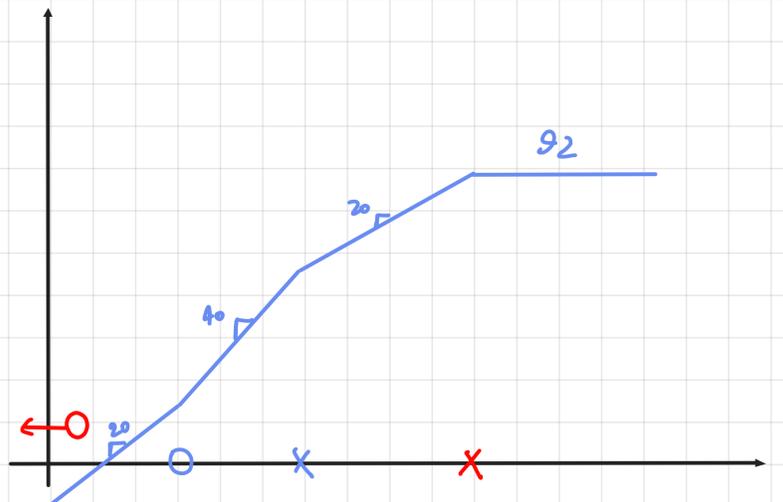
Bode INA



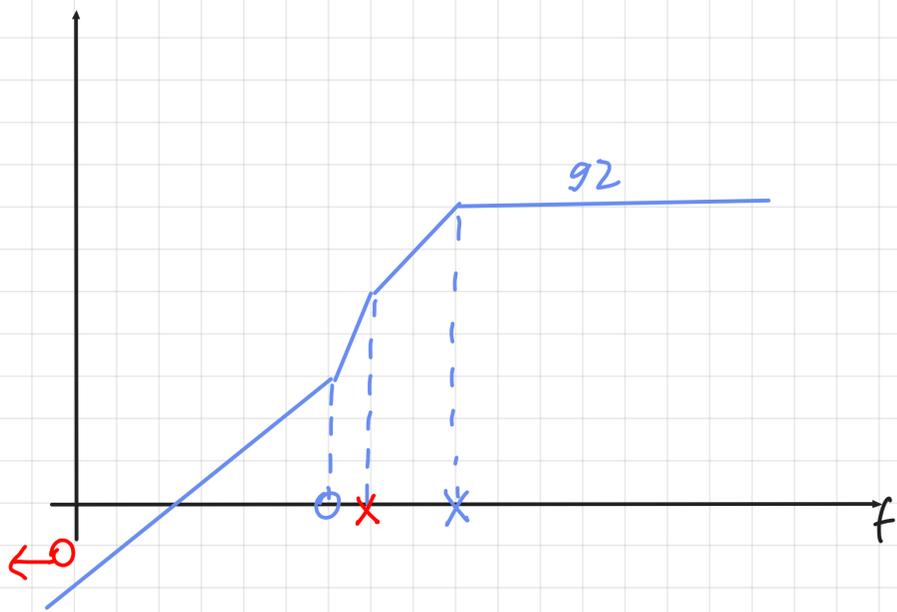
o If  $X < 0$



o If  $X > 0$



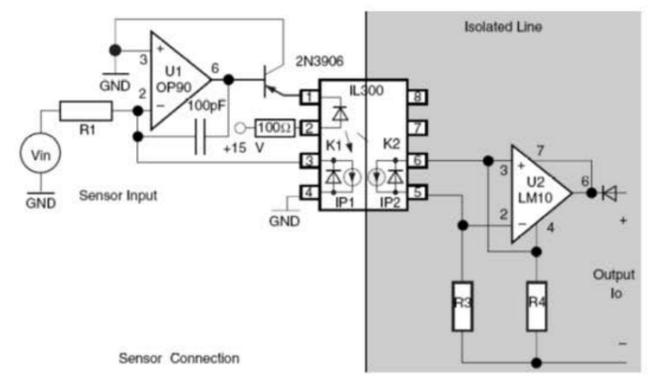
• If  $0 < x < x$



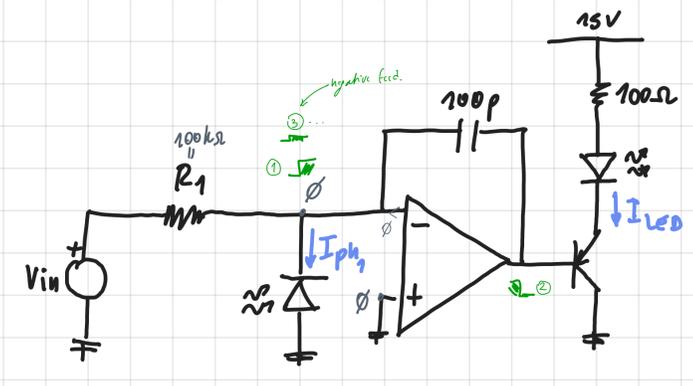
Ex. 1

OpAmp:  $A_0=100\text{dB}$ ,  $\text{GBWP}=10\text{MHz}$ ,  $I_b=10\text{nA}$ ,  $V_{os}=5\text{mV}$ . Dual optocoupler with  $K_1=K_2=I_{\text{photodiode}}/I_{\text{LED}}=0.012$ .  $R_1=100\text{K}\Omega$ .

- a) Compute the relationship  $I_{\text{ph2}}$  vs.  $V_{\text{in}}$ .
- b) Study the role of  $C=100\text{pF}$  on stage stability.
- c) Compute the relationship  $I_o$  vs.  $I_{\text{ph2}}$ .



a)  $\frac{I_{\text{ph2}}}{V_{\text{in}}}$  ?



(Always check first is neg. feedback!)

at DC: (C open)

$$\begin{cases} I_{\text{ph1}} = \frac{V_{\text{in}}}{100\text{k}} \\ I_{\text{ph1}} = I_{\text{LED}} \cdot K_1 \end{cases}$$

from the Opt coupling:  $K_1=K_2 = \frac{I_{\text{ph1}}}{I_{\text{LED}}} = 0.012$

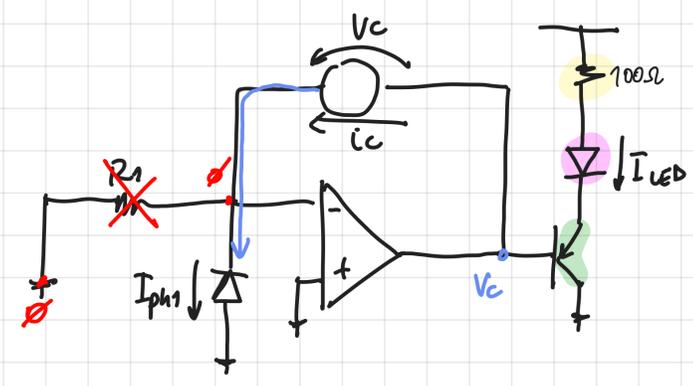
$$I_{\text{LED}} = \frac{V_{\text{in}}}{100\text{k} \cdot K_1} \rightarrow I_{\text{ph2}} = I_{\text{LED}} \cdot K_2 = \frac{K_2}{K_1} \cdot \frac{1}{100\text{k}} \cdot V_{\text{in}}$$

gain DC

$$\frac{I_{\text{ph2}}}{I_{\text{in}}} \Big|_{\text{DC}} = \frac{1}{100\text{k}}$$

at HF: (C short)  $\Rightarrow$  transistor OFF  $\rightarrow$  LED OFF  $\rightarrow$  PHOTO-DIODES OFF

pole



$$I_c = I_{\text{ph1}} = K_1 I_{\text{LED}}$$

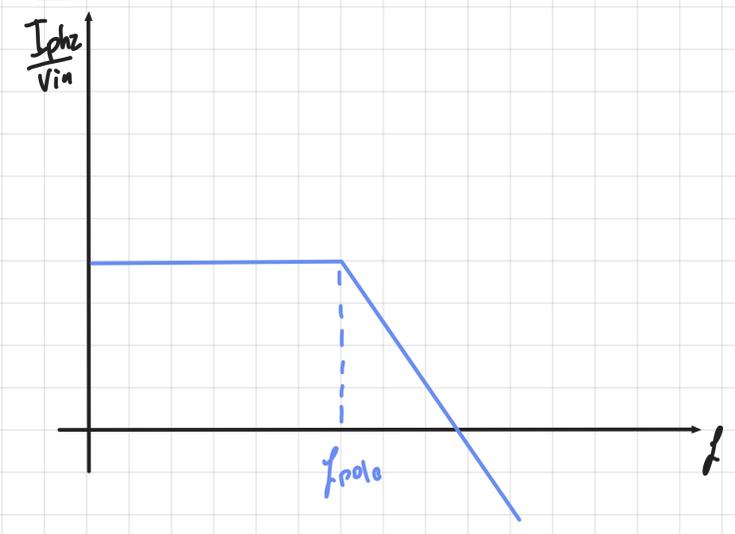
$$I_{\text{LED}} = \frac{V_c}{\frac{1}{g_m} + R_{\text{LED}} + 100\Omega} \approx \frac{V_c}{100\Omega}$$

$$I_c = \frac{K_1}{100\Omega} V_c$$

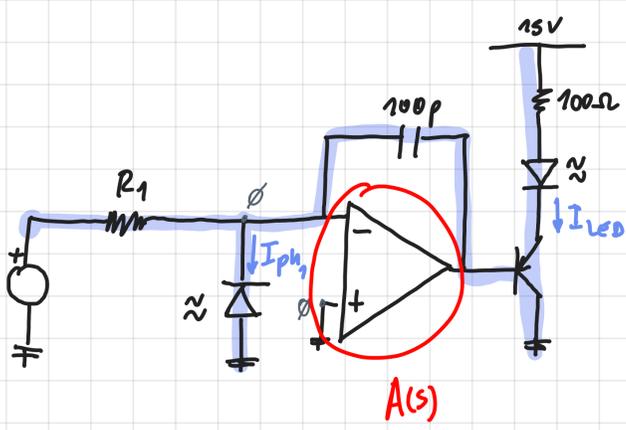
$$\frac{V_c}{I_c} \triangleq R_{\text{eq}} = \frac{100\Omega}{K_1}$$

$$f_{\text{pole}} = \frac{1}{2\pi C R_{\text{eq}}} = 191\text{ kHz}$$

Bode:

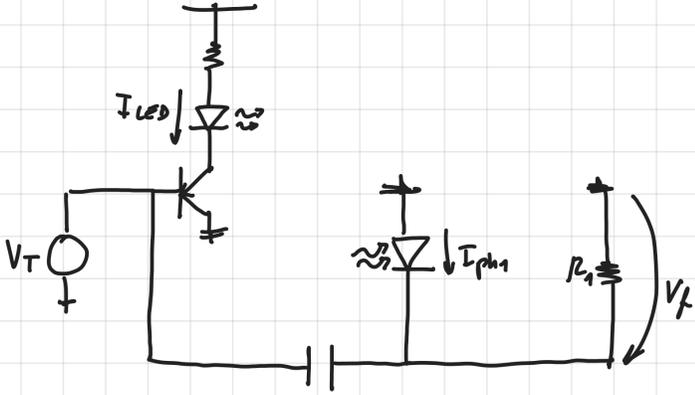


b)



A(s)

→ B(s):



• at DC:  
(C open)

$$V_f = R_1 I_{ph1} = R_1 k_1 I_{LED} \quad I_{LED} = \frac{V_T}{\frac{1}{\beta_{HF}} + R_{LED} + 100\Omega}$$

$$\hookrightarrow V_f = \frac{V_T}{\frac{1}{\beta_{DC}} + R_{LED} + 100\Omega} k_1 R_1$$

$$\hookrightarrow \beta_{DC} = \frac{V_f}{V_T} \approx \frac{k_1 R_1}{100\Omega} \rightarrow \frac{1}{\beta_{DC}} = \frac{1}{12}$$

• at HF:  
(C short)

$$V_f = V_T$$

$$\hookrightarrow \beta_{HF} = 1 \rightarrow \frac{1}{\beta_{HF}} = 1$$

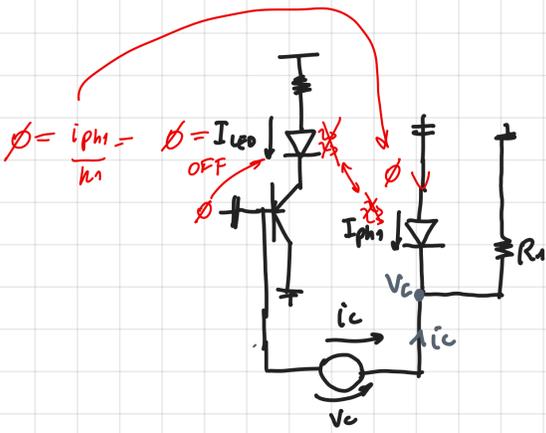
← • pole

$$f_{p\beta} = \frac{1}{2\pi C R_1} = 15.9 \text{ kHz}$$

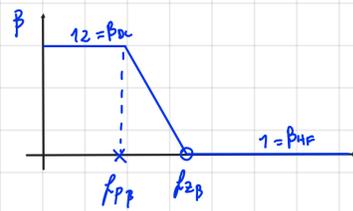
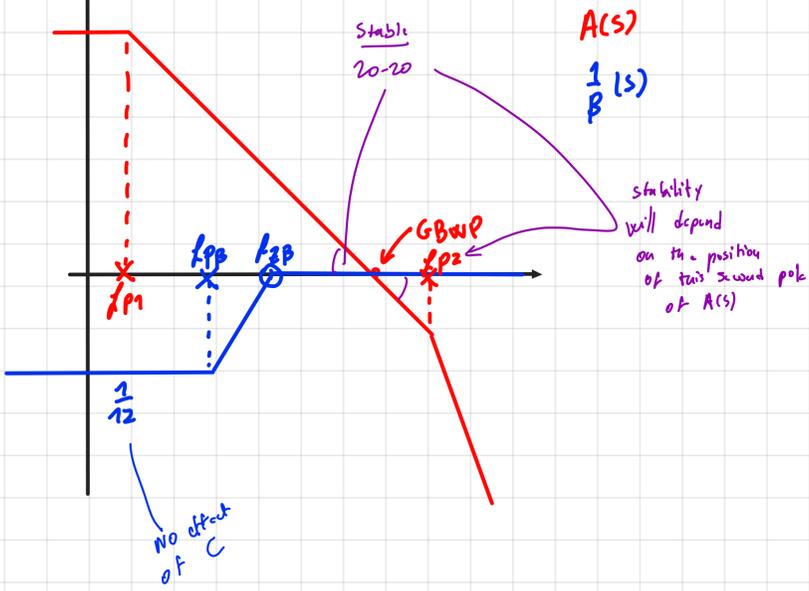
zero  
(symmetrically)

$$f_{z\beta} = f_{p\beta} \beta_{DC} \Rightarrow f_{z\beta} = 1.91 \text{ kHz}$$

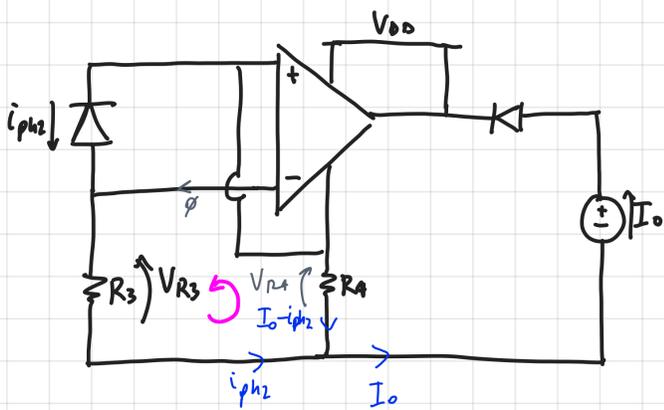
$$\Rightarrow \frac{V_c}{I_c} = R_{eq} = R_1$$



Bode:



c)



$$R_3 i_{ph2} = R_4 (I_0 - i_{ph2})$$

$$\hookrightarrow \frac{I_0}{I_{ph2}} = 1 + \frac{R_3}{R_4}$$

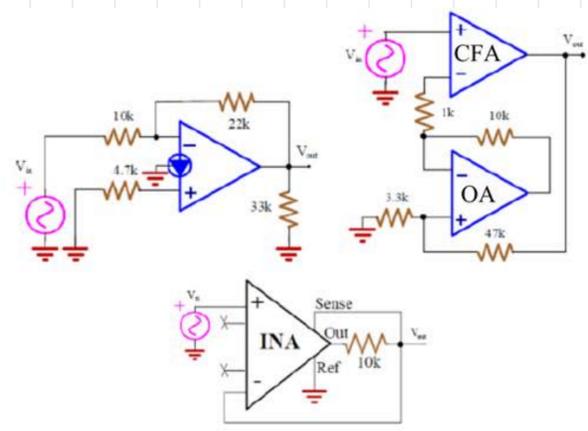
gain

2

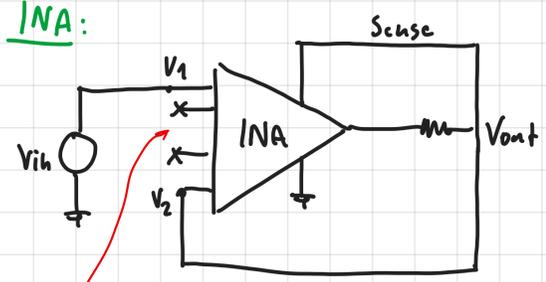
## Ex. 2

The following blocks utilize INA, CFA and Norton amplifiers.

- Compute the INA gain  $V_{out}/V_{in}$  and give an estimation of the output impedance
- Compute the gain  $V_{out}/V_{in}$  of the Norton system, with  $A_i=4$  and give an estimation of the input and output impedances
- Compute the gain  $V_{out}/V_{in}$  of the CFA block.



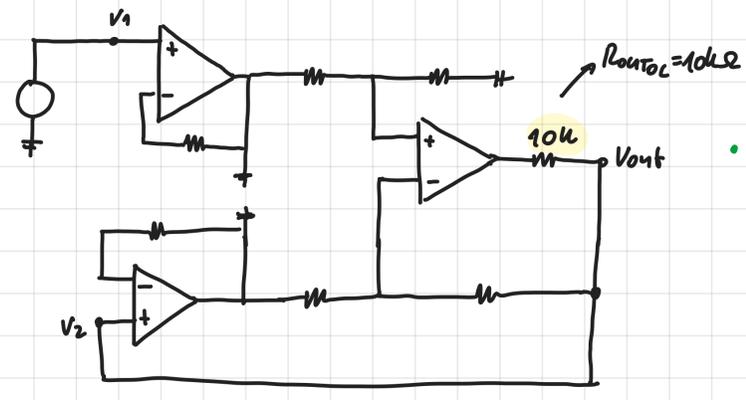
### a) INA:



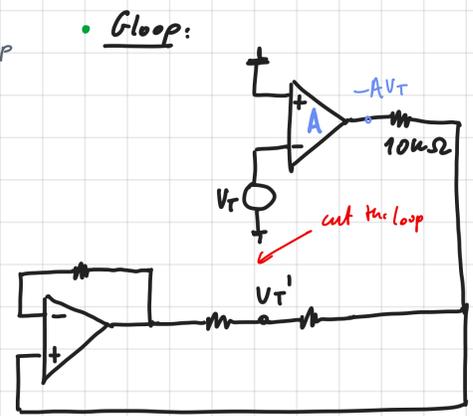
- $V_{out}/V_{in}$  ?
- $Z_{out}$  ?

$G_{diff} = 1$      $V_1 = V_{in}$      $V_2 = V_{out}$     We know:  $V_{out} = G_{diff} (V_1 - V_2) = 1 \cdot (V_{in} - V_{out}) \rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{2}$

For the computation of  $Z_{out}$  we consider the internal stage



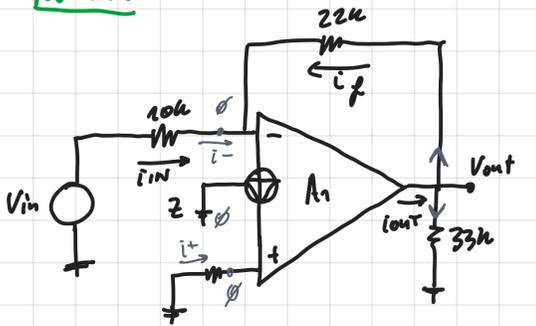
$R_{out} = \frac{R_{out,OL}}{1 - G_{loop}}$



$V_T' = -A_T V_T$   
 $\frac{V_T'}{V_T} = -A = G_{loop}$

$R_{out} = \frac{10k}{1+A} \approx 0$

### b) Norton:



- $V_{out}/V_{in}$  ?
- $Z_{in} \approx ?$
- $Z_{out} \approx ?$

$i^+ = 0$   
 $i^- = i_{in} + i_f = \frac{V_{in}}{10k} + i_f$   
 $i_f = V_{out} \frac{33k}{33k+22k}$  (current divider)  
 $i_{out} = A_i (i^+ - i^-) \rightarrow i_{out} = -A \left( \frac{V_{in}}{10k} + i_{out} \frac{33}{55} \right)$

$\frac{V_{out}}{V_{in}} = -1.55$

$i_{out} \left( 1 + A \frac{33}{55} \right) = -\frac{A}{10k} V_{in}$   
 $\Rightarrow i_{out} = -\frac{A}{10k} \frac{V_{in}}{1 + A \frac{33}{55}}$   
 $V_{out} = i_{out} (33k // 22k) = -1.55 V_{in}$

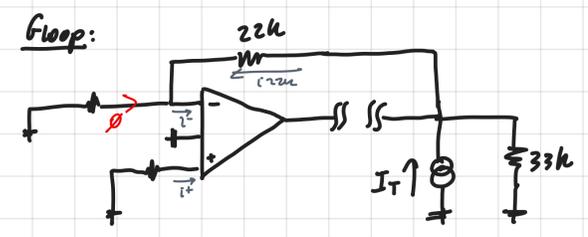
$Z_{in} = R_{in} = 10k\Omega$

$Z_{out} = R_{out} = \frac{R_{out,OL}}{1 - G_{loop}}$

$R_{out,OL} = 22k // 33k$

$Z_{out} = \frac{22k // 33k}{1 + 2.4} = 3.88k\Omega$

### Loop:

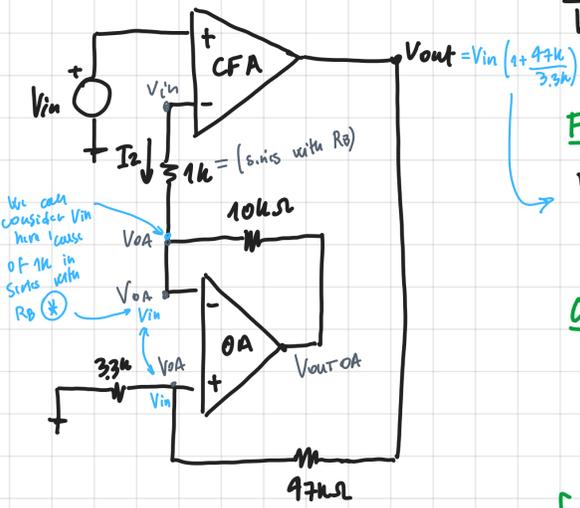


$i^+ = 0$   
 $i^- = i_{22k} = i_T \left( -\frac{33}{55} \right)$   
 $i_{out} = A_1 (i^+ - i^-) = -4 \frac{33}{55} i_T$

$G_{loop} = \frac{i_{out}}{i_T} = -2.4$

c) CFA:

from lesson notes - E507:



$$\frac{V_{out}}{V_{in}} = ?$$

Fast method → Annotations on circuit

$$V_{out} = V_{in} \left( 1 + \frac{47k}{3.3k} \right) \rightarrow \frac{V_{out}}{V_{in}} = 15.24 \checkmark$$

Computational method → Annotations

$$V_{OA+} = V_{OA-} = V_{out} \frac{3.3k\Omega}{47k\Omega + 3.3k\Omega}$$

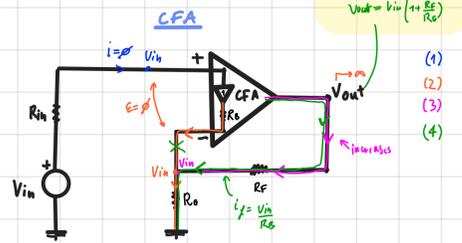
$$V_{outOA} = V_{OA} + \frac{V_{OA} - V_{in}}{1k} \cdot 10k$$

$$\rightarrow V_{outOA} = 10k \left( \frac{V_{in}}{1k} \right) - V_{OA} \left( \frac{1}{1k} - \frac{1}{10k} \right)$$

$$\frac{V_{outOA} - V_{OA}}{10k} = \frac{V_{in} - V_{OA}}{1k} \quad \text{KCL}$$

$$10k \left( \frac{V_{in}}{1k} \right) - V_{out} \frac{3.3k}{47k + 3.3k} \left( \frac{1}{1k} - \frac{1}{10k} \right) - V_{out} \frac{3.3k}{47k + 3.3k} + \frac{V_{out} \frac{3.3k}{47k + 3.3k} - V_{in}}{1k} \cdot 10k$$

$$\rightarrow \frac{V_{out}}{V_{in}} = 15.24 \checkmark$$

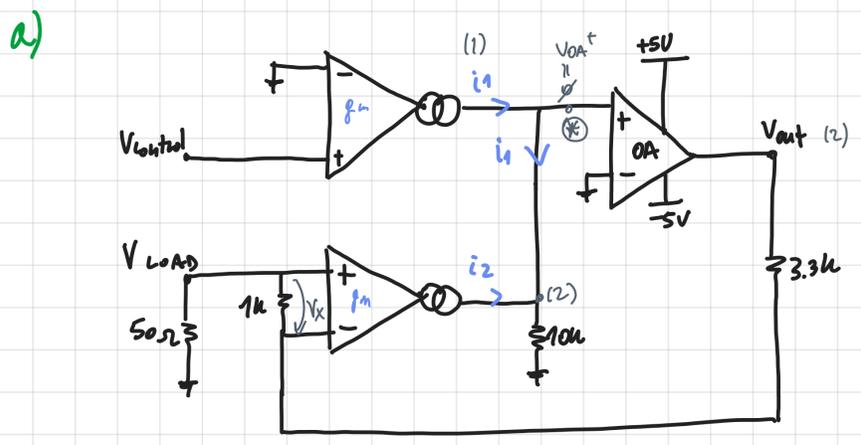
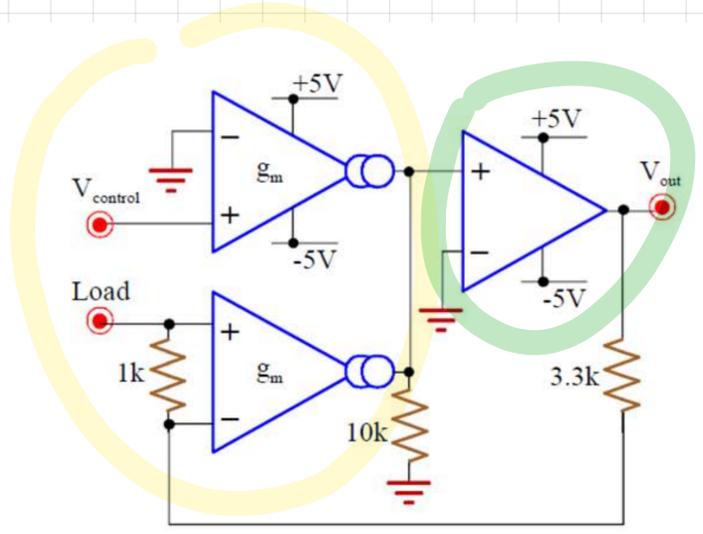


3

**Ex. 3**

OpAmp with  $A_0=100\text{dB}$ ,  $\text{GBWP}=20\text{MHz}$ ,  $I_B=10\text{nA}$ ,  $V_{OS}=0.5\text{mV}$ , **output swing  $-3\text{V} \div +3\text{V}$** .  
 OTAs with  $g_m=10\text{mS}$ , **output swing  $-4\text{V} \div +4\text{V}$** .

- Connect a  $50\Omega$  load at the Load output.  
 a) Compute  $V_{\text{Load}}/V_{\text{control}}$  and  $V_{\text{out}}/V_{\text{control}}$  and describe the function of the circuit.  
 b) Compute the error at the Load, caused by the  $I_B$  of the OpAmp.



(Always check NEG. feedback first)

- $\frac{V_{\text{LOAD}}}{V_{\text{control}}} = ?$
  - $\frac{V_{\text{out}}}{V_{\text{control}}} = ?$
- (1) increase  $i_1$   
 (2) Voltage across  $50\Omega \uparrow$ ,  $V_{\text{out}} \uparrow$   
 (3) Feedback action  $\Rightarrow i_2 \downarrow$  to decrease voltage across  $50\Omega$
- Such that  $i_1 + i_2 = i_R = 0$  (virtual ground)

We have that for OTAs:

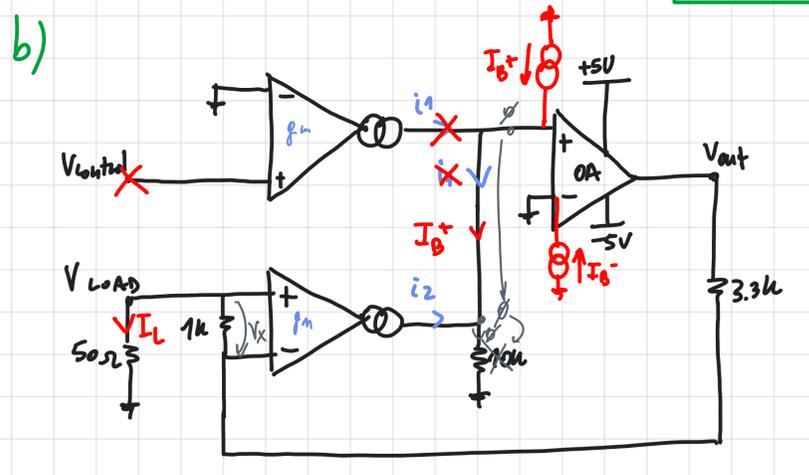
$$\begin{cases} i_1 = V_{\text{control}} \cdot g_{m1} \\ i_2 = V_x \cdot g_{m2} = V_{\text{out}} \frac{1k}{50 + 1k + 3.3k} g_{m2} \end{cases}$$

$$i_1 + i_2 = i_R = 0 \quad i_1 = -i_2 \quad (V_{\text{OA}}^+ = 0V)$$

$$V_{\text{control}} g_{m1} = V_{\text{out}} \frac{1k}{50 + 1k + 3.3k} g_{m2}$$

$$\frac{V_{\text{out}}}{V_{\text{control}}} = \frac{50 + 1k + 3.3k}{1k} = 4.35$$

$$V_{\text{LOAD}} = V_{\text{out}} \frac{50}{50 + 1k + 3.3k} \Rightarrow \frac{V_{\text{LOAD}}}{V_{\text{control}}} = 0.05$$



$\epsilon_{I_B} = 0$  (no error contribution from  $I_B^-$ )

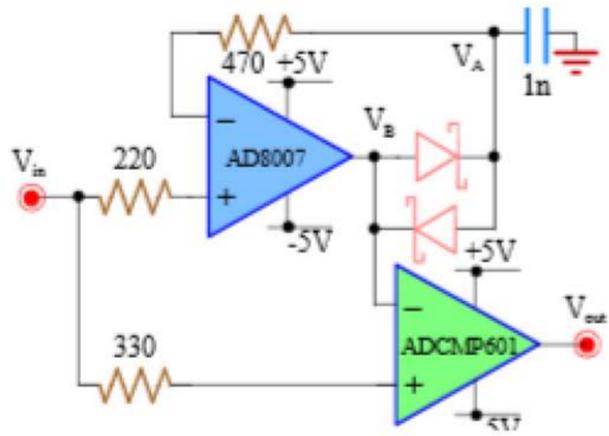
$$I_L = \frac{V_x}{1k} = \frac{i_2/g_{m2}}{1k} = \frac{I_{B^+}/g_{m2}}{1k} = 1\mu\text{A} \quad (\text{error current on the load})$$

④

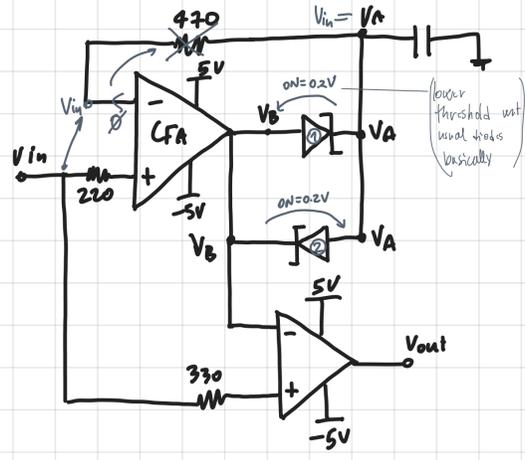
### Ex. 4

The circuit employs a comparator, a CFA amplifier and two Schottky diodes. The  $V_{in}$  input is sinusoidal with 3V peak amplitude.

a) Plot the  $V_A$ ,  $V_B$  and  $V_{out}$  waveforms and describe the function of the circuit.



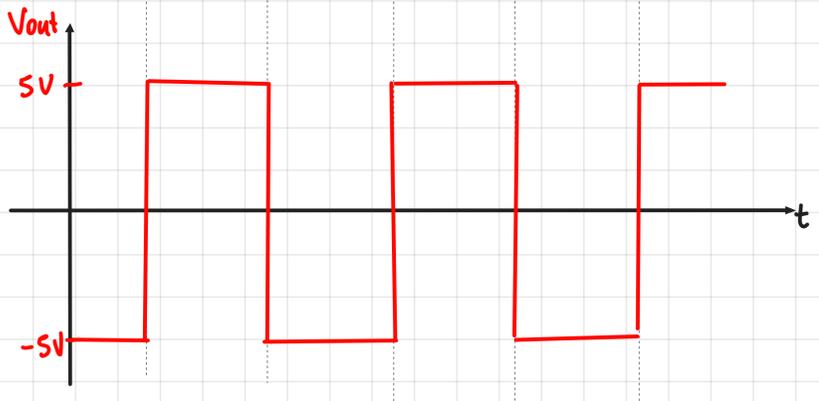
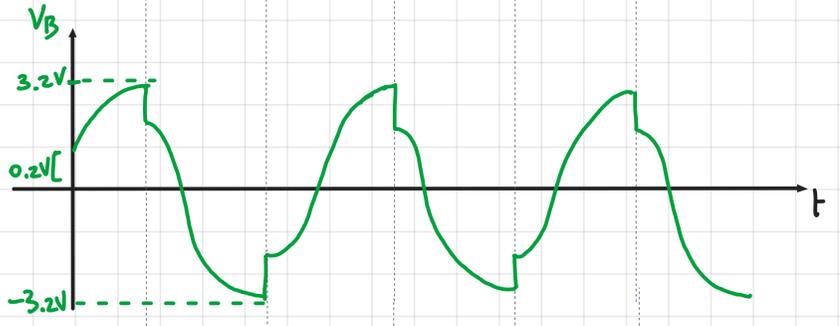
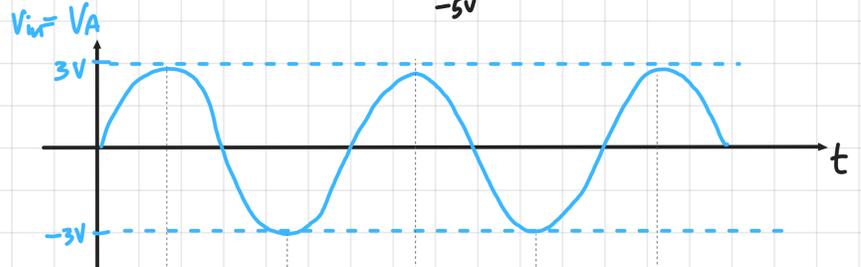
a)



$V_{in}$  sine 3V<sub>peak</sub>

$V_A = V_{in}$

- If  $V_{in} \uparrow \Rightarrow V_A \uparrow$  (Schottky ① is ON)  $\Rightarrow V_B = V_A + 0.2V$
- If  $V_{in} \downarrow \Rightarrow V_A \downarrow$  (Schottky ② is ON)  $\rightarrow V_B = V_A - 0.2V$

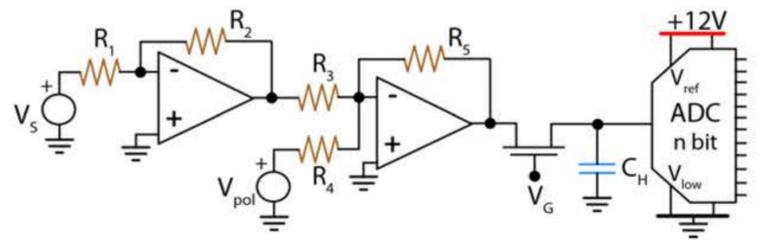


→ every time the input  $V_{in}$  reaches a peak the comparator switches

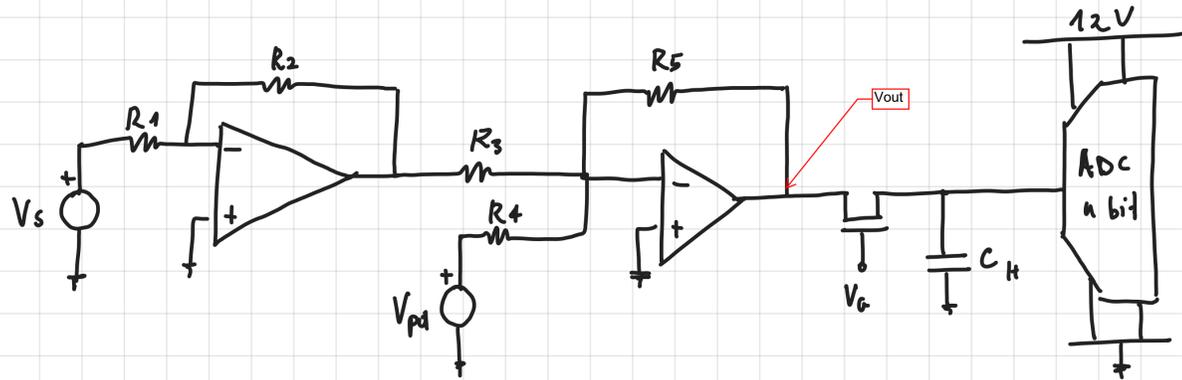
so also acts like a PEAK DETECTOR circuit

① Ex. 1

Input signal from -15mV to +25mV with a 5kHz bandwidth. Required  $V_S$  input resolution of  $10\mu V$ . ADC with  $T_{CONV}=90\mu s$ .  $R_1=4.7k\Omega$ ,  $R_3=3.3k\Omega$ ,  $R_4=15k\Omega$ ,  $R_5=47k\Omega$ ,  $C_H=100nF$ .



- a) Size  $R_2$ ,  $V_{POL}$  and  $n_{bit}$  to fully exploit the ADC input range.
- b) Select  $R_{ON}$  to provide  $< 1/2 LSB$  error during sampling at Shannon limit.



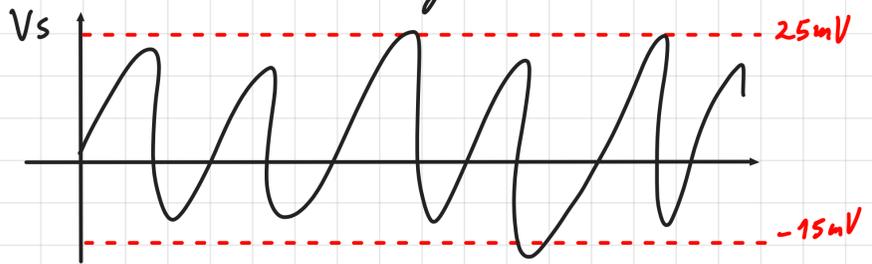
a)  $R_2=?$   $V_{pol}=?$   $n_{bit}=?$  → to exploit the full range of the ADC

→ from superimposition of effects:

$$V_{out} = V_s \left( -\frac{R_2}{R_1} \right) \left( -\frac{R_5}{R_3} \right) + V_{pol} \left( -\frac{R_5}{R_4} \right)$$

so:

From data we know the range of  $V_s$ :



- for  $V_s|_{max}$  →  $V_{out} = 25mV \left( \frac{R_2}{R_1} \right) \left( \frac{R_5}{R_3} \right) - V_{pol} \left( \frac{R_5}{R_4} \right) = 12V$  ← MAX output of ADC
  - for  $V_s|_{min}$  →  $V_{out} = -15mV \left( \frac{R_2}{R_1} \right) \left( \frac{R_5}{R_3} \right) - V_{pol} \left( \frac{R_5}{R_4} \right) = 0V$  ← MIN output of ADC
- } ADC RANGE

$$+ 40mV \left( \frac{R_2}{R_1} \right) \left( \frac{R_5}{R_3} \right) = 12V \rightarrow R_2 = \frac{12V}{40mV} \frac{R_3}{R_5} R_1 = 99k\Omega$$

$$V_{pol} = -15mV \left( \frac{R_2}{R_1} \right) \left( \frac{R_5}{R_3} \right) \frac{R_4}{R_5} = -1.44V$$

↳ we know that:  $LSB_{in} = 10\mu V \Rightarrow LSB_{out} = LSB_{in} \left( \frac{R_2}{R_1} \right) \left( \frac{R_5}{R_3} \right) = 3mV$

↳  $LSB_{out} = \frac{FSR}{2^{n_{bit}}} \Rightarrow n_{bit} = \log_2 \left( \frac{FSR}{LSB_{out}} \right) = 12 \text{ bit}$

b)  $R_{ON}?$  |  $\epsilon_{sampling} < \frac{LSB}{2}$  @  $f_{sampling} = 2f_{max} = 10kHz$  (Shannon limit)

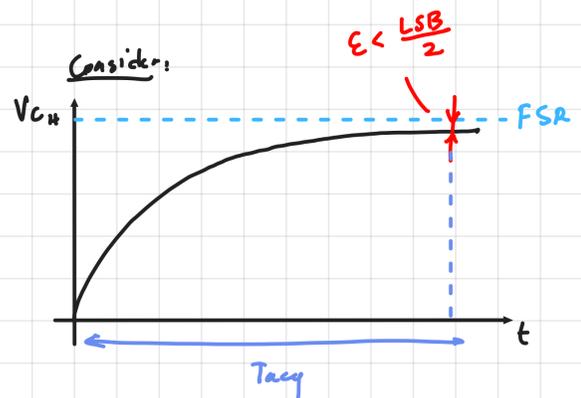
$T_{sampling} = \frac{1}{f_{sampling}} = 100\mu s \rightarrow T_{sampling} = T_{conv} + T_{acy} \rightarrow T_{acy} = 10\mu s$

$\downarrow$  100μs       $\downarrow$  90μs

$\epsilon = FSR - FSR(1 - e^{-\frac{T_{acy}}{\tau}}) < \frac{LSB}{2} \rightarrow FSR e^{-\frac{T_{acy}}{\tau}} < \frac{LSB}{2}$

$\tau = R_{ON} C_H$

↳  $\frac{T_{acy}}{\tau} > \ln(2 \cdot 2^{n_{bit}}) \rightarrow R_{ON} < \frac{T_{acy}}{C_H \ln(2^{n_{bit}})} = 11,11\Omega \rightarrow c.p. R_{ON} = 10\Omega$

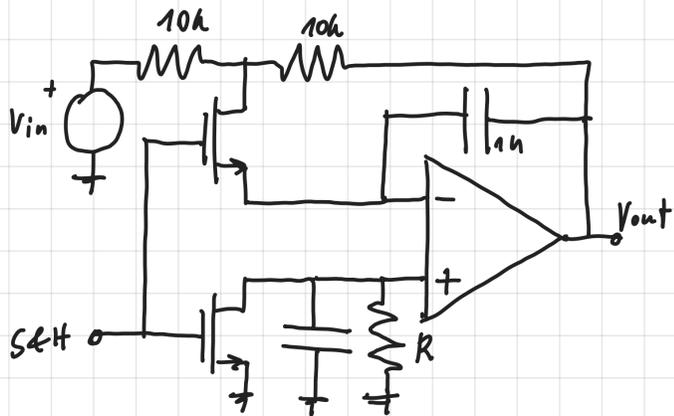
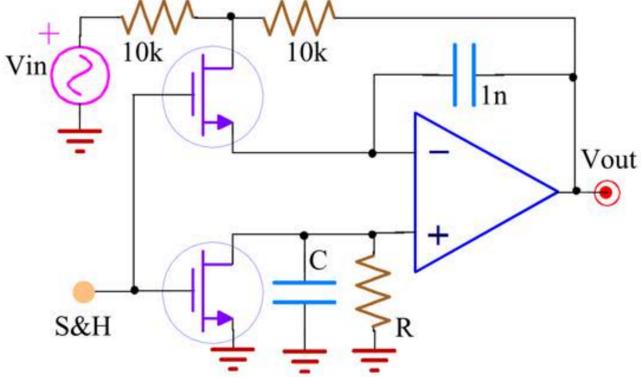


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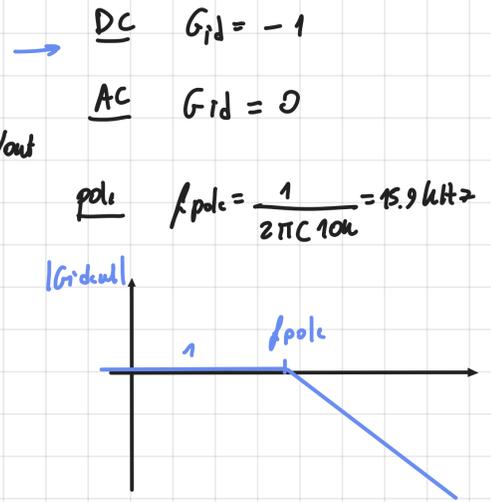
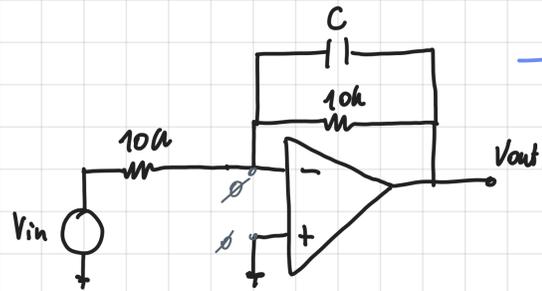
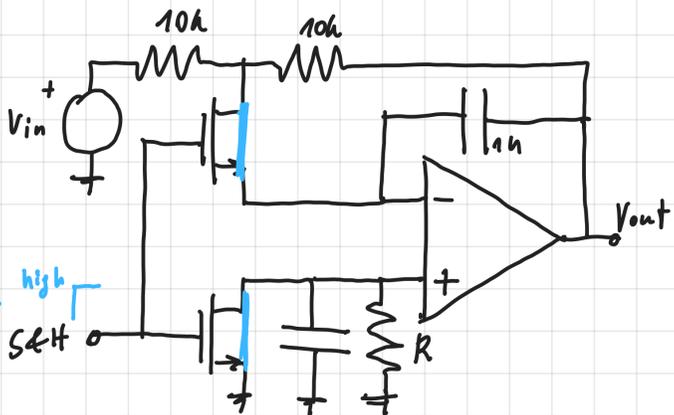
Ex. 2

Uncompensated OpAmp with  $A_0=100\text{dB}$ ,  $f_{\text{high}}=5\text{MHz}$ ,  $A_{\text{min}}=20\text{dB}$  and  $I_B=200\text{nA}$  (inward going).

- a) Compute the acquisition time for a 12 bit ADC.
- b) Consider  $V_{\text{in}}=+100\text{mV}$ ,  $C=1\text{nF}$  and  $R=100\text{k}\Omega$ , the plot the  $V_{\text{out}}(t)$  waveform during the Hold phase, lasting 1ms.
- c) Properly modify and size the RC network in order to **compensate charge injection and bias currents**.

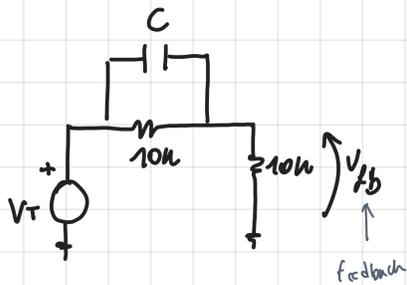


a)  $T_{\text{acq}}=?$  (12 bit ADC)



Let's now study the real gain, consider:

$\beta(s)$

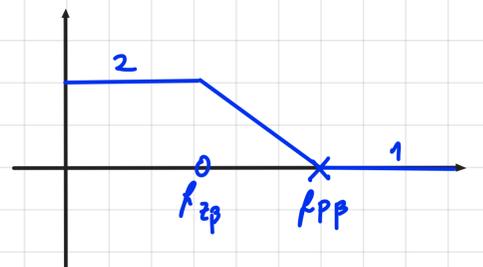


$\xrightarrow{\text{DC}} \beta(s)|_{\text{DC}} = \frac{V_{\text{fb}}}{V_{\text{T}}} = \frac{10k}{10k+10k} = \frac{1}{2}$

$\xrightarrow{\text{AC}} \beta(s)|_{\text{AC}} = 1$

$\text{pole } f_{\text{p}\beta} = \frac{1}{2\pi C (10k\Omega || 10k\Omega)} = 31.8 \text{ kHz}$

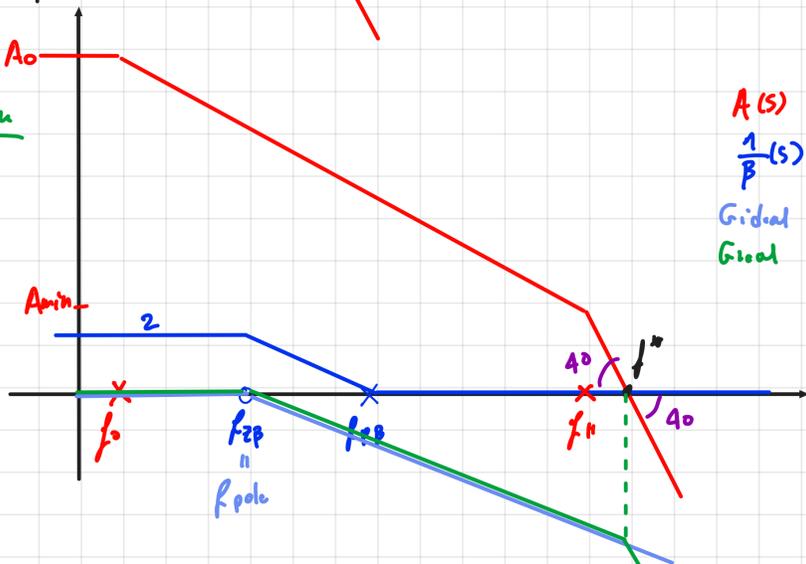
$\text{zero } f_{\text{z}\beta} = \frac{1}{2\pi C 10k\Omega} = 15.9 \text{ kHz}$



$A(s)$



Real gain



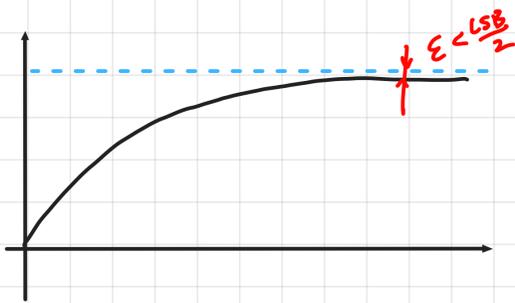
cosine angle:  $40-40$  (UNSTABLE)

$f^* = \sqrt{A_{\text{min}} f_{\text{H}}} = 15.8 \text{ MHz}$

PHASE MARGIN:

$\phi_m = 180^\circ - \arctan\left(\frac{f^*}{f_0}\right) + \arctan\left(\frac{f^*}{f_{z\beta}}\right) - \arctan\left(\frac{f^*}{f_{p\beta}}\right) - \arctan\left(\frac{f^*}{f_{\text{H}}}\right) = 17.56^\circ$

So, now we consider the error:



$$\epsilon = FSR e^{-\frac{T_{\text{acq}}}{\tau}} < \frac{LSB}{2}$$

$$T_{\text{acq}} = \tau \ln \left( \frac{\Delta V_{H_{\text{max}}}}{\epsilon H} \right)$$

$$\frac{\Delta V_{H_{\text{max}}}}{\epsilon H} = \frac{FSR}{\frac{LSB}{2}} = \frac{FSR}{\frac{FSR}{2^{13} \cdot 2}} = 2^{13}$$

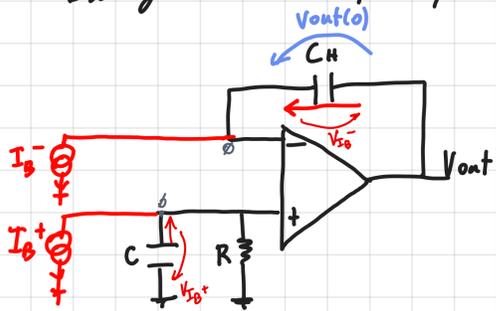
$$\tau = \frac{1}{2\pi f_{\text{pole}}} = 10 \mu\text{s}$$

15.9 kHz

$$\Rightarrow T_{\text{acq}} = 90.1 \mu\text{s}$$

b)  $V_{in} = 100 \text{ mV}$   $C = 1 \mu\text{F}$   $R = 100 \text{ k}\Omega$   $\rightarrow V_{out}(t) = ?$

During the HOLD phase, the circuit:



$$\rightarrow V_{out}(0) = -100 \text{ mV}$$

now let's consider the effect of the bias currents:

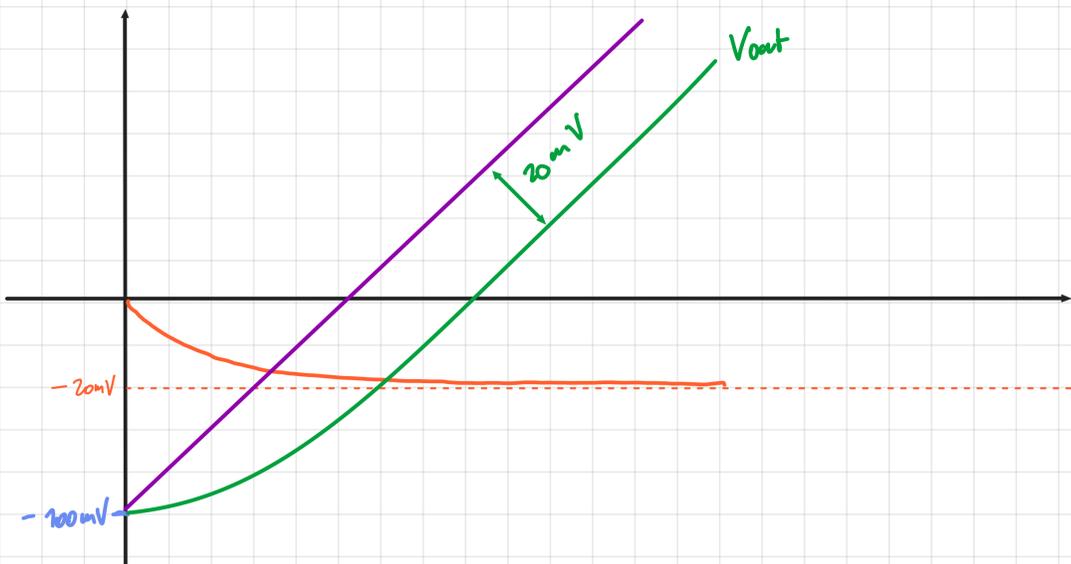
$I_{B-}$ :  $\rightarrow$  We'll have a voltage drop ( $V_{I_{B-}}$ ) due to the current  $I_{B-}$  going through  $C_H$ :

$$\frac{dV_{out}}{dt} = \frac{I_{B-}}{C_H} = 200 \frac{V}{s}$$

$I_{B+}$ :  $\rightarrow$  We'll have a voltage drop ( $V_{I_{B+}}$ ) on  $C$ :

$$\rightarrow V^+ = \frac{I_{B+} R}{20 \text{ mV}} (e^{-\frac{t}{\tau}} - 1)$$

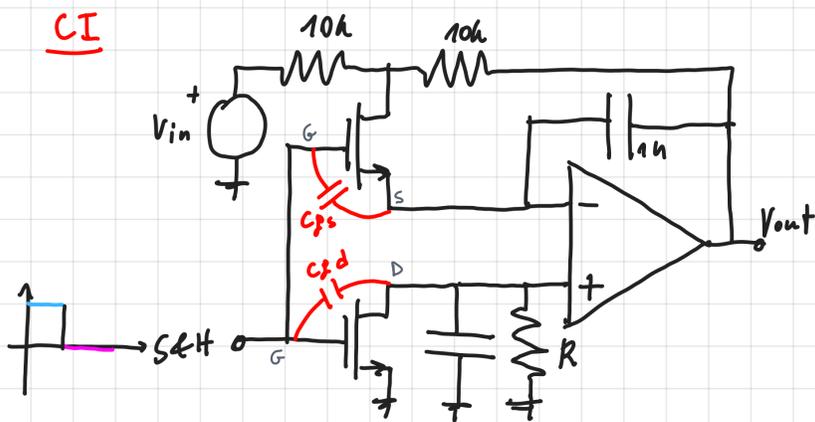
$$V_{out}(t) = V_{out}(0) + \frac{dV_{out}}{dt} \cdot t + 20 \text{ mV} (e^{-\frac{t}{\tau}} - 1)$$



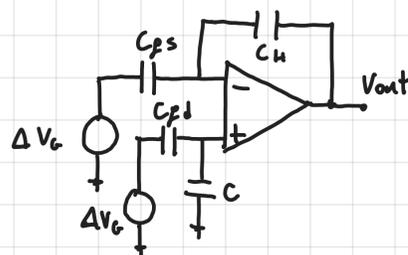
$\rightarrow$  then for  $t = t_{\text{hold}} = 1 \mu\text{s}$   
we can exactly compute  $V_{out}(1 \mu\text{s})$

c)  $R = ?$   $C = ?$  | compensate charge injection (CI) & bias current ( $I_B$ )

CI



$\rightarrow$  We can simplify the circuit to consider this effect:



error coming from CI:

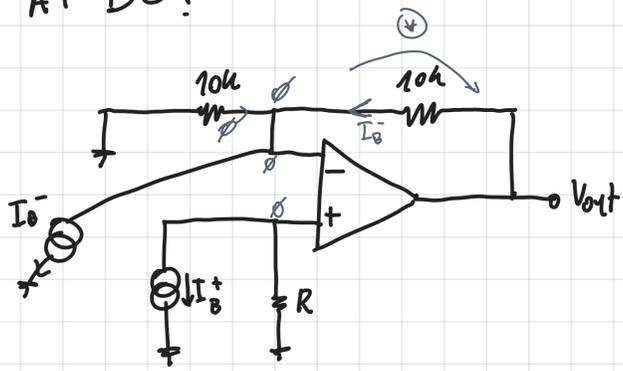
$$\epsilon_{CI} = -\Delta V_g \frac{C_{ps}}{C_H} + \Delta V_g \frac{C_{pd}}{C_{pd} + C} \left( 1 + \frac{C_{ps}}{C_H} \right)$$

$\rightarrow$  now let's consider  $C_{ps} \approx C_{pd}$  (Hp.) so this expression simplifies: (and  $C = C_H$ )

$$\epsilon_{CI} = -\Delta V_g \frac{C_{ps}}{C_H} + \Delta V_g \frac{C_{pd}}{C_{pd} + C} \left( \frac{C_H + C_{ps}}{C_H} \right) = -\Delta V_g \frac{C_{ps}}{C_H} + \Delta V_g \frac{C_{pd}}{C_H} = 0$$

Compensated ✓  
if  $C_{pd} = C_{ps} + C = C_H$

$I_B$  At DC:



$I_B^+$   $\rightarrow$  no contribution on the out.

$I_B^-$   $\rightarrow \epsilon = I_B^- 10k\Omega \oplus$

$$\epsilon = I_B^- 10k\Omega - I_B^+ R \left( 1 + \frac{10k\Omega}{10k\Omega} \right) \rightarrow \text{for } \epsilon = 0 \Rightarrow R = 5k\Omega$$

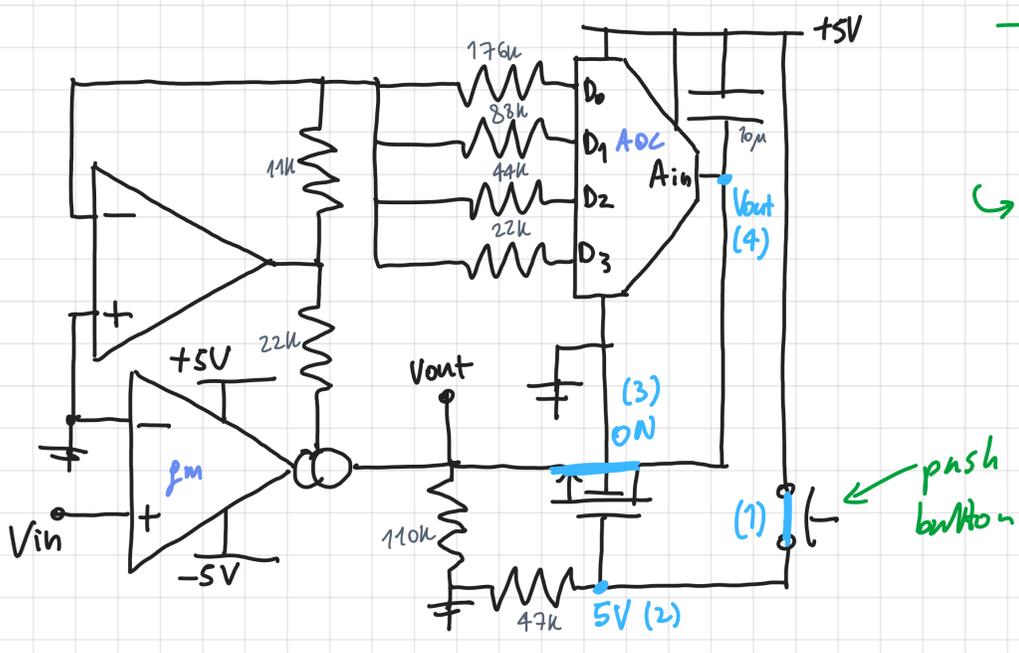


$i_{out} = V_{in} \cdot f_m$

$V_{out} = 110k\Omega \cdot f_m V_{in} = V_{in} \cdot 9mS \left(1 - \frac{D_{out}}{16}\right) 110k\Omega$

$\Rightarrow \frac{V_{out}}{V_{in}} = 1000 \left(1 - \frac{D_{out}}{2^4}\right)$  so the gain also depends on the ADC output digital word

b)



→ What happens if we press for a short amount of time the push-button?

↳ By pressing the button

(4): ADC will convert  $V_{out}$  voltage

↳  $A_{in} = V_{out}(0)$

→ We've seen that the gain  $\frac{V_{out}}{V_{in}}$  is dependent on the digital output of the ADC

↳ So if we compute the digital output in this case:

$D_{out} = \frac{A_{in}}{5V} 2^4 = \frac{V_{out}(0)}{5V} 2^4 \rightarrow V_{out} = V_{in} \cdot 1000 \left(1 - \frac{V_{out}(0)}{5V}\right)$

↳  $V_{out}$  depends on  $V_{out}(0)$

⇒ By pressing instantaneously the button we're modifying the T.F. between in and out

c) Push-button is kept press for a long time ⇒ it's like we're closing the feedback

PUSH-BUTTON ON ⇒ key feedback

( $A_{in} = V_{out}$  always, not just for some instant)

$D_{out} = \frac{A_{in}}{5V} \cdot 2^4$

↳  $V_{out} = V_{in} \cdot 1000 \left(1 - \frac{V_{out}}{5V}\right)$

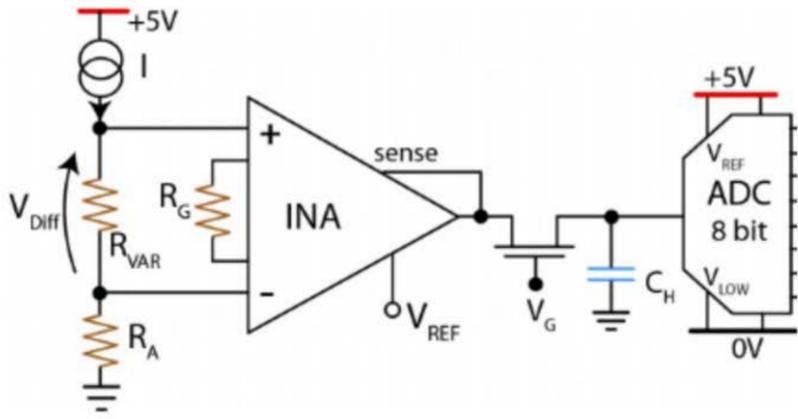
↳  $V_{out} = V_{in} \cdot 1000 - V_{out} V_{in} \frac{1000}{5}$

I.F. (gain):  $V_{out} = \frac{V_{in} \cdot 1000}{1 + V_{in} \cdot 200}$

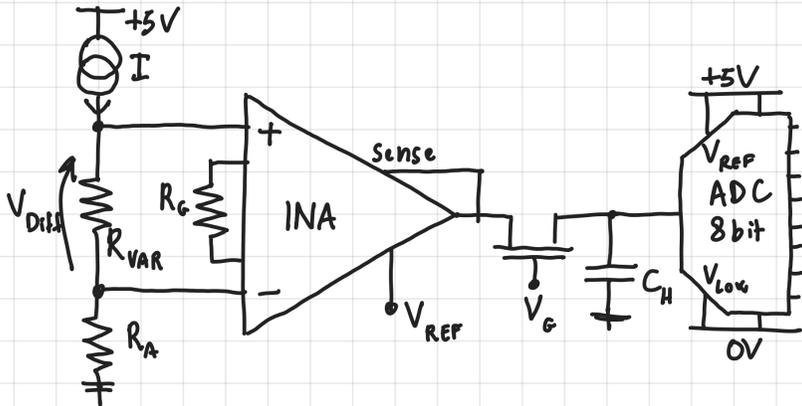
4

Ex. 4

A breath sensor measures the chest extension through an  $R_{VAR} = 10k\Omega \cdot (1 + \alpha \cdot d)$ , where  $\alpha = 0.02mm^{-1}$  and  $d$  is in the 0-50 mm range.  $R_A = 10k\Omega$ ,  $I = 100\mu A$ ,  $C_H = 10nF$ . INA's internal resistors are  $R_f = 10k\Omega$ . The SAR ADC has 50nA input leakage.



- a) Size  $R_G$  and  $V_{REF}$  for exploiting the whole ADC's FSR.
- b) With  $V_G$  swinging between 0V and 10V and an n-MOSFET with  $V_T = 1V$  and  $C_{gs} = 100pF$ , compute aperture charge-induced error range over  $R_{VAR}$ .
- c) Select  $T_{conv}$  and  $T_{clock}$  to achieve a precision better than  $1/2LSB$



$R_{VAR} = 10k(1 + \alpha d)$       $\alpha = 0.02 mm^{-1}$       $d = 0 \div 50mm$

a)  $R_G = ?$ ,  $V_{REF} = ?$  | exploits FSR of ADC

$R_{VAR}|_{MIN} = 10k(1 + 0.02 mm^{-1} \cdot 0) = 10k\Omega$

$R_{VAR}|_{MAX} = 10k(1 + 0.02 mm^{-1} \cdot 50mm) = 20k\Omega$

*we can now compute  $V_{diff\ min}$  &  $V_{diff\ max}$*

$V_{diff\ min} = R_{VAR}|_{MIN} I = 1V$

$V_{diff\ max} = R_{VAR}|_{MAX} I = 2V$

$\Delta V_{diff} = V_{diff\ max} - V_{diff\ min} = 1V$

At the output we'll have a variation:

$\Delta V_{out} = FSR = 5V$

*Variations of the INPUT*

From the INA formulas:

$\Delta V_{out} = \left(1 + \frac{2R_f}{R_G}\right) \Delta V_{diff} = 5V \implies R_G = 5k\Omega$

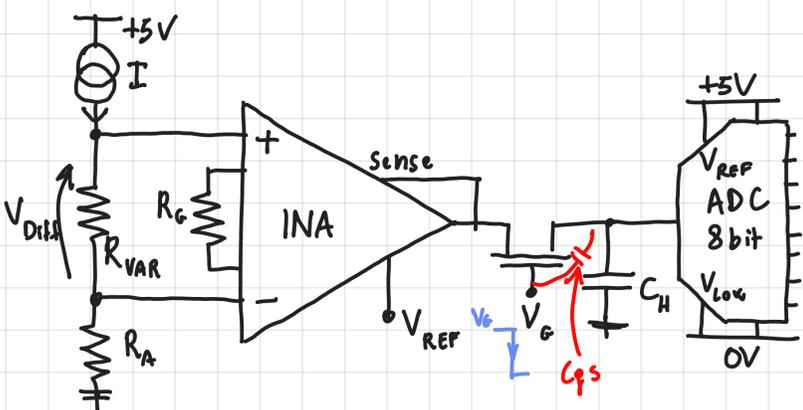
Now we have to fix the bias such that when we use  $R_{VAR}|_{min} \implies V_{out} = 0$

$R_{VAR}|_{max} \implies V_{out} = 5V$

We can do it by choosing  $V_{REF}$ :

$V_{out} = V_{diff\ min} \cdot G_{INA} + V_{REF} = 0 \implies V_{REF} = -5V$

b)



*Vg during sampling phase depends on the out of the ina  $V_o + V_T$*

*ECI*  
 $\Delta V_G = V_o + V_T - V_{G\ hold}$

**BEST CASE**  $V_{out} = 0V \implies \Delta V_G = +1V$

$\epsilon_{out} = \Delta V_G \frac{C_{gs}}{C_{gs} + C_H} = 10mV \implies \epsilon_{RVAR} = \frac{\epsilon_{out}}{G_{INA}} = 2mV$

**WORST CASE**  $\Delta V_G = 6V$

$\epsilon_{out} = 60mV \implies \epsilon_{RVAR} = 12mV$

c)  $T_{conv} = ?$   $T_{clk} = ?$  | precision  $< \frac{LSB}{2}$

$$LSB = \frac{FSR}{2^{nbit}} = \frac{5V}{2^8} = 19.5 mV$$

$$T_{conv} = \frac{LSB}{2 \cdot C_H \cdot I_{leakage}} = 1.95 ms$$

$$[T_{conv} = (n+1)T_{clk}] \implies T_{clk} = \frac{T_{conv}}{9} = 216.6 \mu s$$

5) ex on phot solutions

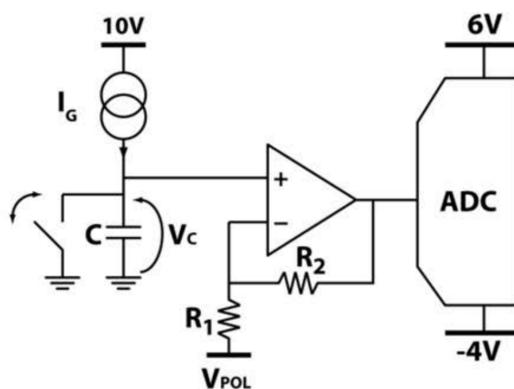
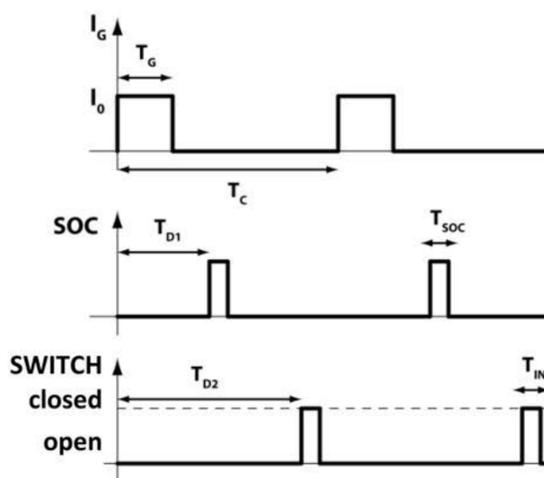
### Ex. 5

Let us measure the C value with the timings shown in figure. The ADC conversion is triggered by the SOC rising-edge and lasts  $T_{CONV} = 1 \mu s$ .

$R_1 = 4.7 k\Omega$ ,  $R_2 = 100 k\Omega$ ,  $V_{POL} = 0.25V$ ,  $T_{D1} = 2 \mu s$ ,  $T_{D2} = 4 \mu s$ ,  $T_{SOC} = 0.2 \mu s$ ,  $T_{INT} = 0.2 \mu s$ .

The  $I_G = 0.5mA$  current generator is pulsed with duration  $T_G = 1.2 \mu s$  and period  $T_C = 5 \mu s$ . OpAmp biased at  $\pm 10V$ .

- Plot  $V_C(t)$  for  $C = 2nF$  (be  $V_C(0) = 0V$  for  $t = 0s$ ).
- Determine the range of measurable C values.

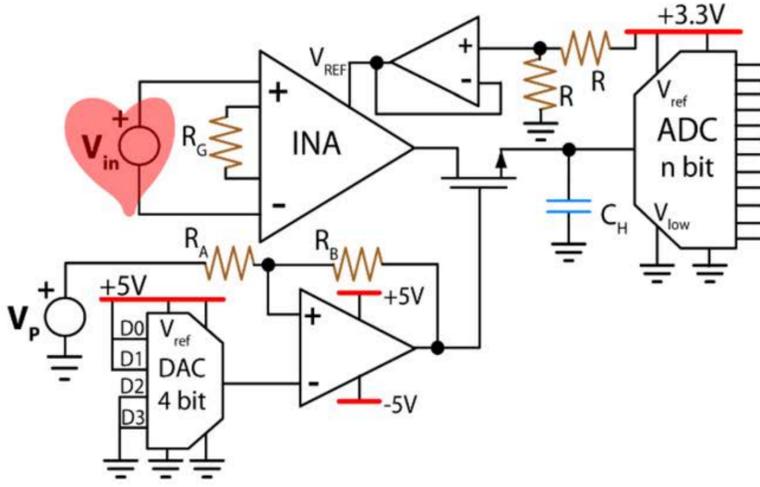


E04 (Burrho)

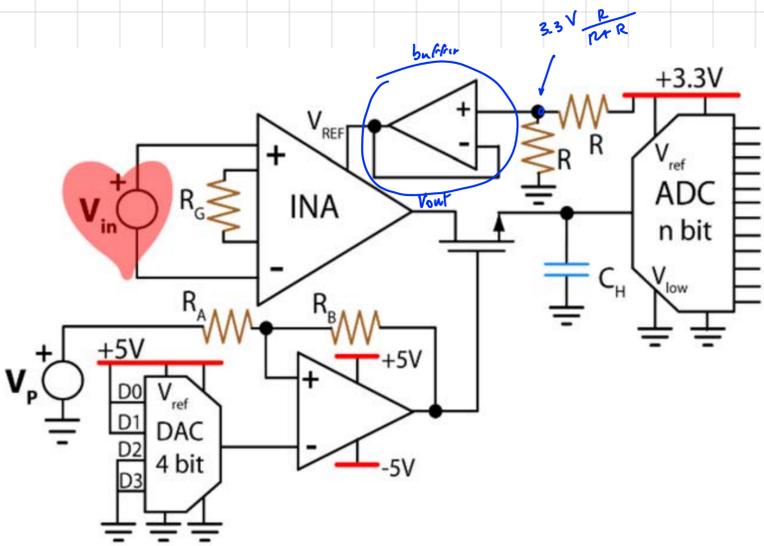
Ex. 1

$R_A=1k\Omega$ ,  $R_B=2k\Omega$ ,  $C_H=100nF$ . INA with internal  $R_F=10k\Omega$ . Input ECG signal  $V_{in}$  ( $-1mV \div +9mV$ ) sampled when pulse oximetry signal  $V_P$  reaches the threshold set by the DAC. MOSFET with  $V_T=0.8V$  and  $R_{DS\ on} < 100\Omega$ .

- Properly size  $R_G$  so to exploit ADC dynamics at its best and choose the number  $n$  of bits to provide 50ppm FSR resolution.
- Compute the  $V_P$  values that start sampling and hold phases, respectively, of the S&H.
- Choose the parasitic  $C_{gs\ max}$  that ensures a charge injection lower than  $100\mu V$ .



a)  $R_G=?$   $n\ bit=?$  50ppm FSR



$V_{out} = V_{in} G_{INA} + V_{ref}$   
 $V_{ref} = 3.3V \frac{R}{R+R} = 1.65V$   
 $V_{out\ max} = V_{in\ max} G_{INA} + V_{ref} \rightarrow G_{INA} = \frac{V_{out\ max} - V_{ref}}{V_{in\ max}} = 183.3$

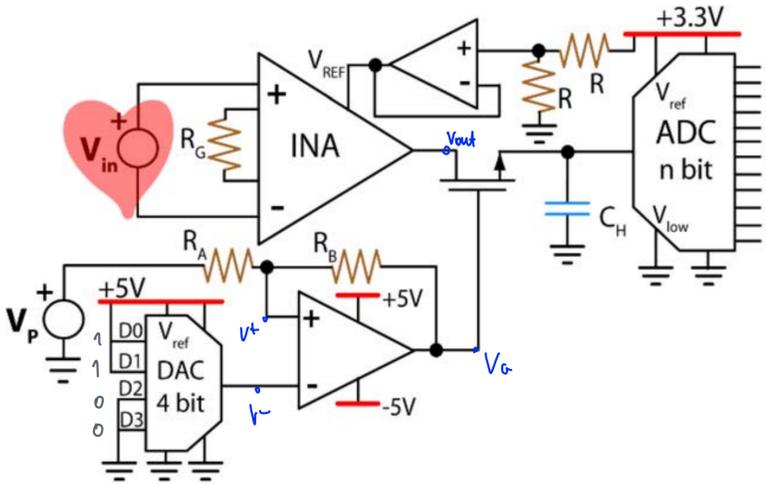
We know, from theory, that:  $G_{INA} = 1 + \frac{2R_F}{R_G} \rightarrow R_G = \frac{2R_F}{G_{INA}-1} = 110\Omega$

LSB = 50ppm FSR

$\frac{FSR}{2^{n\ bit}} = LSB = \frac{50}{10^6} FSR \rightarrow n\ bit \geq \log_2 \left( \frac{10^6}{50} \right) = 14.28$

$\rightarrow n\ bit = 15\ bit$

b)  $V_{ps}=?$   $V_{ph}=?$



- $V^+ > V^- \rightarrow V_G = +5V \rightarrow MOS\ ON \rightarrow S$
- $V^+ < V^- \rightarrow V_G = -5V \rightarrow MOS\ OFF \rightarrow H$

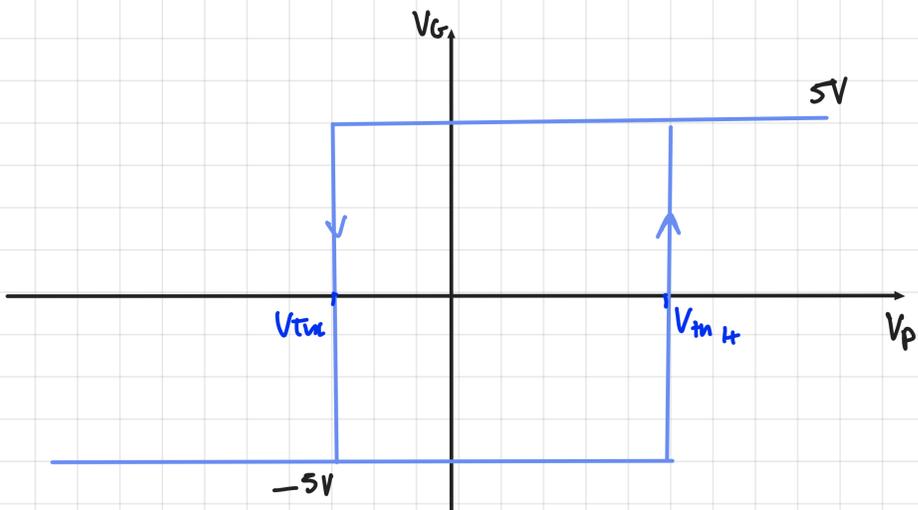
Compute the thresholds of the comparator:

$V^- = \frac{V_{ref}}{2^4} D_{in} = \frac{V_{ref}}{2^4} (D_0 2^0 + D_1 2^1 + D_2 2^2 + D_3 2^3) = 937.5\ mV$   
 $0011b = 3_{10}$

$V_{thH} \quad V_G = -5V$   
 Superposition effect:  
 $V^+ = \underbrace{V_P \frac{R_B}{R_A+R_B}}_{V_P\ ON, V_G\ OFF} - \underbrace{5V \frac{R_A}{R_A+R_B}}_{V_P\ OFF, V_G\ ON} \geq 937.5\ mV$   
 $\rightarrow V_P \geq 937.5\ mV \frac{(R_A+R_B)}{R_B} + 5V \frac{R_A}{R_B} = 3.91V \quad V_{thH}$

$V_{thL} \quad V_G = 5V$   
 $V^+ = V_P \frac{R_B}{R_A+R_B} + 5V \frac{R_A}{R_A+R_B} < 937.5\ mV$

$\rightarrow V_P < -1.09V \quad V_{thL}$



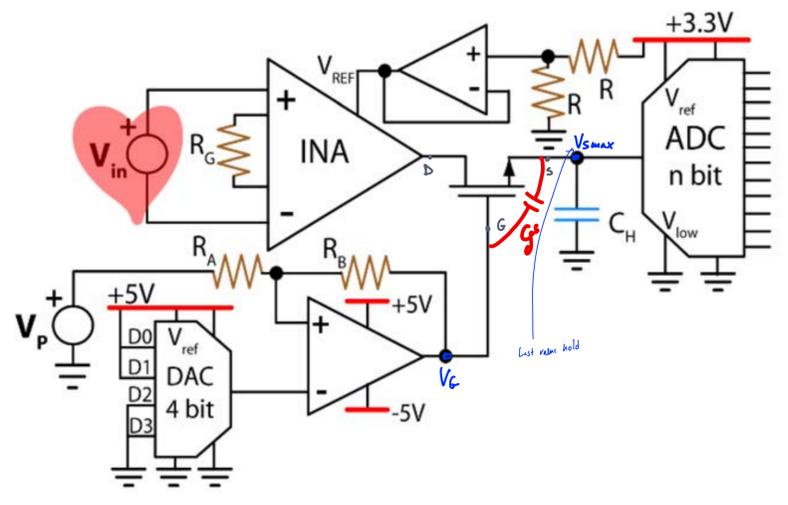
c)  $E_{ci} < 100 \mu V$   $C_{psmax} = ?$

•  $\Delta V_G = V_s^{max} + V_T - (-5V) = 9.1 V$

$\uparrow$              $\uparrow$              $\uparrow$   
 3.3V        0.8V         $V_G$

•  $E_{ci} = \Delta V_G^{max} \frac{C_{ps}}{C_{ps} + C_H} < 100 \mu V$

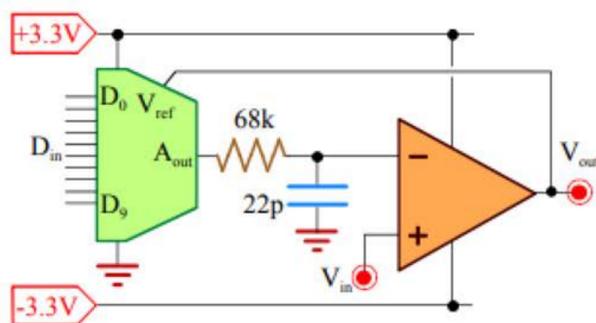
$\hookrightarrow C_{ps} < C_H \left( \frac{1}{\frac{\Delta V_G^{max}}{E_{ci}} - 1} \right) \longrightarrow C_{ps} < 1.1 pF$



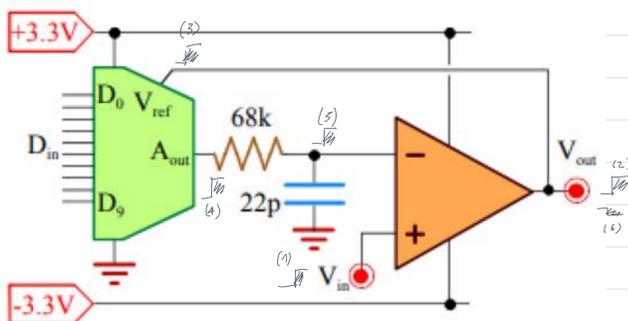
2

Ex. 2

- OpAmp biased at  $\pm 3.3V$ . DAC single-battery operated.
- Plot the  $V_{out}$  vs.  $V_{in}$  relationship and its dependence on the 10bit  $D_{in}$  digital bus content.
  - Discuss bandwidth and stability when  $A_0=100dB$  and  $GBWP=10MHz$  depending on the  $D_{in}$  value.



a) 10 bit,  $V_{out}$  vs  $V_{in}$



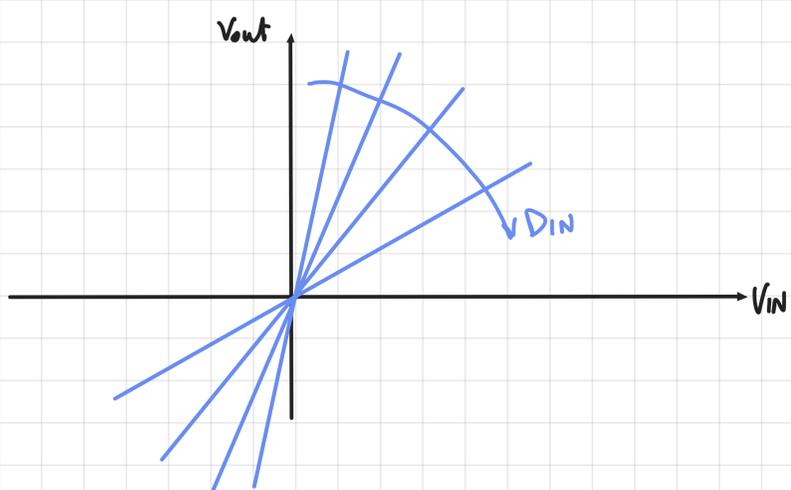
- $V_{out} = V_{ref}$   
(negative feedback)
- Neg. feed  $\rightarrow A_{out} = V_{in}$

$$A_{out} = \frac{V_{ref}}{2^{10}} D_{in}$$

$$V_{in} = \frac{V_{out}}{2^{10}} D_{in} \rightarrow V_{out} = V_{in} \frac{2^{10}}{D_{in}}$$

$$D_{in}^{max} = 2^{10} - 1 \rightarrow V_{out} = V_{in} \text{ (Buffer)}$$

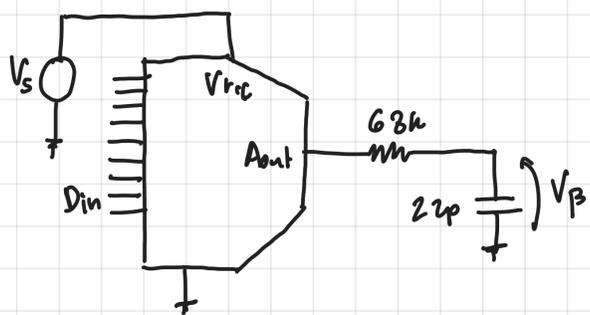
$$D_{in}^{min} = 0 \rightarrow G \rightarrow \infty \text{ (comparator)}$$



b)  $A_0 = 100dB$   $GBWP = 10MHz$  Stability?

$A(s) \rightarrow OA$

$\beta(s) \rightarrow$

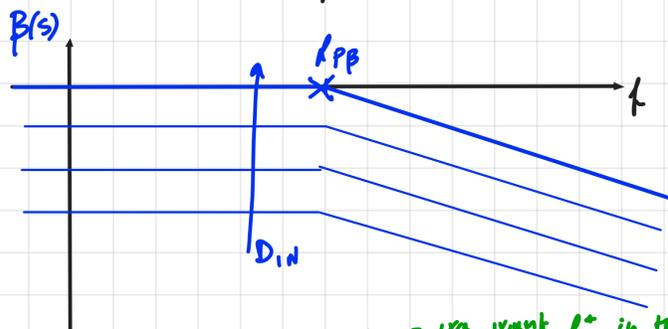


$$A_{out} = \frac{V_s}{2^{10}} D_{in}$$

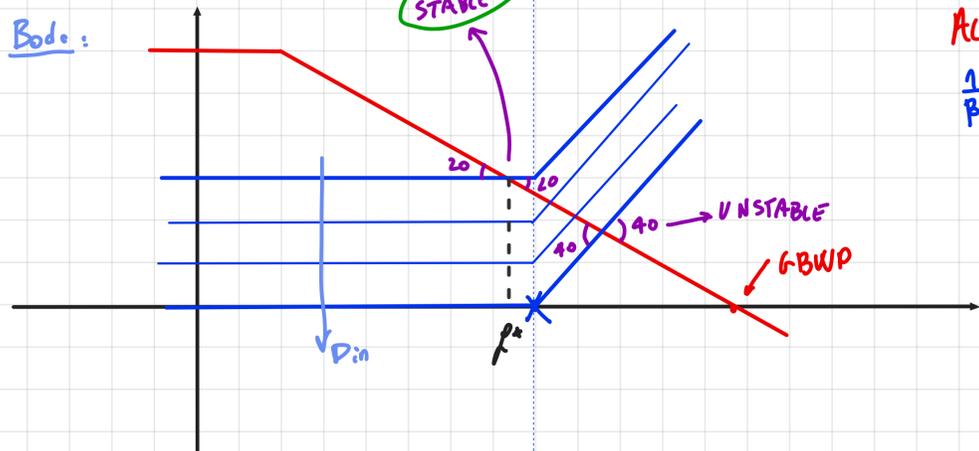
$$\beta(s) = \frac{D_{in}}{2^{10}}$$

$$\beta(\infty) = 0$$

$$f_{PB} = \frac{1}{2\pi \cdot 22p \cdot 68k} = 106 \text{ kHz}$$



we want  $f^*$  in this intersection point



$A(s)$

$\frac{1}{\beta(s)}$

$f^* < f_{PB}$  (for stability)

$$\frac{GBWP}{\frac{1}{\beta(s)}} \leq f_{PB} \rightarrow \frac{GBWP}{2^{10}} D_{in} \leq f_{PB}$$

$$D_{in} \leq \frac{f_{PB} \cdot 2^{10}}{GBWP} = 10.85$$

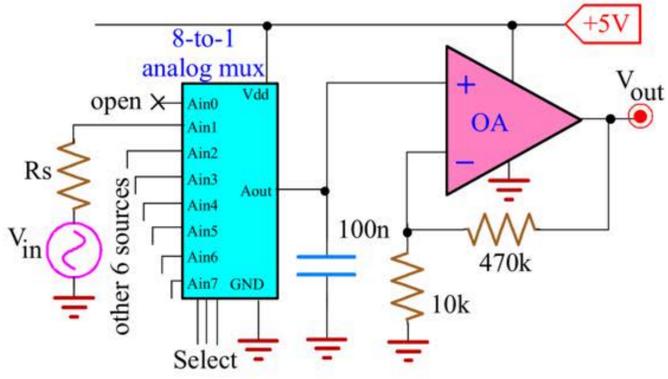
$\Rightarrow D_{in,max} = 10$

3

**Ex. 3**

The circuit acquires 7 signals in the range 0÷100mV.  $R_S$  is in the 500Ω÷2kΩ range. The OpAmp has  $I_{bias}=1nA$ ,  $V_{OS}=1mV$ ,  $A_0=50V/mV$ . The mux has  $R_{on}=2÷20Ω$ ,  $R_{off}=3MΩ÷30MΩ$  and  $I_{leakage}=4nA$ .

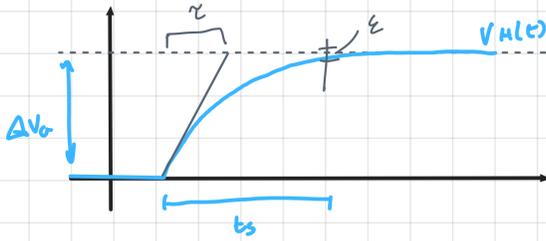
- a) Compute the sampling time (when Sel=001) and the hold time (Sel=000) to attain an accuracy of 16bit.
- b) Determine the maximum static error at the output.



d)  $t_s = ?$  (Sel=001)  $t_H = ?$  (Sel=000) | Accuracy of nbt=16 bit

*Sampling* *Hold*

Consider the sampling phase:



$$V_H(t) = \Delta V_H (1 - e^{-\frac{t}{\tau}})$$

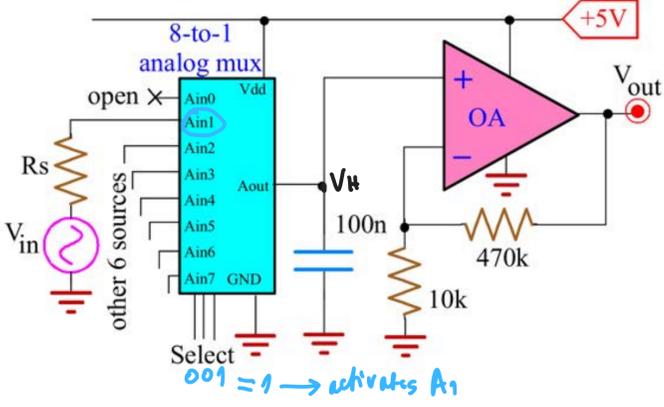
$$\Delta V_H (1 - e^{-\frac{t_s}{\tau}}) = \Delta V_H - \epsilon$$

$$\epsilon = \Delta V_H e^{-\frac{t_s}{\tau}} < \bar{\epsilon}_s \rightarrow \bar{\epsilon}_s = \frac{LSB}{2}$$

*max error:*

$$t_s > \tau \ln \left( \frac{\Delta V_H}{\bar{\epsilon}_s} \right)$$

$$\ln \left( \frac{\Delta V_H}{\bar{\epsilon}_s} \right) < \frac{t_s}{\tau}$$



$$\tau = C_H \left[ (R_S^{max} + R_{on}^{max}) \parallel \frac{R_{off}^{max}}{6} \right] \approx 100nF \cdot 2k\Omega = 200\mu s$$

*# Channels*

We have to consider the worst case (for  $R_S^{max}$ ,  $R_{on}^{max}$ ,  $R_{off}^{max}$ )

to compute the minimum sampling time  $t_s$

$$\Delta V_H^{max} = 100mV$$

$$\bar{\epsilon}_s = \frac{LSB}{2} \cdot \frac{1}{G}, \quad LSB = \frac{FSR}{2^{16}} = \frac{5V}{2^{16}} = 76.3\mu V$$

*Hp. of the possible following ADC (seeing that the P.S are always max 5V)*

$$G = \left( 1 + \frac{470k}{10k} \right) = 48$$

*OpA: non inverting stage*

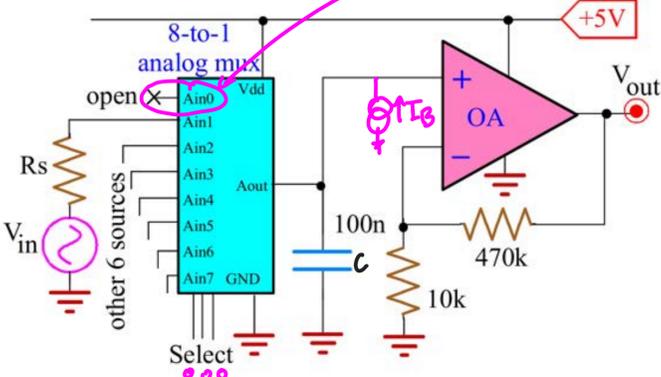
$$\bar{\epsilon}_s = \frac{LSB}{2} \cdot \frac{1}{G} = 795\mu V$$

$$t_{s_{min}} = \tau^{max} \ln \left( \frac{\Delta V_H^{max}}{\bar{\epsilon}_{s_{out}}} \right) = 2.37ms \quad (Sel=001)$$

Consider now the holding phase for Sel=000

Don just for Ain0, (from Ain1 - Ain7 all in Roff)

2 leakage currents (one for each side of the switch)



$$\frac{I_{leak_{TOT}}}{C} \cdot t_H = \epsilon < \bar{\epsilon}_H = \frac{LSB}{2} \rightarrow t_H < \frac{C \bar{\epsilon}_H}{I_{leak_{TOT}}}$$

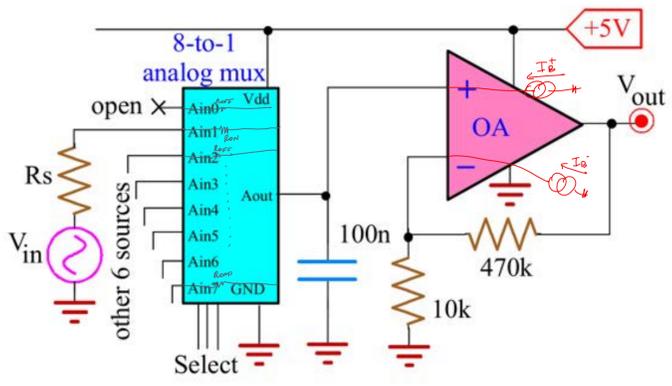
linear discharge of the C capacitance

$$I_{leak_{TOT}} = I_B + (7+2) I_{leak}$$

*bias current*  
*OFF (Ain1-7)*  
*2 for ON (Ain0)*

$$t_H^{max} = \frac{C_H \bar{\epsilon}_H}{I_B + 9I_{leak}} = 2.15\mu s$$

b) Static errors



• Static error due to  $R_{OFF}$  voltage divider (during sampling phase)

$$\epsilon_H^I = 100mV \frac{R_s^{max} + R_{ON}^{max}}{R_s^{max} + R_{ON}^{max} + \frac{R_{OFF}^{min}}{6}} = 460\mu V$$

$$\epsilon_{Hout}^I = \epsilon_H^I G = 22mV$$

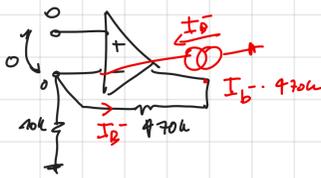
• Static error due to bias current  $I_B^+$  and  $I_{leak}$

$$\epsilon_{Hout}^n = G \epsilon_n^n = 48 (I_B^+ + 9 I_{leak}) \left[ \frac{R_{OFF}^{max}}{6} \parallel (R_s^{max} + R_{ON}^{max}) \right] = 3.59mV$$

↑  
 same previous reasoning → 2 load for Ain0  
 1 for the rest

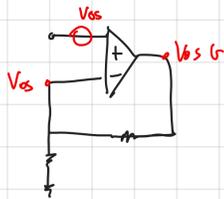
• Static error due to bias current  $I_B^-$

$$\epsilon_{Hout}^{III} = I_B^- \cdot 470k = 470\mu V$$



• Static error due to offset  $V_{os}$

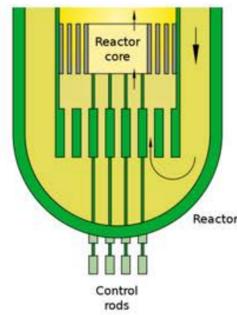
$$\epsilon_{Hout}^{IV} = V_{os} G = 48mV$$



4

### Ex. 4

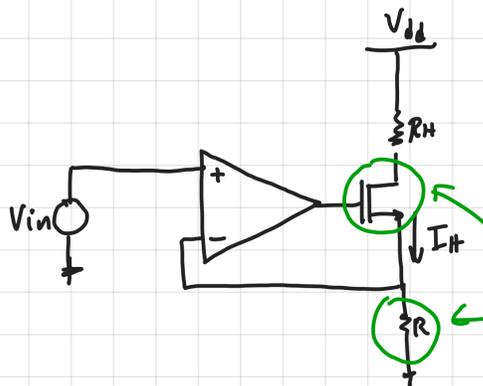
A reactor heater must be **biased at constant current**, within the 50mA-500mA range. The heater has a **nominal**  $R_{heater}=10\Omega$ . OpAmps have just 10 mA *output driving capability*.



- a) **Linearly** drive the heater through a voltage input, from  $V_{in}=0V$  (providing  $I_{heater}=50mA$ ) to  $V_{in}=5V$  ( $I_{heater}=500mA$ ) and all other values in the between (with linear relationship), after filtering with **three 100Hz low-pass poles**.
- b) Turn on a red LED whenever the voltage across the pump differs from the expected nominal value (**at the set current**) by more than 100mV.

a) We know that just the OpAmp is not enough to drive our circuit

We have:



$$I_H = \frac{V_{in}}{R}$$

- $V_{in}=0 \rightarrow I_H = 50 \text{ mA}$
- $V_{in}=5V \rightarrow I_H = 500 \text{ mA}$
- 3 @ 100Hz

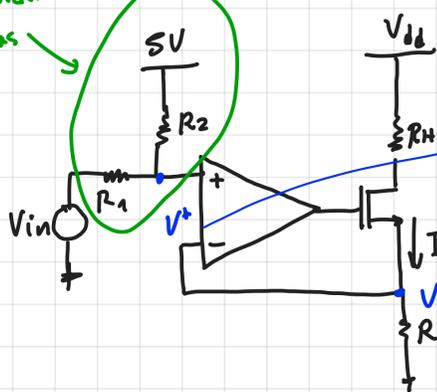
Add this to provide the current in the correct range

↪ bias basic current 50mA +

$$\begin{cases} \Delta I_H = 0 \text{ mA} @ V_{in}=0 \\ \Delta I_H = 450 \text{ mA} @ V_{in}=5V \end{cases}$$

we have to add a bias point

Voltage divider for the bias current



↪ For superposition of effect:

$$V^+ = V_{in} \frac{R_2}{R_1+R_2} + 5V \frac{R_1}{R_1+R_2}$$

$$V^+ = R(I_H + \Delta I_H)$$

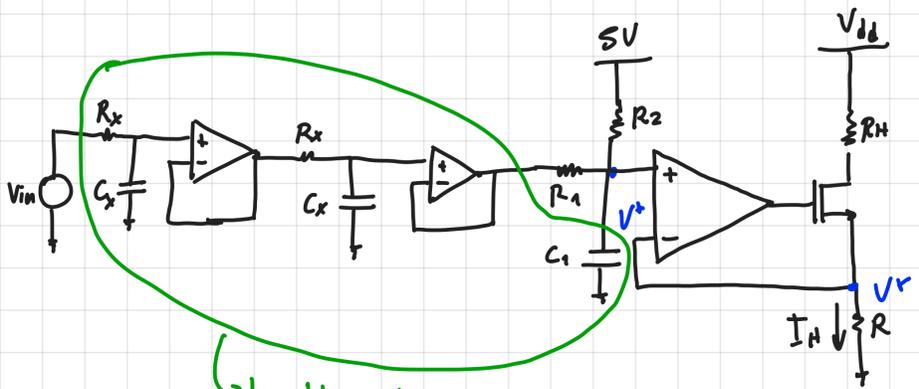
↪ @  $V_{in}=0$   $5V \frac{R_1}{R_1+R_2} = R \cdot \underbrace{I_H}_{50 \text{ mA}}$

↪ @  $V_{in}=5V$   $5V \left( \frac{R_2}{R_1+R_2} + \frac{R_1}{R_1+R_2} \right) = R \left( \underbrace{I_H}_{50 \text{ mA}} + \underbrace{\Delta I_H}_{450 \text{ mA}} \right)$

$$\begin{cases} R_1 = 1k \\ R_2 = 9k \end{cases}$$

↪  $R = 10 \Omega$

↪ Now we want to add 3 poles @ 100 Hz



↪ to add 3 poles

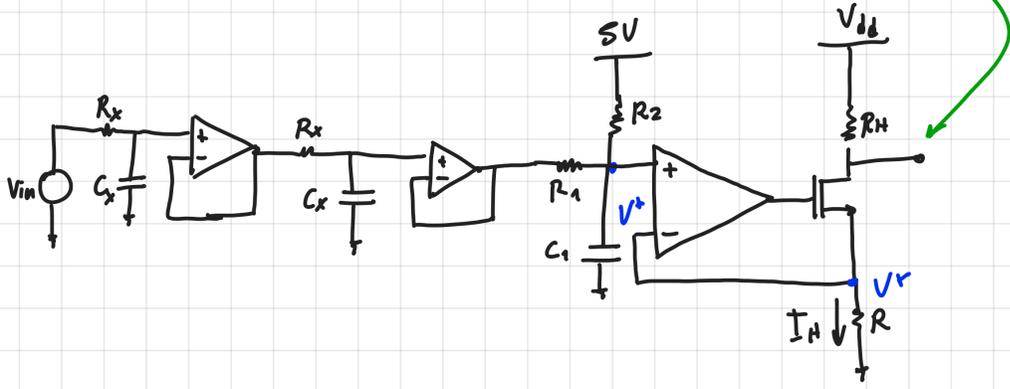
$$I_H = \frac{V_{in}}{R}$$

$$f_p = \frac{1}{2\pi C_x R_x} = \frac{1}{2\pi C_x (R_1 || R_2)} = 100 \text{ Hz}$$

$$R_{\bar{x}} (R_1 || R_2) = 900 \Omega$$

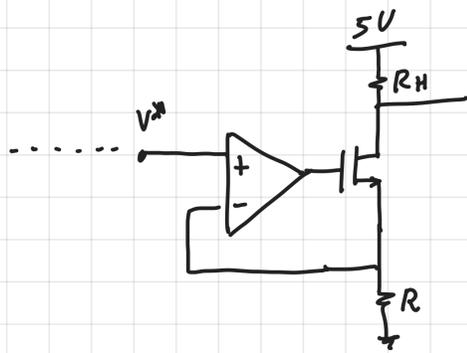
$$C_1 = C_x = 1.77 \mu F$$

b) We should expect a nominal value here due to nominal  $R_H$  but nominal value in reality can change

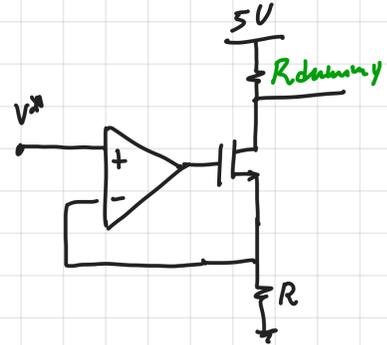


we want to detect these changes and activate a LED when this happens

So considering:  $|\Delta V| > 100\text{mV} \rightarrow \text{LED ON}$

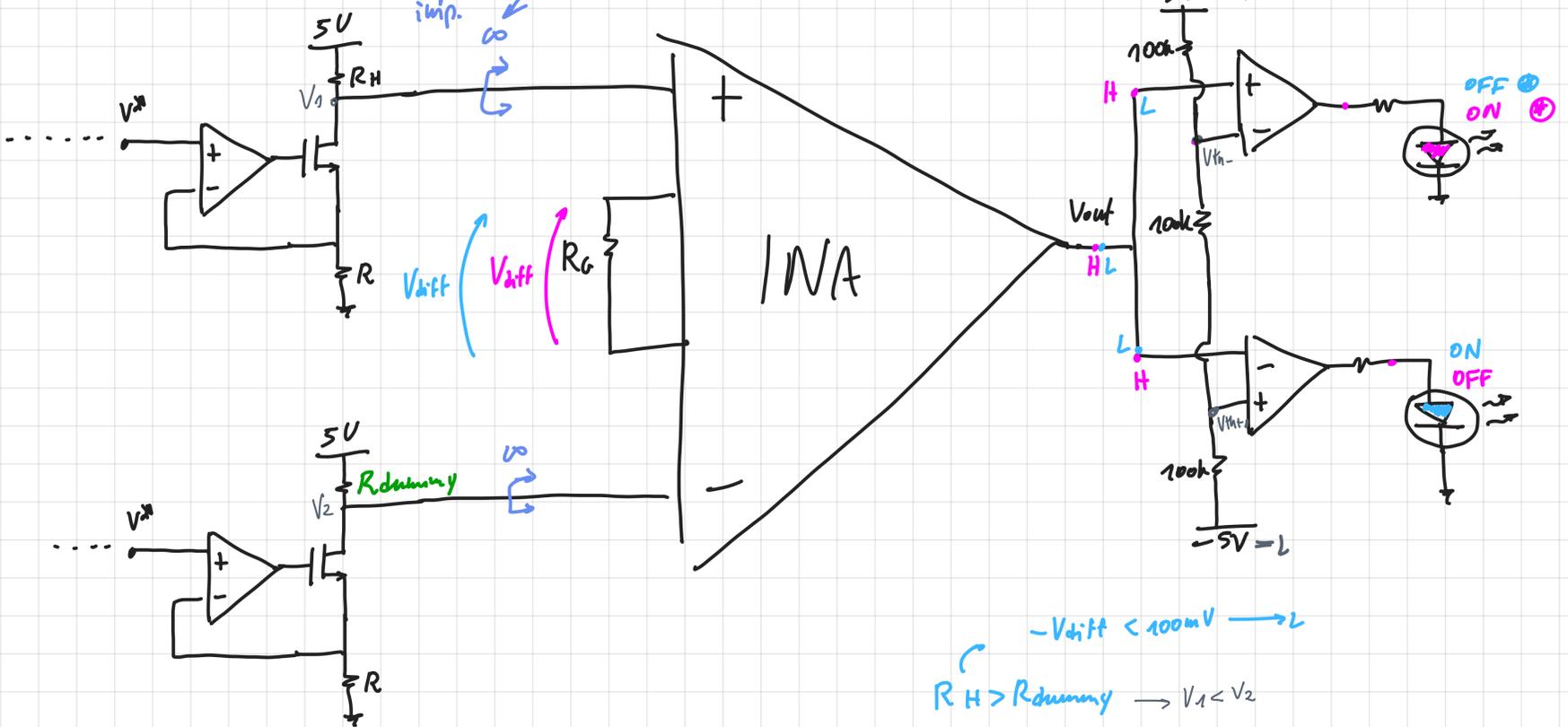


we then consider it with a "dummy" R to check the difference



to compute the difference we can use a INA

because it has high input imp.



$-V_{diff} < 100\text{mV} \rightarrow L$   
 $R_H > R_{dummy} \rightarrow V_1 < V_2$   
 $R_H < R_{dummy} \rightarrow V_1 > V_2$

when  $V_{diff} > 100\text{mV} \rightarrow H$   
 $\Delta V \rightarrow$  if  $V_{out} > V_{th-}$

Now we have to compute the INA gain

$$V_{TH} = \frac{5V \cdot \frac{2}{3}}{\text{due to } 5V} - \frac{5V \cdot \frac{1}{3}}{\text{due to } -5V} = 1.67V$$

$$V_{TL} = \frac{-5V \cdot \frac{2}{3}}{\text{due to } -5V} + \frac{5V \cdot \frac{1}{3}}{\text{due to } +5V} = -1.67V$$

$$G_{INA} = \frac{1.67V}{100\text{mV}} = 16.66 \Rightarrow R_{INA} = 100k\Omega$$

$$G_{INA} = 1 + \frac{2M\Omega}{R_G} \rightarrow R_G = 12.77k\Omega$$

Voltage difference

when this  $V_{diff} = \pm 100\text{mV}$  happens at the input we want to switch  $\rightarrow$  output  $= \pm 1.67V =$  thresholds

$\rightarrow$  in order to switch on the LEDs as described

5

**Ex. 5**

Monitor the laser power emitted by a laser curing device, by means of a photodiode, whose photocurrent  $I_{in}$  must stay in the  $10 \div 70 \mu A$  range, with a nominal value of  $40 \mu A$ . The photodiode is operated reverse-biased at  $5V$ .



- a) Provide a  $V_{out}$  in the  $0 \div 5V$  range, proportional to the **absolute unbalance** away from the nominal value (i.e.  $0V @ 40 \mu A$  and  $+5V @ 10 \mu A$  and  $+5V @ 70 \mu A$ ).
- b) Switch on an LED when the photocurrent dose exceeds  $2.4 mC$  and reset the measurement every  $5 min$ .

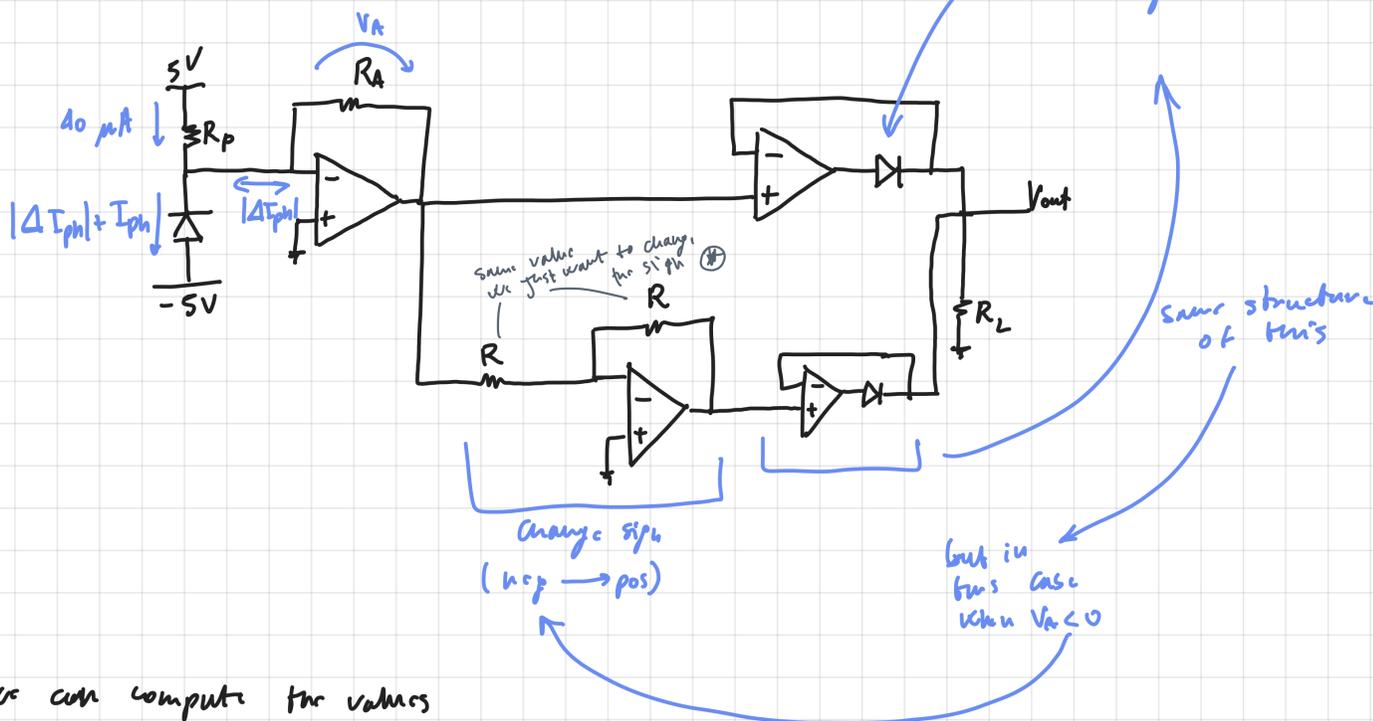
a)  $V_{out} = 0 - 5V$

$$\left\{ \begin{array}{l} \text{photo diode current } I_{in} = 10 \div 70 \mu A \text{ [nominal value } 40 \mu A] \\ \text{photo diode reverse bias } V_{ph} = 5V \end{array} \right.$$

$|\Delta I_{ph}|_{max} = 30 \mu A$

$$\hookrightarrow \text{We want: } \left\{ \begin{array}{l} @ 40 \mu A \rightarrow V_{out} = 0V \\ @ 10 \mu A \rightarrow V_{out} = 5V \\ @ 70 \mu A \rightarrow V_{out} = 5V \end{array} \right.$$

Consider the photodiode



Now we can compute the values

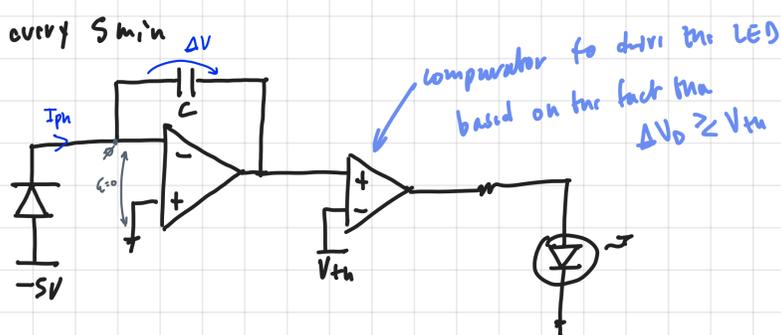
$R_p = \frac{5V}{40 \mu A} = 125 k\Omega$

$|V_A| = R_A |\Delta I| \rightarrow R_A = \frac{5V}{30 \mu A} = 166.66 k\Omega$

( $R$  is not important the important thing is  $\oplus$ )

b)  $Q_{ph} \geq 2.4 mC$   $\leftarrow$  to check it we can use an integrator stage

change  
+  
reset every 5 min



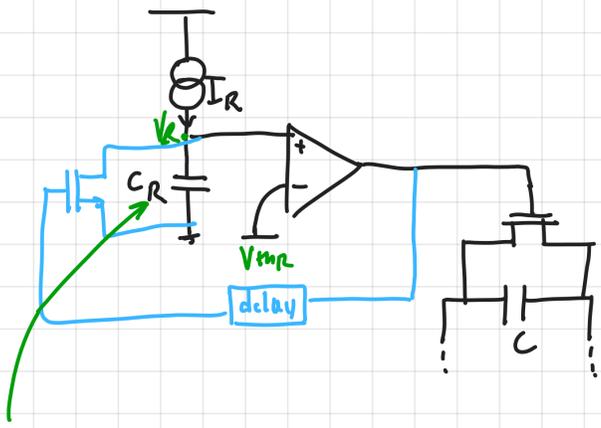
$I_{ph} = C \frac{dV}{dt}$

$Q_{ph} = C \Delta V$   $\leftarrow$  this value depends on the  $V_{th}$  value we fix

$\hookrightarrow$  If e.g. we consider  $V_{th} = 5V = \Delta V$  to turn on the LED

$C = \frac{Q_{ph}}{\Delta V} = \frac{2.4 mC}{5V} = 480 \mu F$

↳ For the reset circuit we need a switch (MOS) to reset the circuit every 5 min



We can increase the value across the capacitance  $C_R$  with a const. current

change the  $C_R$  with a const. current

↳ when the value  $V_R > V_{thR}$  the comparator reset the circuit

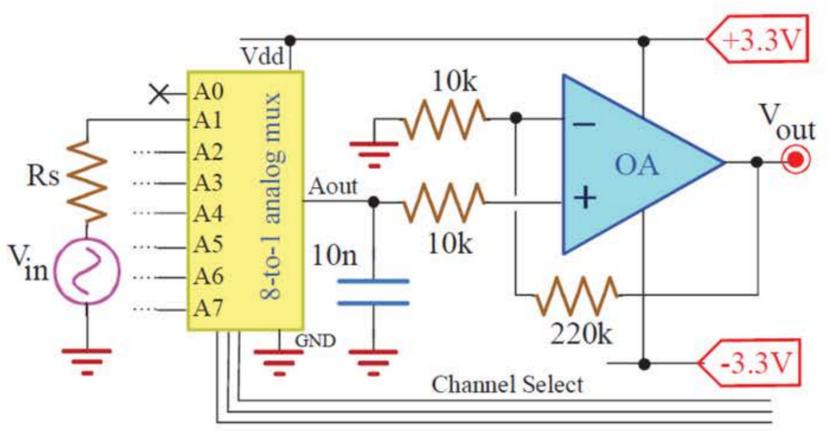
↳ then we need to reset also  $C_R$  → branch with delay with a switch to reset also  $C_R$  → so when we reset  $C$  we have also reset  $C_R$

↳ Now we can compute the values:

$$\frac{\Delta V_R}{\Delta t} = \frac{I_R}{C_R} \rightarrow \frac{I_R}{C_R} = \frac{5V}{5 \cdot 60s} = 16.67 \frac{mV}{s}$$

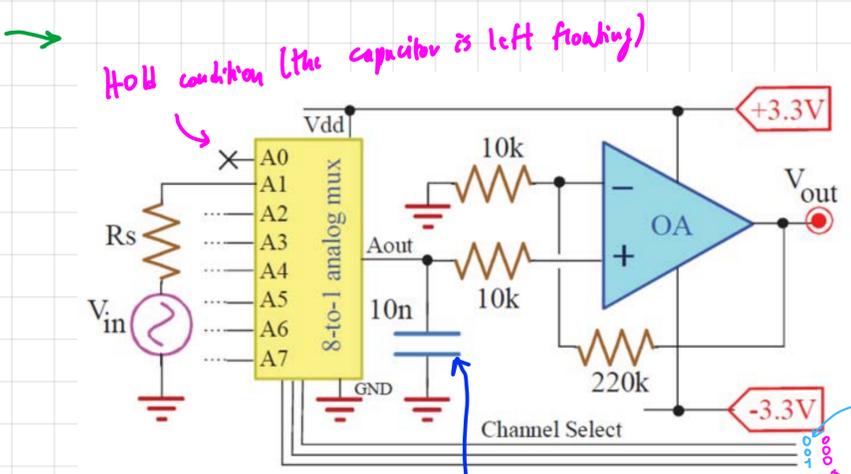
$I_R = 167 \mu A$   
 $C_R = 100 \mu F$

①



Seven inputs with  $R_s=100\Omega$  **1k $\Omega$** . OpAmp with sourcing  $I_B=1nA$  and  $V_{os}=0.2mV$ . Mux with  $R_{on}=5-50\Omega$   $R_{off}=2M\Omega-20M\Omega$  and  $I_{leak}=5nA$ .  
 a) Compute **sampling** and **hold** times for 12 bit resolution and  $v_{in,max}=\pm 50mV$  and specify if they are max or min values.  
 b) Compute all static errors and properly add them to compute the total output error in LSBs.

Worst case conditions



Hold condition (the capacitor is left floating)

the capacitor will work as a S&H where the S and H phases are commanded directly with the switches from the MUX

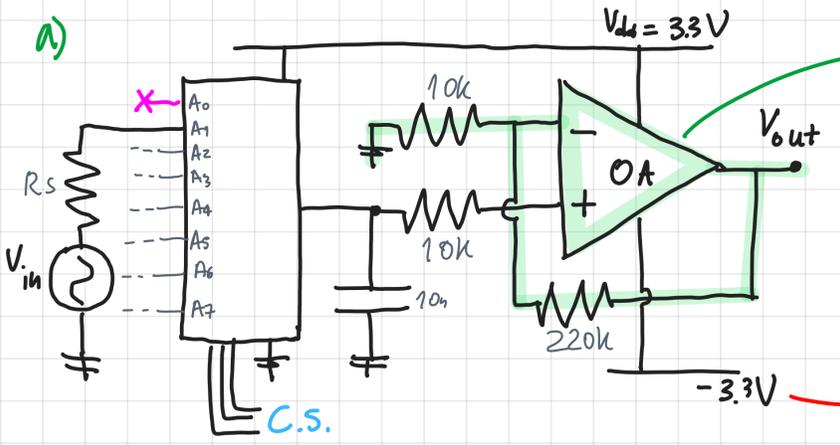
Gain OpAmp: The OpAmp is connected in a std non-inverting config.

$$G_{OA} = 1 + \frac{R_2}{R_1} = 1 + \frac{220k}{10k} = 23$$

$$V_{out,max} = \pm V_{in,max} \cdot G = \pm 50mV \cdot 23 = \pm 1.15V$$

output swing

$$FSR = 2 \cdot 3.3V = 6.6V$$

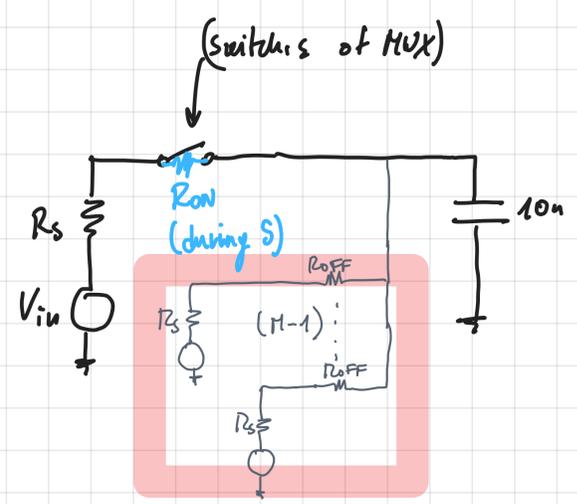


$$LSB = \frac{FSR}{2^n} = \frac{6.6V}{2^{12}} = \frac{6.6V}{4096} \approx 1.6\mu V$$

12 bit

The constraints

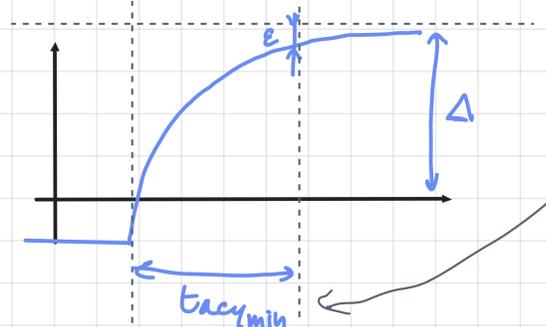
Consider the S&H circuit:



M=7

It may seem misleading but with this  $t_{max}$  we indicate the max constraint which in case of acquisition time the max extreme constraining condition is  $t_{acq,min}$

$$t_{acq,max} = \tau_{max} \ln \frac{\Delta_{max}}{\epsilon_{min}}$$

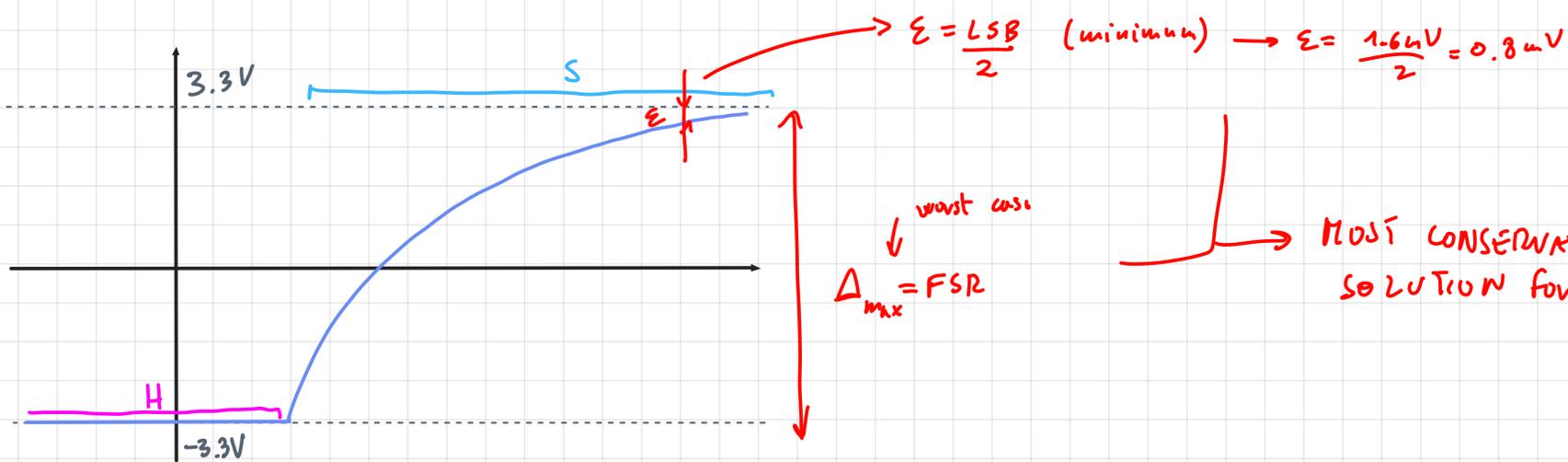


Considering also the effect of the other switches (that are OFF) and the C component tolerance:

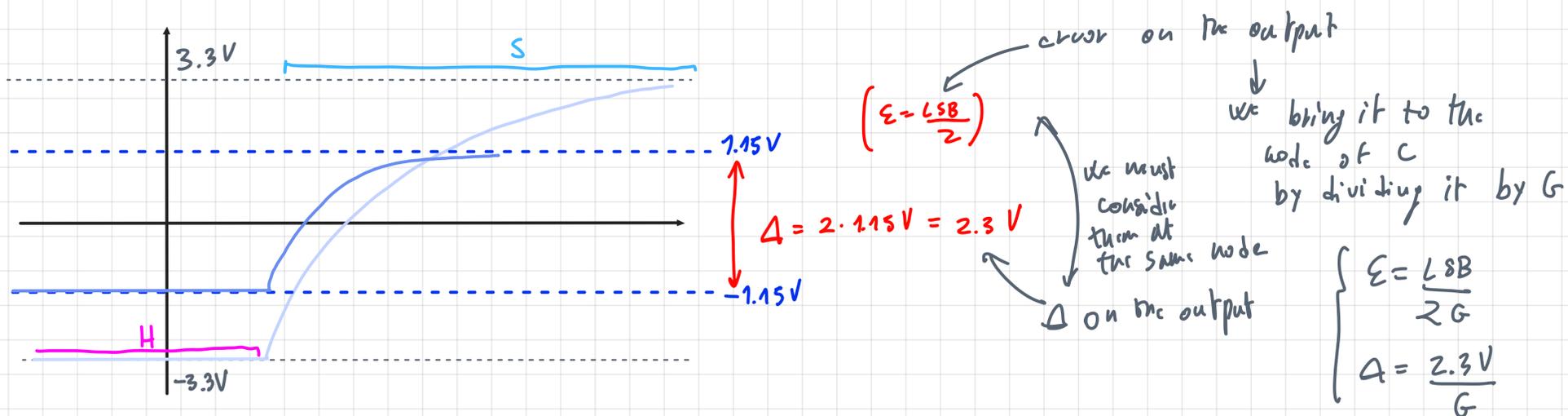
$$t_{acq,min} = \tau_{max} \ln \frac{\Delta_{max}}{\epsilon_{min}} = \left[ \left( R_{s,max} + R_{on,max} \right) \parallel \left( \frac{R_{off} + R_s}{6} \right) \right] C (1 + \text{toll.}) \cdot \ln \frac{\Delta_{max}}{\epsilon_{min}}$$

$R_{s,max} \uparrow 1k\Omega$      $R_{on,max} \uparrow 50\Omega$      $R_{off} \uparrow 20M\Omega$      $R_s \uparrow 1k\Omega$      $C \downarrow 10nF$      $\text{toll.} \downarrow 0.2$

For  $\Delta_{max}$  and  $\epsilon_{min}$  we can consider:



Instead of this most conservative solution we would like to consider the actual output swing of  $\pm 1.15V$



So in the previous expression

$$t_{acy\ min} = \tau_{MAX} \ln \frac{\Delta_{max}}{\epsilon_{min}} = \left[ \left( R_{s\ max} + R_{on\ max} \right) \parallel \left( \frac{R_{off} + R_s}{6} \right) \right] \cdot C \cdot (1 + \text{roll.}) \cdot \ln \frac{\Delta_{max}}{\epsilon_{min}} =$$

↑ 14Ω      ↑ 50Ω

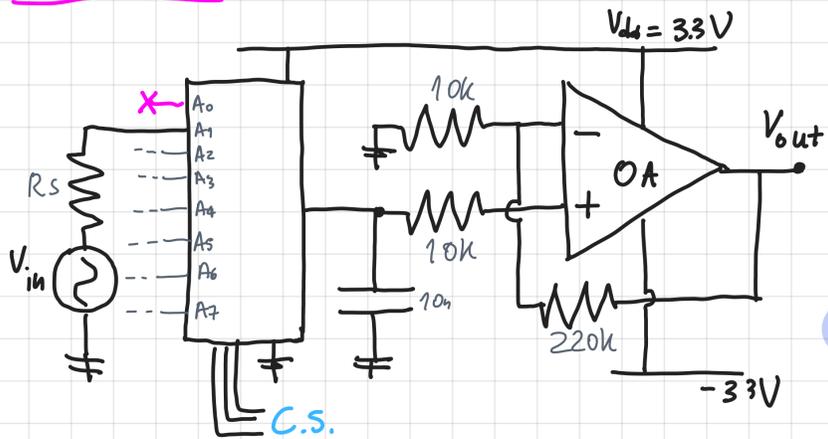
↑ 10n      ↓ 0.2

= 1050Ω · 12nF · ln  $\frac{2.3V}{0.8mV} \approx 100\mu s$

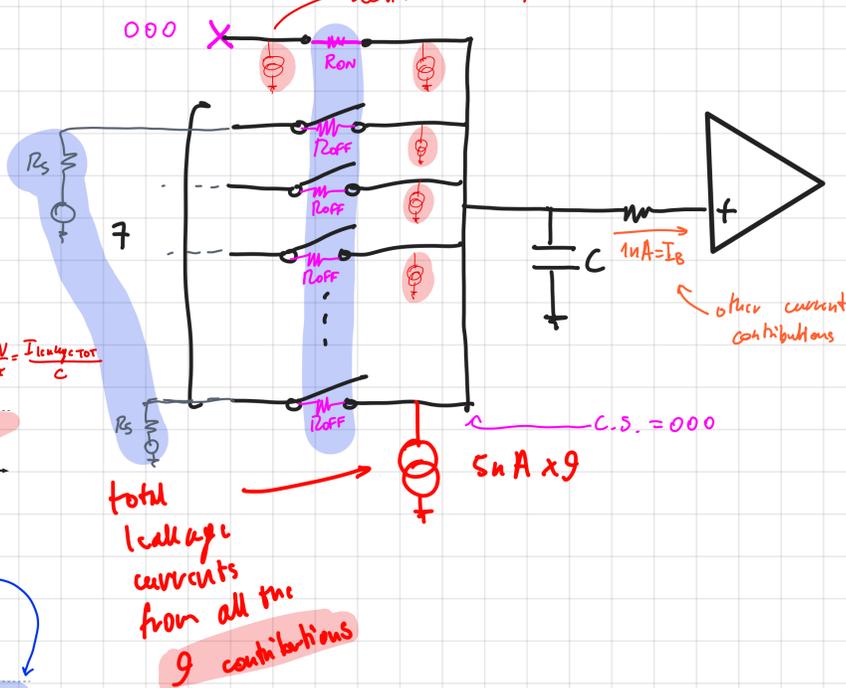
7.96

SAMPLING TIME

Hold time:



the first channel to will be connected  
 additional leakage on source of the switches cause it's closed



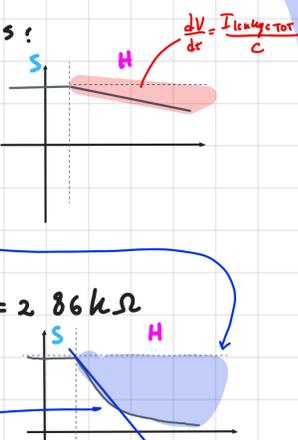
currents ( $I_{leakage} + I_{bias}$ ) contributions:

$$I_{leakage\ tot} = 9 \cdot I_{leak} + I_B = 46\mu A$$

resistors contributions → discharges C:

$$R_{off\ min\ tot} = \frac{R_{off\ min} + R_{s\ min}}{7} = \frac{2\ M\Omega}{7} = 286\ k\Omega$$

to simplify let's consider a linear discharge



$$I_{OFF} = \frac{V_{max}}{R_{offmin}} = \frac{1.15V}{286k\Omega} = 4\mu A$$

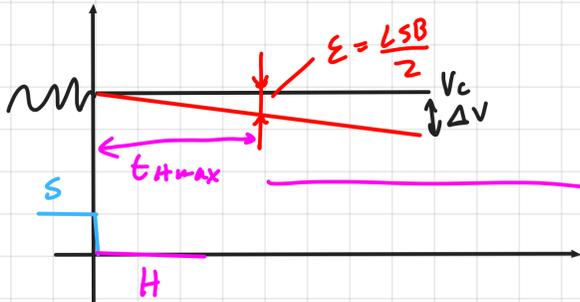
for resistors

So we can see that:  $I_{OFF} = 4\mu A$ ,  $I_{leakage, TOT} = 46\mu A$

So we can define the droop rate:  $droop = \frac{dV}{dt} \Big|_{t_{max}} = \frac{I_{OFF}}{C_{min}} = \frac{4\mu A}{8nF} = 500 \frac{V}{s}$

$(10nF)(1 - \text{toll}) = 8nF$

For the hold time consider:



time it takes to reach an error of  $\epsilon = \frac{LSB}{2}$

so for an ADC in series to the S/H we will need  $t_{conv} < t_{Hmax}$

$$t_{Hmax} = \frac{\Delta V}{\frac{I_{OFF}}{C}} = \frac{\frac{LSB}{2}}{500 \frac{V}{s}} = \frac{0.8mV}{500 \frac{V}{s}} = 1.6 \mu s$$

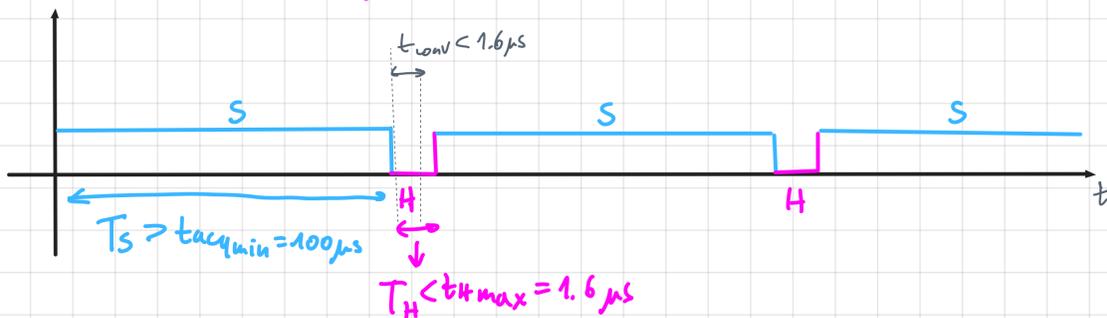
So we discovered that:

Sampling time:  $t_{acqmin} = 100 \mu s$

Holding time:  $t_{Hmax} = 1.6 \mu s$

Pretty BAD S/H

↳ wait long with close switches (not very high speed)  $f_s \oplus$



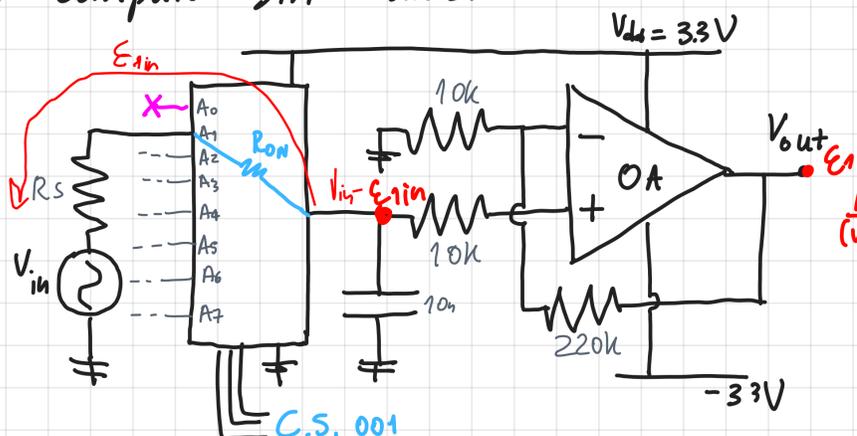
↳ expensive ADC for  $t_{conv} < 1.6 \mu s$

$$f_s = \frac{1}{T_s + T_H} = \frac{1}{102 \mu s + 1 \mu s} = 9.7 \text{ ksp/s}$$

for ex.

Bandwidth (Shannon):  $f_{max} \leq \frac{f_s}{2} \leq 4.7 \text{ kHz}$  (To avoid aliasing)

b) Compute static errors:



When sampling we connect one of the 7 channel bus  $A_0 \rightarrow A_7$

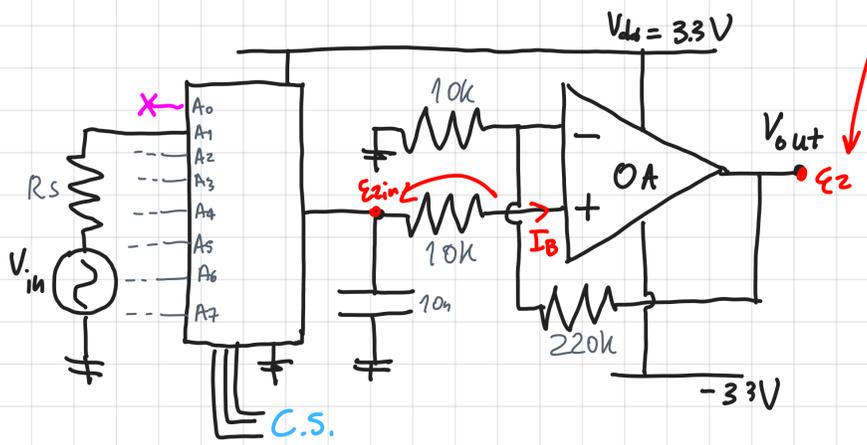
e.g. we connect  $A_1 \rightarrow$  it will have a voltage drop across  $R_s + R_{on}$

$$\text{first error: } \epsilon_1 = V_{in} \cdot \frac{(R_s + R_{on})_{MAX}}{(R_s + R_{on})_{MAX} + \left( \frac{R_{OFF} + R_s}{6} \right)_{MIN}} \cdot 23 = \pm 50mV \cdot \frac{1050}{1050 + \frac{2M}{6}} \cdot 23 \approx \pm 3.6mV$$

$\epsilon_{in}$

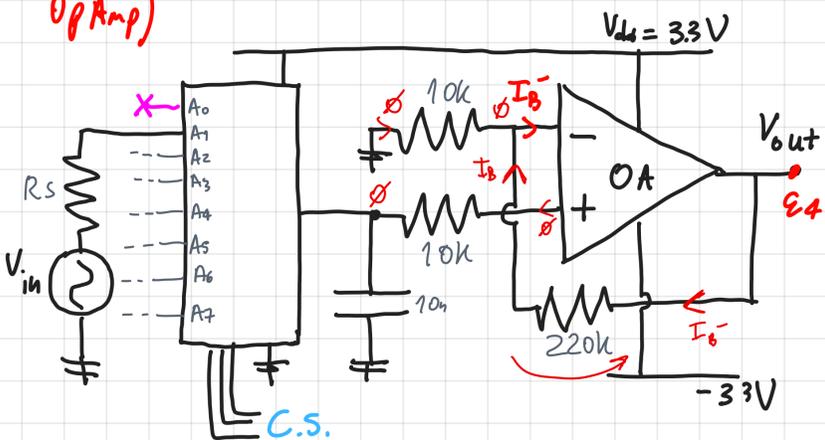
other 6 non active switches

→ Second error :  $\epsilon_2 = -I_B \cdot 10k \cdot G = -1n \cdot 10k \cdot 23 = -230 \mu V$   
 (Voltage drop across resistor 10k due to  $I_B$ )  
 $\epsilon_{2in}$  at the output

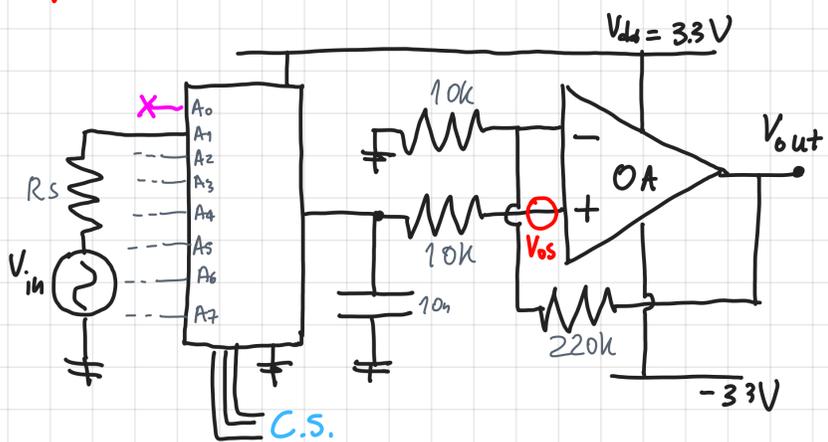


→ Third error :  $\epsilon_3 = (9 I_{leak} \cdot I_B) \left[ (R_s + R_{on}) \parallel \frac{(R_{off} + R_s)}{6} \right] \cdot G = 4.6nA \cdot 1050 \cdot 23 = 1.1 mV$   
 (Voltage drops due to leakages)  
 (usually the sign of the leakages is not known)  
 AS seen before in sampling time

→ Fourth error :  $\epsilon_4 = I_B^- \cdot 220k = 220 \mu V$   
 ( $I_B^-$  of the OpAmp)



→ Fifth error :  $\epsilon_5 = \pm V_{os} \cdot G = \pm 0.2 mV \cdot 23 = \pm 4.6 mV$   
 (opamp offset)



⇒ TOTAL ERROR at the output  $\sum_{i=1}^5 \epsilon_i = \pm 3.6 mV - 230 \mu V \pm 1.1 mV + 220 \mu V \pm 4.6 mV = \pm 9.3 mV$

Obs. • We can try to change components to reduce this error

• Or after the ADC conversion we can try to remove this error, indeed:

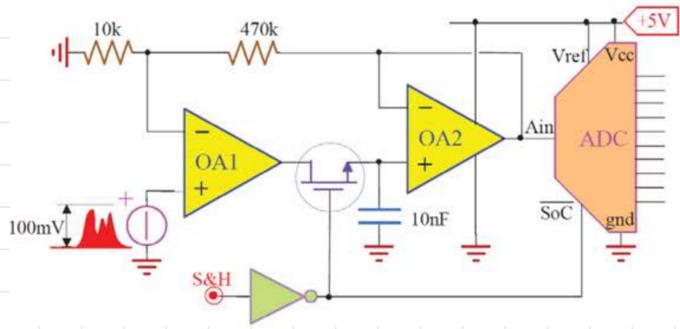
Considering that:

$$\frac{9.3 mV}{LSB} = \frac{9.3 mV}{1.6 mV} \approx 6 \rightarrow D_{out} = D_{out,ideal} \pm 6$$

↑  
digital level

(e.g. If we're supposed to be in  $2B_H$  will be in the range of  $2B_H \pm 6 = 25_H \div 31_H$ )

(Prod different → wrong computation?)  
 $(B = 11_{decimals})$   
 $2B_H = 0010 \quad 1011_B = 43_D$   
 $25_H = 0010 \quad 0101_B = 37_D$   
 $31_H = 0011 \quad 0001_B = 49_D$

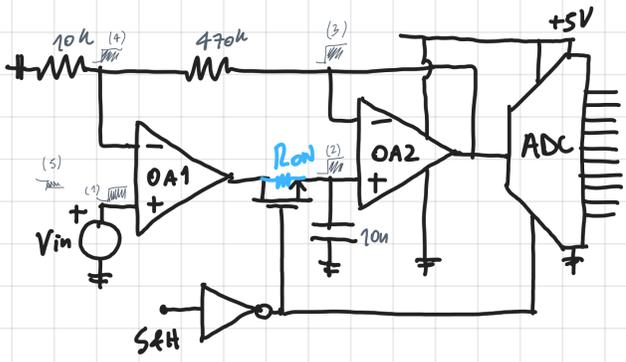


OpAmps:  $A_0=100\text{dB}$ ,  $\text{GBWP}=100\text{MHz}$ .

MOS:  $R_{on}<100\Omega$ ,  $V_T=0.8\text{V}$ . ADC: 12bit.

a) Compute the acquisition time.

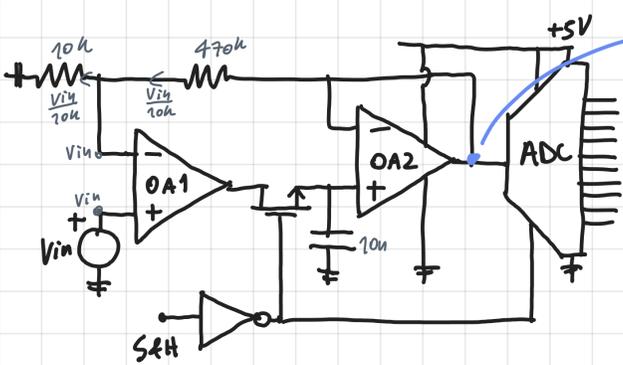
b) Compute static error on  $A_{in}$  in LSB, due to  $I_B=5\text{nA}$  (sink) and  $V_{os}=5\text{mV}$ .



During sampling (S)

→ (1) → (5) (NEGATIVE FEEDBACK LOOP)

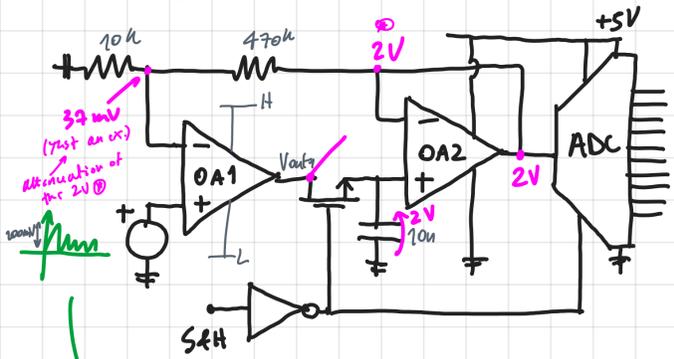
Gain



$$V_{out} = \frac{V_{in}}{10k} (10k + 470k)$$

$$G = 1 + \frac{R_2}{R_1} = 1 + \frac{470k}{10k} = 48$$

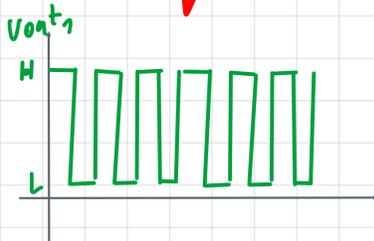
a) Acquisition time



During hold phase

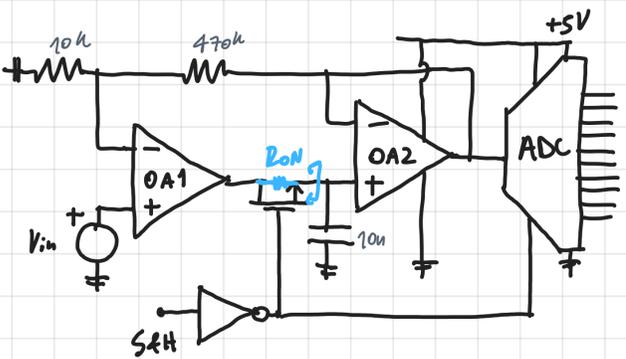
BAD BEHAVIOUR!

(OA1 has no feedback, switch open, so it keeps saturating between high and low values of P.S.)



If  $V_{in} > V_{OA1}^- \Rightarrow V_{out1} (H)$

If  $V_{in} < V_{OA1}^- \Rightarrow V_{out1} (L)$



During sampling  $\Rightarrow$  OA1 has now feedback!

$$\tau_{acq} = \tau \ln \frac{\Delta}{\epsilon}$$

$\tau \neq R_{on} \cdot C_{H}$  ← cause we have feedback

$$\tau = \frac{1}{2\pi f_{pole}}$$

We need to compute the BW of the closed loop config.

f<sub>pole</sub>

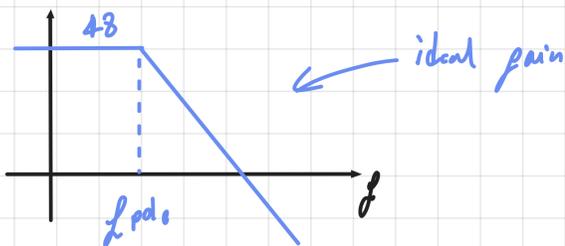
Since we have a feedback circuit C

$$\text{See as } \tau_{eq} = \frac{R_{on}}{1 - G_{loop}}$$

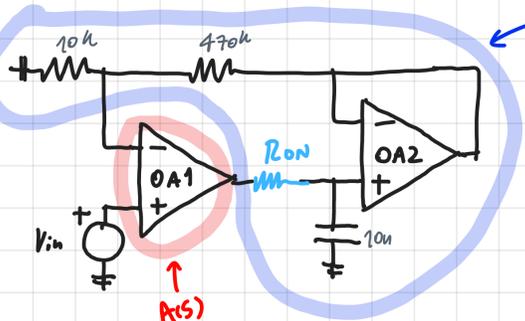
$$f_{pole} = \frac{1}{2\pi C \frac{R_{on}}{1 - G_{loop}}}$$

↳ better to study the gain directly in the total Bode diagram →

Bode:



Consider:

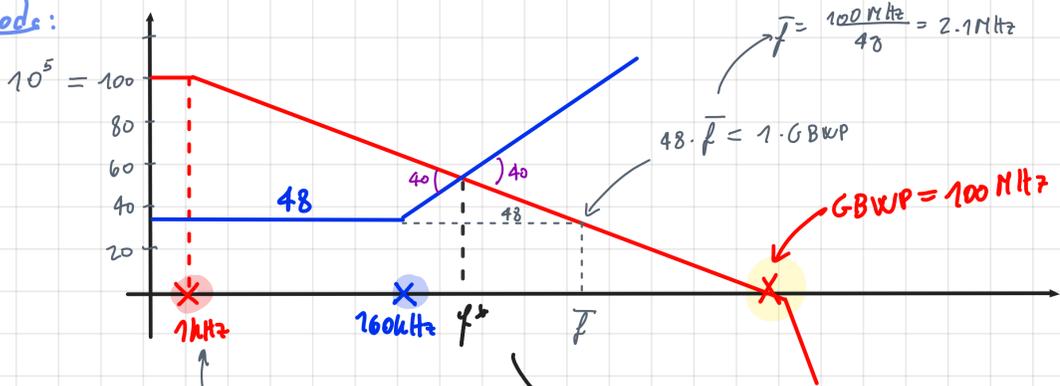


$$\beta(0) = \frac{10k}{10k + 470k} \cdot 1 \rightarrow \frac{1}{\beta(0)} = 48$$

$$\beta(\infty) = 0 \rightarrow \frac{1}{\beta(\infty)} = \infty$$

$$f_{pole} = \frac{1}{2\pi R_{on} C} = 160\text{kHz}$$

Bode:



$A(s)$

$\frac{1}{\beta}(s)$

ideal gain

real gain

$$10^5 \cdot f_{PM(s)} = 1 \text{ GBWP}$$

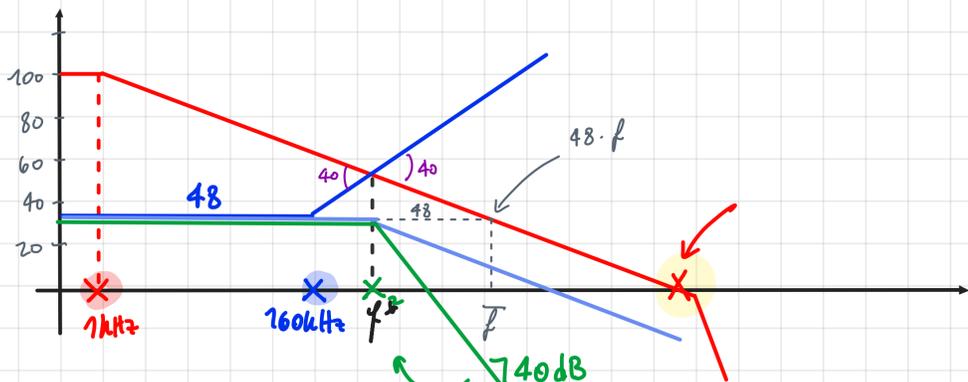
$$f_{PM(s)} = \frac{100 \cdot 10^6}{10^5} = 1 \text{ MHz}$$

$$f^* = \sqrt{f_{pole} \cdot \bar{f}} = \sqrt{160 \text{ kHz} \cdot 2.1 \text{ MHz}} = 580 \text{ kHz}$$

The closure angle is 40-40 but  $f^*$  is near 160 kHz but maybe we're not already unstable:

compute the  $PM = 180^\circ - 90^\circ - \text{arg } \frac{1}{\beta}(s) = 90^\circ - \text{arg } \frac{1}{\beta}(s) = 15^\circ \rightarrow \text{too low} \rightarrow \text{circuit is UNSTABLE}$

Let's avoid compensation in this case, and just analyze the real gain response:



$A(s)$

$\frac{1}{\beta}(s)$

ideal gain

real gain

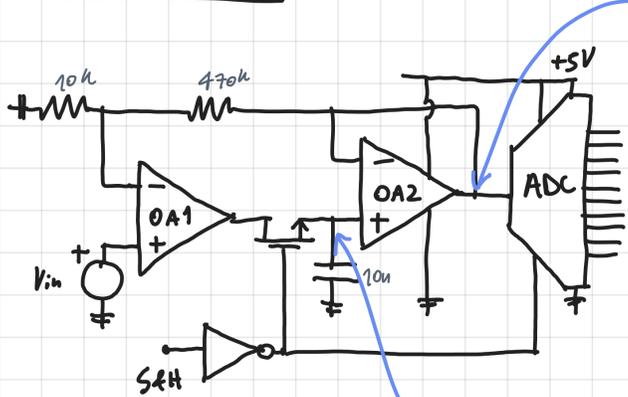
740 dB  
so 2 poles in  $f^*$  (unstable)

we can't realistically say that:  $\tau = \frac{1}{2\pi f^*}$

But we make an approx.  $\tau \approx \frac{1}{2\pi f^*} = \frac{1}{2\pi \cdot 580 \text{ kHz}} = 275 \text{ ns}$  (At the written test it will specify if we can use this approx.)

$$t_{acq} \approx \tau \ln \frac{\Delta}{\epsilon}$$

For  $\Delta$  and  $\epsilon$  Consider:



• FSR = 5V (from P.S. 0-5V)

•  $LSB = \frac{FSR}{2^{12}} = 1.2 \text{ mV} \rightarrow \epsilon = \frac{LSB}{2} = 0.6 \text{ mV}$

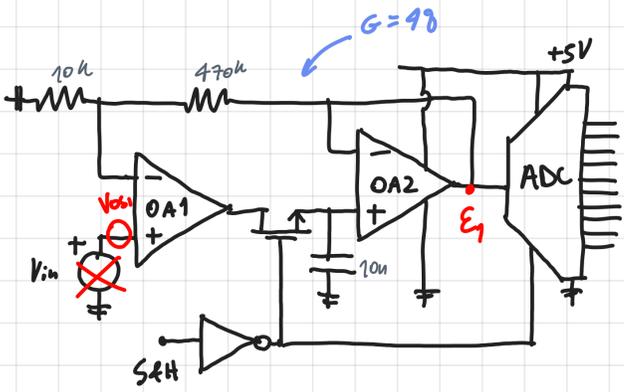
$\Delta$ :  $\rightarrow$  worst case:  $\Delta = 5 \text{ V}$

$\rightarrow$  If we instead consider the real max input of 100mV

$\rightarrow V_{outmax} = 100 \text{ mV} \cdot 48 = 4.8 \text{ V} \approx 5 \text{ V}$  (we can consider this)

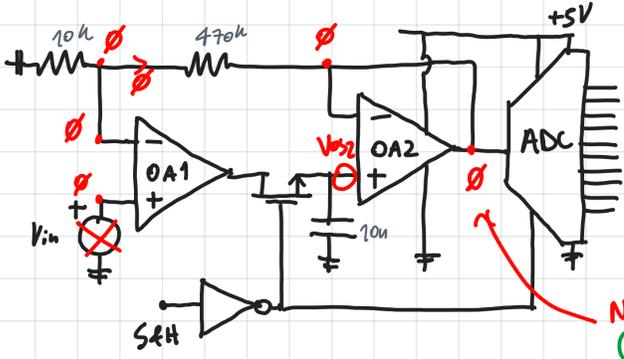
$$\Rightarrow t_{acqmin} = \tau \ln \frac{\Delta_{max}}{\epsilon_{min}} = 275 \text{ ns} \ln \frac{5 \text{ V}}{0.6 \text{ mV}} = 2.5 \mu\text{s}$$

b) Computation of static errors



$V_{os1}$   $\epsilon_1 = V_{os1} \cdot 48 = 240mV! = \pm 200$  digits

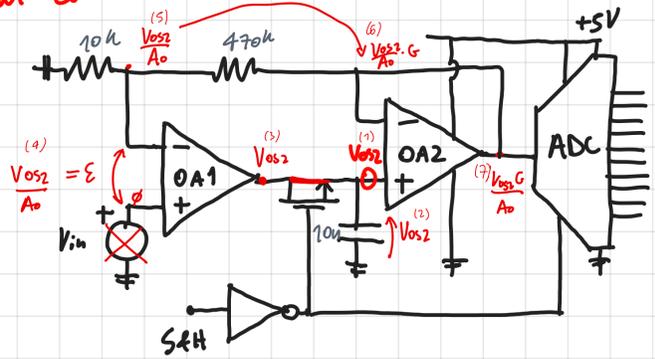
cf.  $F_{3B_H}$   $\left[ \begin{matrix} F_{3B_H} + 200 \\ F_{3B_H} - 200 \end{matrix} \right] \leftarrow$  Very inaccurate!  
 12 bit  
 4x3  
 3 hexadec. digits



$V_{os2}$   $\epsilon_2 = 0$

NO EFFECT (ideally)

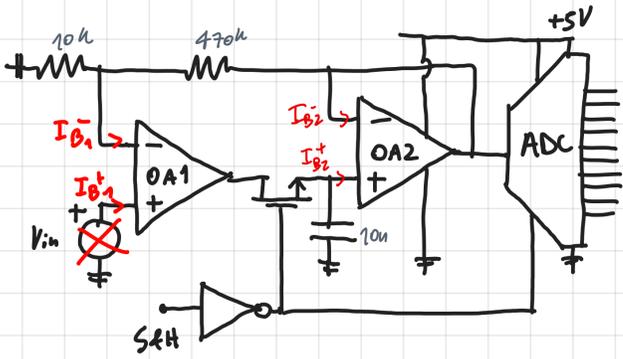
If we consider the real circuit ( $\epsilon \neq 0$ )



so  $\epsilon_2 = \frac{V_{os2}}{A_o} \cdot G \approx 0$   
 (negligible indeed)

Considerations direction:

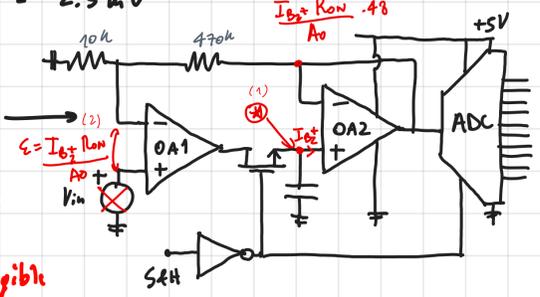
- (1)  $V_{os2}$  applied
- (2)  $V_{os2}$  on C
- (3)  $V_{os2}$  must be provided by OpAmp
- (4) To be provided  $V_{out1} = V_{os2}$  it means the diff input of OA1 is  $\epsilon = \frac{V_{os2}}{A_o}$   
 Since  $V_{out1} = \epsilon \cdot A_o = V_{os2}$
- (5)  $V_{os2}/\epsilon$  on the node
- (6)  $\frac{V_{os2}}{\epsilon} \cdot G$
- (7)



$I_{B1}^+$  (no effect)  $\epsilon_3 = 0$

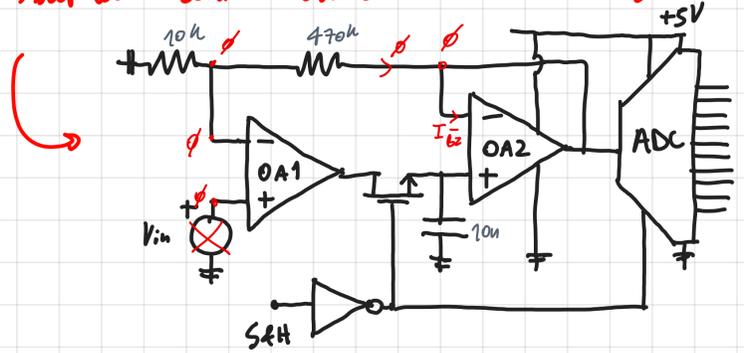
$I_{B1}^- \rightarrow \epsilon_4 = I_{B1}^- \cdot 470k = 2.3mV$

$I_{B2}^+ \rightarrow \oplus I_{B2}^+ \cdot R_{on}$   
 $\epsilon_5 = \frac{I_{B2}^+ \cdot R_{on}}{A_o} \cdot 48 \approx 0$   
 negligible



$I_{B2}^-$  (no effect)  $\epsilon_6 = 0$

ideally  $\rightarrow$  but we've seen these contributions are negligible

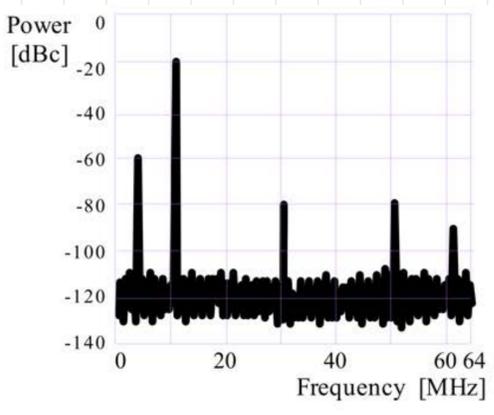


EX07\_DAC

①

The 12bit DAC has 5V FSR. The measured output spectrum has a 2kHz bin-width. There is a distortion tone at 5MHz.

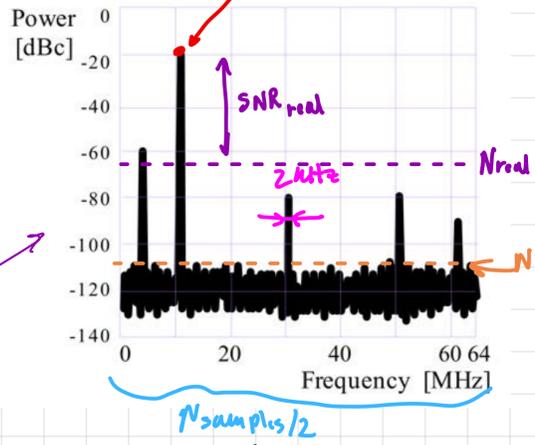
- a) Compute  $SNR_{ideal}$ ,  $SNR_{theor}$ ,  $SNR_{real}$ , and ENOB.
- b) Compute the THD and the SiNAD.



a)  $FSR = 5V$     $h = 12$  bit

$SNR_{ideal} = 6.02h + 1.76 = 74dB$

But we can see here that  $S_{real} \neq S_{max} = 0dBc \rightarrow S_{real} = -20dBc$  (cause we take  $\Delta S = (S_{max} - S_{real})_{dB} = 20dB$ )



Theoretical SNR:  $SNR_{th} = SNR_{ideal} - \Delta S = 74dB - 20dB = 54dB$

Attenuation:  $\Delta S = 20dB$

$N_{real} = ?$

We can compute that:

$N_{ideal} = N_{min} = N_{quantization} = \frac{LSB^2}{12} = \dots = -SNR_{ideal} = -74dBc$

We know that:  $\frac{f_s}{2} = 64MHz$     $f_s = 2 \cdot 64M = 128Mps$    bin-width = 2kHz

$N_{samples} = \frac{f_s}{bin-width} = \frac{128MHz}{2kHz} = 64k$  samples

$N_{real} = NF_{real} \cdot \frac{N_{samples}}{2} = -110dBc + 10 \log_{10} \frac{N_{samples}}{2} = -110dBc + 45dB = -65dBc$

↑  
the result has the power dimension (dBc)

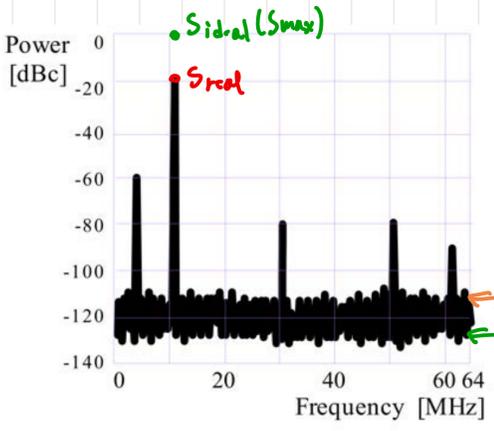
$SNR_{real} = \frac{S_{real}}{N_{real}} = -20dBc - (-65dBc) = 45dB$

↑  
a ratio btw powers is a dimensioned ratio → dB

EXTRA NOISE:

our DAC is not ideal

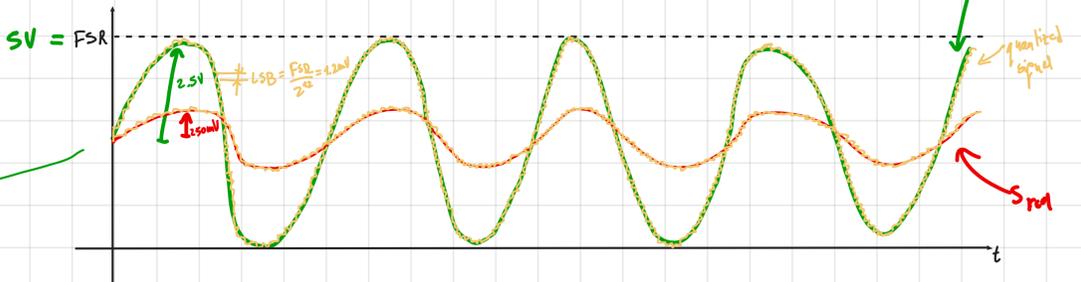
Then we know that:  $SNR_{real} = SNR_{ideal} - \Delta S - \Delta N \rightarrow \Delta N = 74dB - 20dB - 45dB = 9dB$



$N_{ideal} = \frac{N_{ideal}}{\frac{N_{samples}}{2}} = -74dBc - 10 \log_{10} \left( \frac{N_{samples}}{2} \right) = -119dBc$

↑  
if we would have amplify for the all FSR

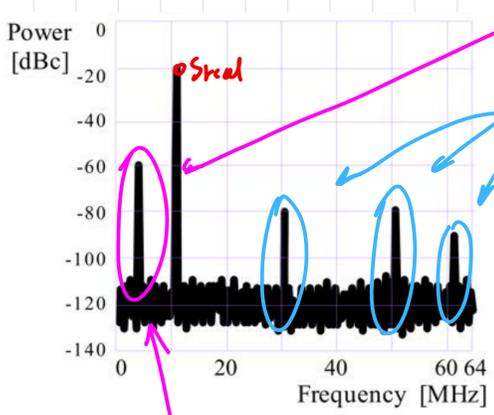
So in time domain we could have had:



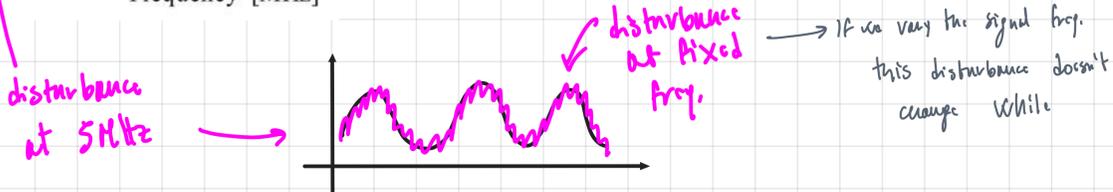
- |                              |                                                |
|------------------------------|------------------------------------------------|
| <u>ideally</u>               | <u>in reality</u>                              |
| $S_{max} = 0dB$              | $S_{real} = 250mV \rightarrow SNR_{th} = 54dB$ |
| $V_p = 2.5V$                 | $V_p = 250mV$                                  |
| $N_{min} = \frac{LSB^2}{12}$ | $N_{real} = N_{min} + \Delta N$                |
| $SNR_{ideal} = 74dB$         | $SNR_{real} = 45dB$                            |

$ENOB = \frac{SNR_{total} + 1.76}{6.02} = 7.8 \text{ bit} \rightarrow 8 \text{ bit}$

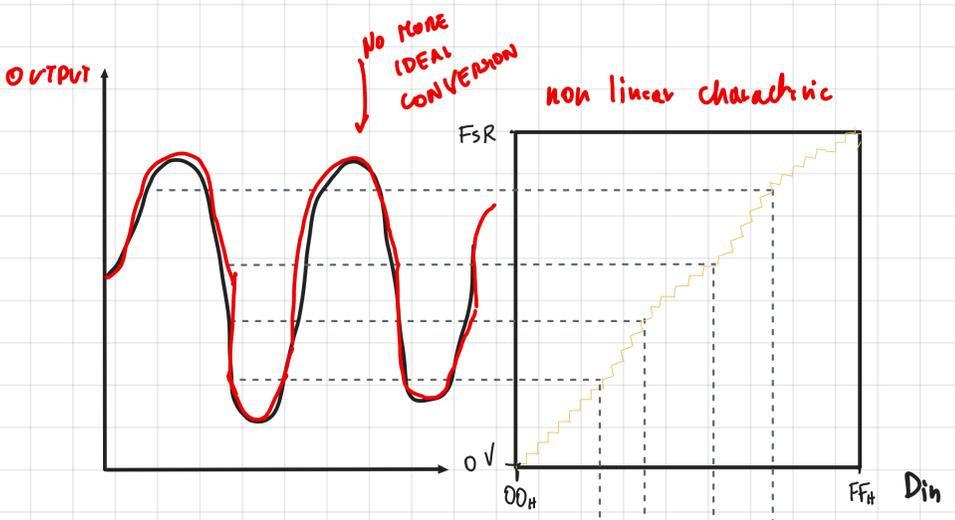
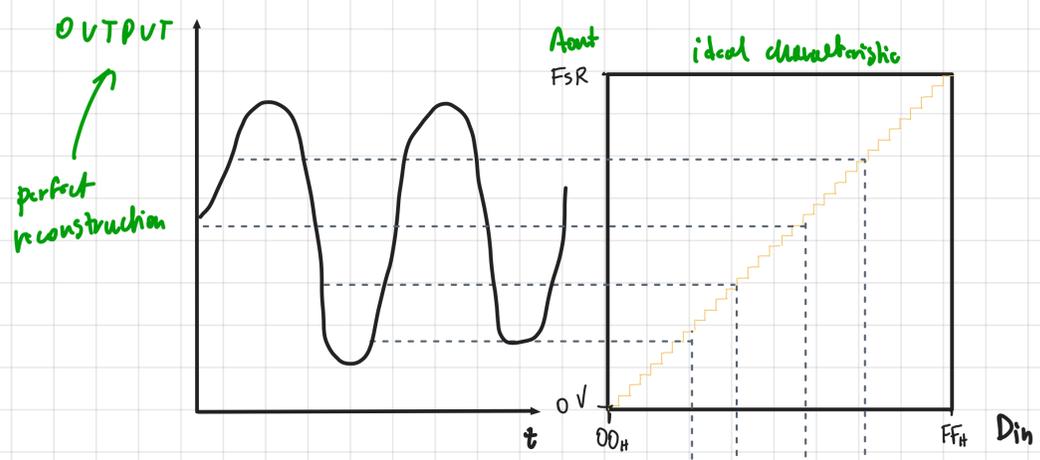
b) We now want to consider the other tones



These are harmonics  $\rightarrow$  harmonic distortion  $\rightarrow$  change with signal freq.  
 Usually due to DAC non-linearities  $\otimes$

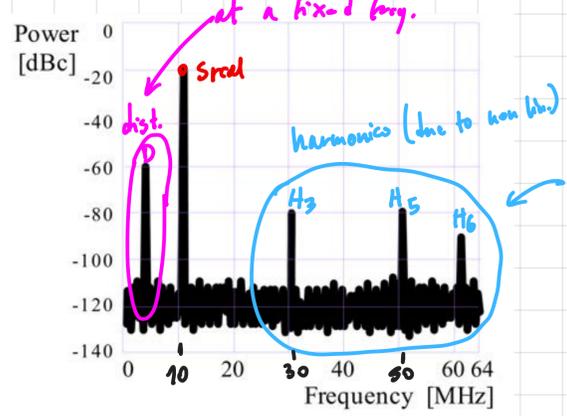


$\otimes$  So observing the converter characteristics:



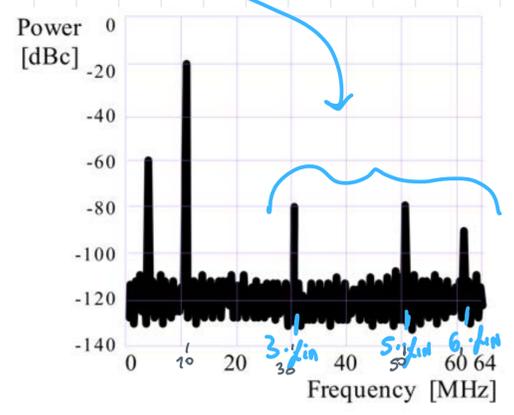
in the freq. spectrum we'll have all the other spurious harmonics of the tones

So given the experimental plots:



So if we change the input freq. the harmonics distortions

all the freq. move away



$S_{sig} = -20 \text{ dBc}$

$D = -60 \text{ dBc}$

signal to (disturbance) distortion ratio:  
 $SDR = -20 \text{ dBc} - (-60 \text{ dBc}) = 40 \text{ dB}$

$H = \sum H_i = H_3 + H_5 + H_6 = 10 + 10 + 10 = 30 \text{ dBc}$   
 $H = 2.1 \cdot 10^{-8} = -77 \text{ dBc}$

Sum powers (it's not a product so not the sum in dB)

$H_3 = -80 \text{ dBc}$   
 $H_5 = -80 \text{ dBc}$   
 $H_6 = -90 \text{ dBc}$

So the Total Harmonic Distortion is:

$$THD = \frac{H}{S_{rid}} = -77dBc - (-20dBc) = 57dB$$

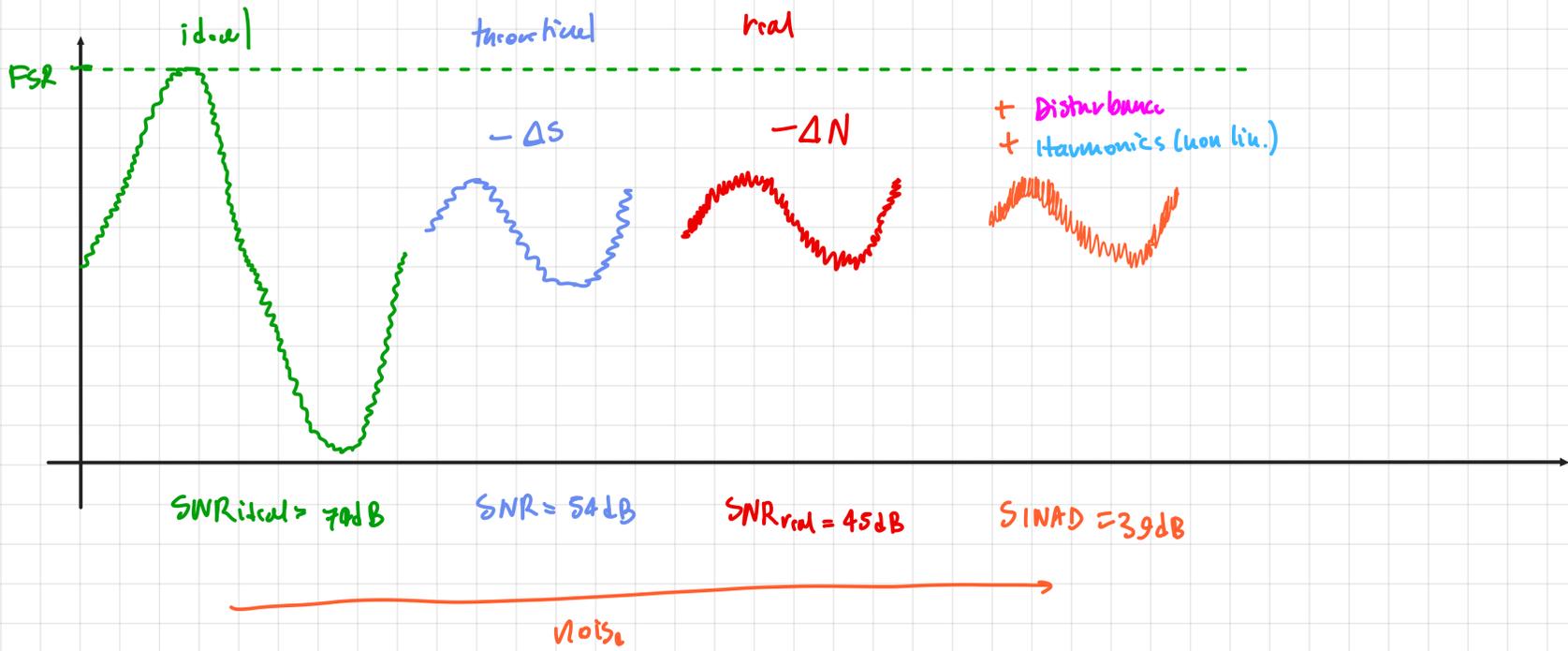
We can also define the SINAD ratio:  $SINAD = \frac{S}{N+D+H} = \frac{S_{real}}{N_{real}+D+H}$

↑  
not a sum in dB

$$= -20dBc - 10 \log_{10} \left[ 10^{\frac{-65dBc}{10}} + 10^{\frac{-60dBc}{10}} + 10^{\frac{-77dBc}{10}} \right] = 39dB$$

+59dBc

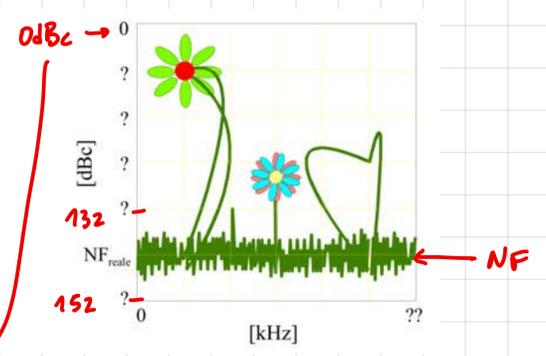
So representing the signal through the various noise considerations:



2

A 16bit DAC with FSR=5V receives a 2MSPS stream of a sinusoidal signal at  $f_c=400\text{kHz}$  with  $200\text{mV}_{\text{peak}}$  amplitude. A noise is superimposed to the signal, thus lowering the SNR by 20dB. The number of samples used for the FFT is 512,000.

- a) Compute  $\text{SNR}_{\text{ideal}}$ ,  $\text{SNR}_{\text{real}}$ , ENOB and real NoiseFloor.
- b) Draw the spectrum, properly quoted in [Hz] and [dBc], adding also a harmonic at  $3 \cdot f_c$  due to a THD=-70dB.



$n=16$     $\text{FSR}=5\text{V}$     $S_{\text{max}}^2 = \left(\frac{\text{FSR}}{2\sqrt{2}}\right)^2 = \dots = 0\text{dBc}$

$\text{SNR}_{\text{ideal}} = 6.02n + 1.76 = 98\text{dB}$     $(\text{SNR}_{\text{ideal}} = \frac{S_{\text{max}}}{N_{\text{min}}})$

$N_{\text{min}} = N_{\text{quant}} = \frac{\text{LSB}^2}{12} = \dots = -98\text{dBc}$

$\text{NF}_{\text{ideal}} = \frac{N_{\text{ideal}}}{N_{\text{sample}}} = -98\text{dBc} - 10 \log \frac{512000}{2} = -98\text{dBc} - 54\text{dB} = -152\text{dBc}$

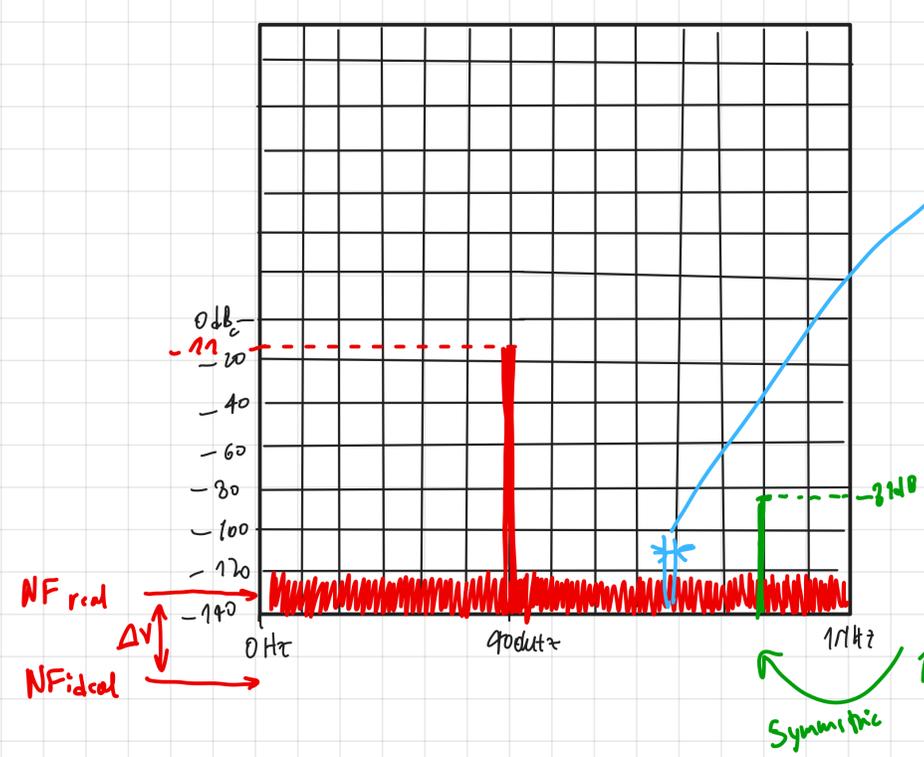
$\Delta N = ?$     $\text{NF}_{\text{real}} = \text{NF}_{\text{ideal}} + \Delta N = -152\text{dBc} + 20\text{dB} = -132\text{dBc}$

$\Delta S = \frac{S_{\text{pp}}}{0.1\text{pp}} = 11\text{dB}$

$\text{SNR}_{\text{th}} = \text{SNR}_{\text{ideal}} - \Delta S = 98\text{dB} - 11 = 87\text{dB}$

$\text{SNR}_{\text{real}} = \text{SNR}_{\text{th}} - \Delta N = 87\text{dB} - 20\text{dB} = 67\text{dB}$

$f_{\text{in}} = 400\text{kHz}$     $f_s = 2\text{MSPS}$     $\frac{f_s}{2} = 1\text{MSPS}$



$\text{bin-width} = \frac{f_s}{N_{\text{sample}}} = \frac{f_s}{\frac{N_{\text{sample}}}{2}} = \frac{2\text{MHz}}{512k}$

$\text{ENOB} = \frac{\text{SNR}_{\text{real}} + 1.76}{6.02} = \frac{67 + 1.76}{6.02} \approx 11.4 \Rightarrow 12\text{ bits}$

$\text{THD} = \frac{H}{S_{\text{real}}} = -70\text{dB}$

$f_{3\text{rdH}} = f_{\text{H3}} = 3 \cdot f_c = 1.2\text{MHz}$

$H_3 = -11\text{dBc} - 70\text{dB} = -81\text{dBc}$

Notes: On the computations:

$\frac{\text{Numerator}}{\text{Denominator}} = \frac{\text{Power}}{\text{attenuating factor}} = \frac{70\text{dBc}}{20\text{dB}} = 70\text{dBc} - 20\text{dB} = 50\text{dBc}$

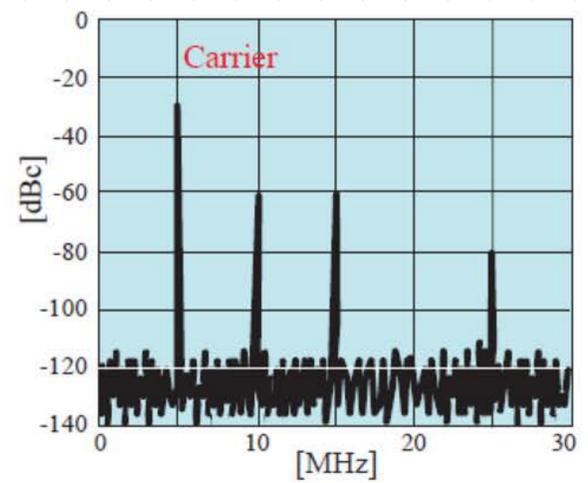
$H_1 + H_2 + D + N = -80\text{dBc} - 90\text{dBc} - 85\text{dBc} - 69\text{dBc} = 10 \log [10^{-8} + 10^{-9} + 10^{-8.5} + 10^{-6.9}] = -65\text{dBc}$

3

14 bit ADC with 3.3V FSR, spectrum with 680ms acquisition.

a) Compute  $\text{SNR}_{\text{ideal}}$ ,  $\text{SNR}_{\text{th}}$ ,  $\text{SNR}_{\text{real}}$ , SiNAD, and THD.

b) In case the 4<sup>th</sup> harmonic power is  $10^6$  lower than the carrier, compute the required duration of the acquisition stream for detecting the harmonic with an  $\text{SNR}=10\text{dB}$  over the noise floor.

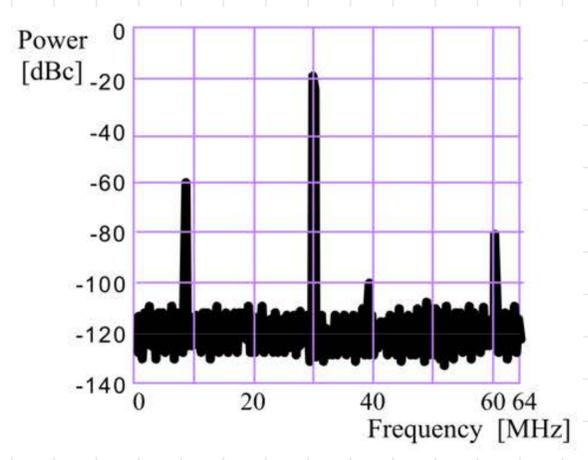


(not done)

4

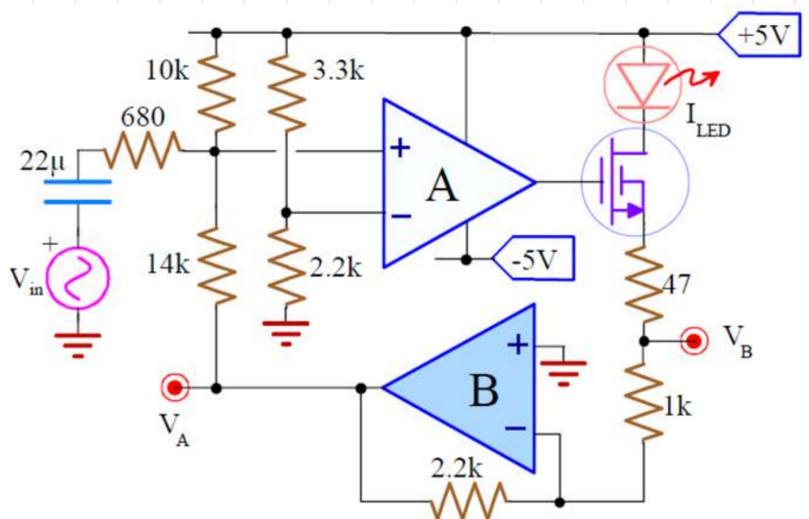
16bit ADC with +3.3V FSR. Measured output spectrum with 2kHz bin-widths. The signal is at 30MHz.

- a) Compute  $SNR_{ideal}$ ,  $SNR_{real}$  and the real noise (in  $V_{rms}$ ).
- b) Compute the THD and the SiNAD.
- c) Compute the 3<sup>rd</sup> harmonic rms and explain why it is at 38MHz.



(not done)

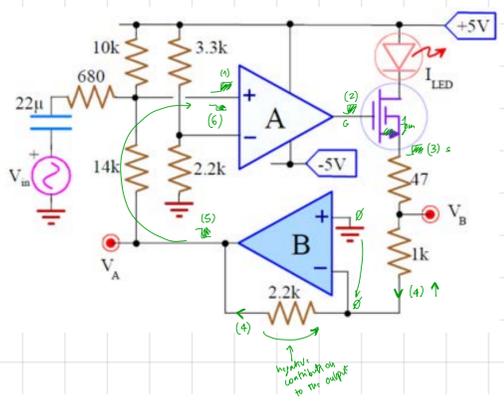
EXO6\_EXAMS



OpAmps with  $A_0=50V/mV$ ,  $V_{OS}=5mV$  and  $I_B=200pA$ . MOSFET with  $k=10mA/V^2$  and  $V_t=0.8V$ .

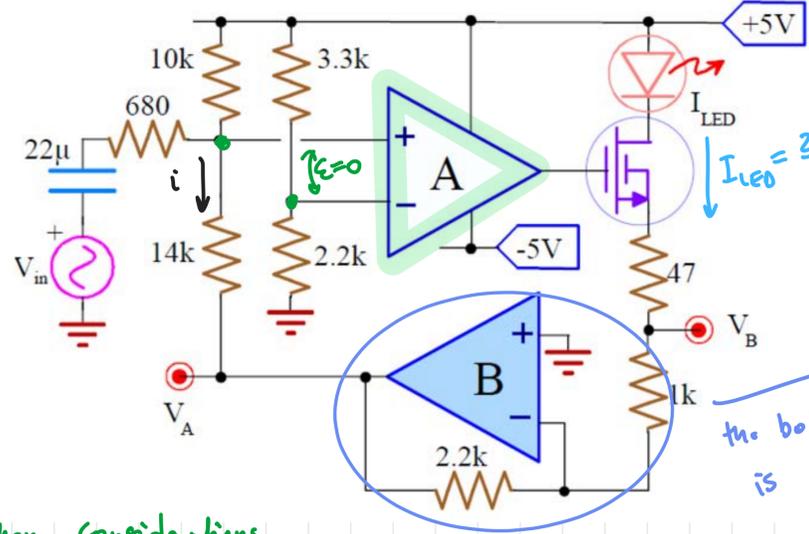
- a) Compute the relationships of  $V_A$ ,  $V_B$  and  $I_{LED}$  vs.  $V_{in}$ .
- b) Compute the input pole and plot  $V_A(t)$  and  $I_{LED}(t)$  waveforms when the input is a high frequency  $200mV_{pp}$  sinusoid.
- c) Compute the maximum  $V_A$  static error due to  $V_{OS}$  and  $I_B$  of the OpAmps.

a) (Negative feedback check)



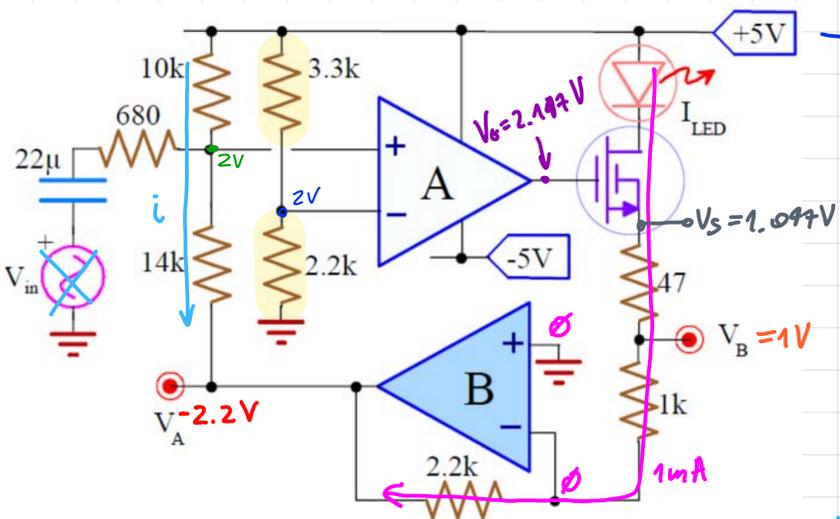
Negative feedback → virtual ground  
 ↓  
 so an OpAmp should have infinite gain

OpAmp A will provide infinite gain  
 ↓  
 $\epsilon = 0$   
 (VIRTUAL GROUND)



$G = \frac{-R_2}{R_1} = -\frac{2.2k}{1k+47}$  ← Finite gain

Other considerations:



Due to the P.S., thanks to the resistors we reach a voltage on the ⊖ pin of 2V

$$V^- = \frac{2.2k}{2.2k+3.3k} 5V = 2V$$

↳ thanks to neg. feedback we'll have virtual ground  $\epsilon=0$  so also on the ⊕ pin will have 2V

↳ At DC (Open) we should have the same current  $i$  across the 10k and 14k resistors →  $i = \frac{5-2}{10k} = 0.3mA = 300\mu A$

$$V_A = 5 - i(10k + 14k) = -2.2V$$

↳ in order to provide  $V_A = -2.2V$  having virtual ground in OpAmp B in inverting config. it means we'll have a current across the 2.2k resistor (this current will pass also through the LED) so it is:

↳  $I_{LED} = \frac{2.2V}{2.2k} = 1mA$  → (LED is always ON to provide  $V_A$ )

↳ We can then compute also  $V_B$  as:

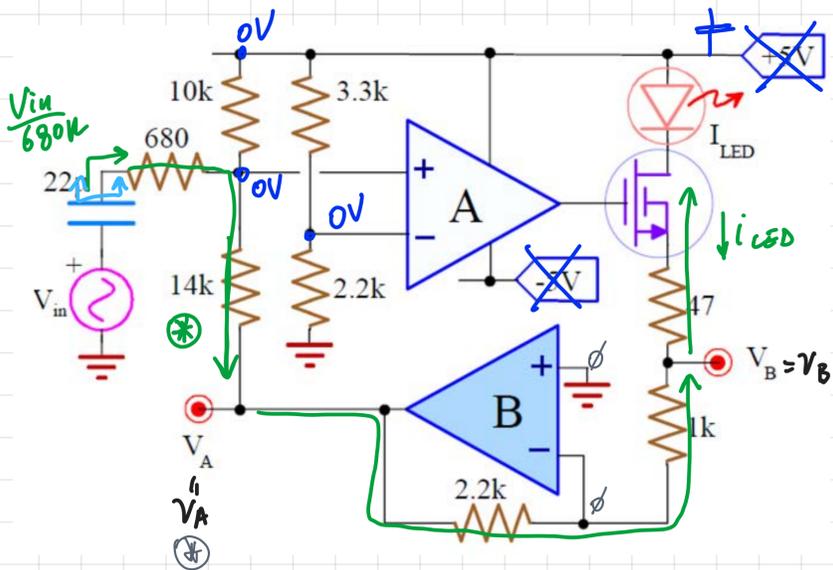
$V_B = 1k \cdot I_{LED} = 1V$  (At the MOS source →  $V_S = 1.047V$ )

↳  $V_G$  (GATE MOS): From the eq.  $I_D = k \cdot (V_G - V_t)^2$   
 $V_{GS} = \sqrt{\frac{I_D}{k}} = \sqrt{\frac{1mA}{10mA/V^2}} + 0.8V = 1.1V$  →  $V_G = 2.147V$

- ⇒ DC
- $i = 300 \mu A$
  - $V_A = -2.2 V$
  - $I_{LED} = 1 mA$
  - $V_B = 1 V$
  - $V_S = 1.047 V$
  - $V_G = 2.147 V$

→ Now we study the signal (so when C is not open)

We can consider the P.S. turned OFF → so at ⊖ and ⊕ are ∅



$$pole_{in} = \frac{1}{2\pi \cdot 22\mu \cdot 680} = 10.6 \text{ Hz}$$

$z_{c_{in}} = \emptyset$  → since at pin ⊕ we have ∅ →  $R_{eq} = 680\Omega$   
 → the capacitor stops the DC ( $\approx 0 \text{ Hz}$ )

→ since across 10k we'll have ∅ voltage

∅ current →  $\frac{V_{in}}{680\Omega}$  goes all in one direction \*

so we can have 0 output applying  $V_{in} \neq 0$  at 0 Hz freq. ↓ zero in 0 Hz!

$$V_A = -\frac{V_{in} \cdot 14k}{680} \Rightarrow \frac{V_A}{V_{in}} = -\frac{14k}{680} = -20.6$$

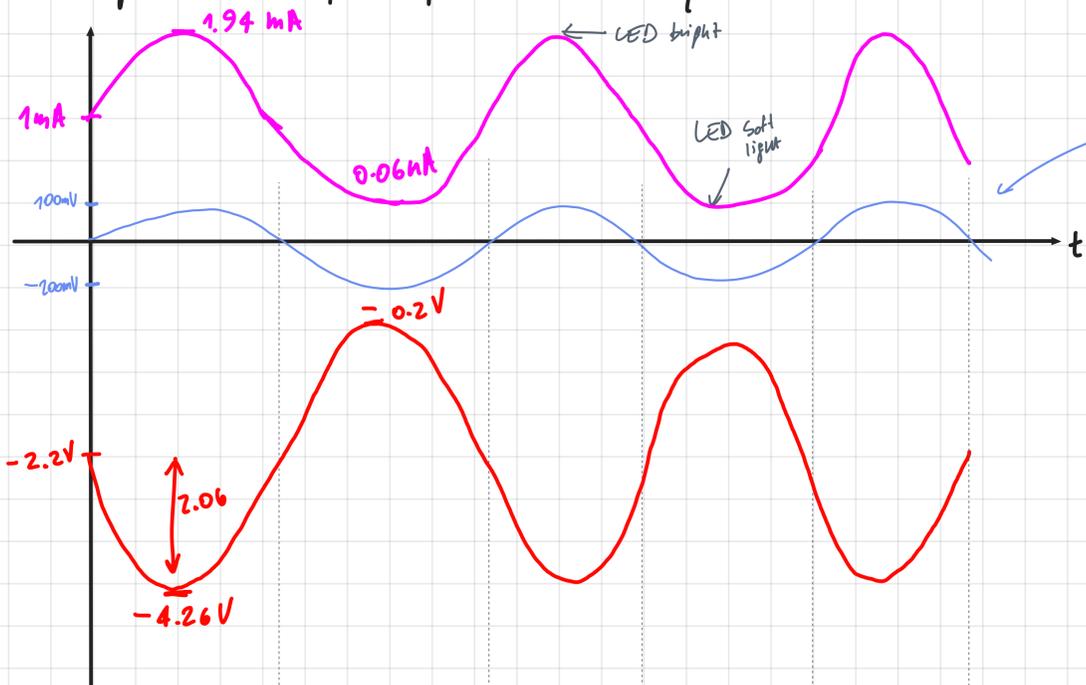
$$-V_B \cdot \frac{2.2k}{1k} = V_A \rightarrow V_B = \frac{14k}{680} \cdot \frac{1k}{2.2k} V_{in} \rightarrow \frac{V_B}{V_{in}} = \frac{14k}{680} \cdot \frac{1k}{2.2k} = 9.4$$

Gain of opamp B

$$i_{LED} = -\frac{V_A}{2.2k} \rightarrow \frac{i_{LED}}{V_{in}} = \frac{14k}{680} \cdot \frac{1}{2.2k} = 9.4 \frac{mA}{V}$$

Note: Since we're analyzing just the signal contribution is better to write the voltages in small letters and use the capital one for the total signal that sums the signal + polarization (DC)

b) • input pole already computed →  $pole_{in} = 10.6 \text{ Hz}$



input signal  $V_{in}$

$V_A = (-2.2V \text{ polarization}) + \text{signal contribution (Gain} = -20.6)$

↳ (peak  $-4.26 V$  is reachable with P.S.  $\pm 5V$ )

$I_{LED} = (1mA \text{ polarization}) + \text{signal contribution (Gain} = 9.4 \frac{mA}{V})$

↳ We should check if the LED saturates

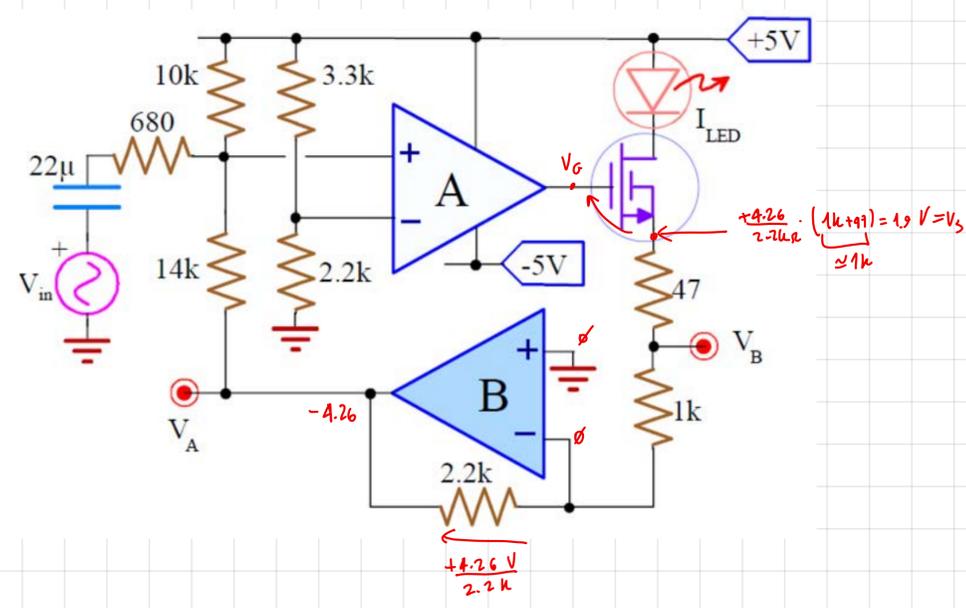
↳ We check if we can actually reach  $-4.26V$  and  $1.94mA$

(From the previous plot)

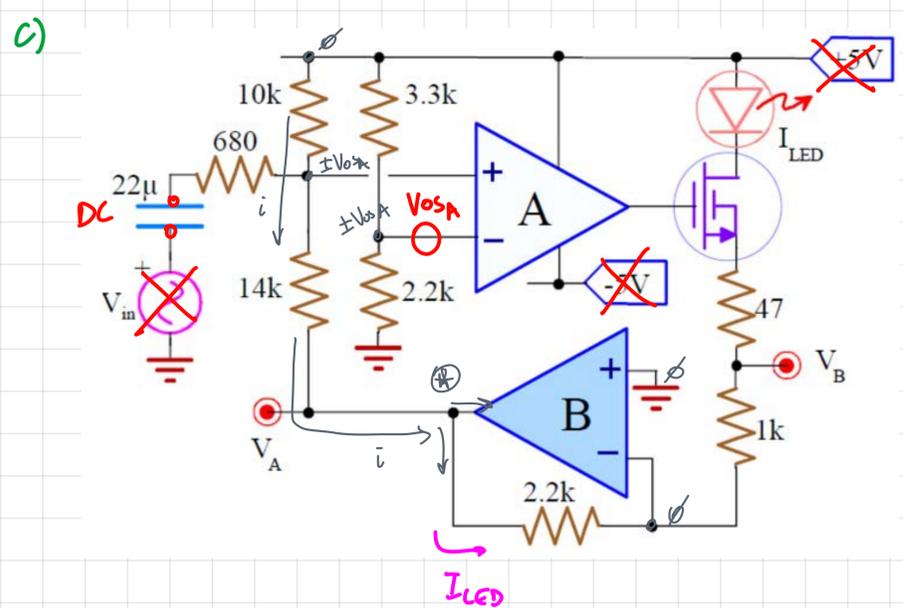
When  $V_A = -4.26$   $I_{LED} = 1.94mA$

$V_{GS} = \sqrt{\frac{I_{LED}}{K}} + V_T = 1.24V$

$V_G = 1.24 + \frac{1.9}{V_S} = 3.14 < 5 \checkmark$



We should also check that we can reach  $\approx -0.2V$  and  $0.06mA$  → they're both really low so the MOS will be near interdiction and  $V_G \approx V_T$  (reasonable V)



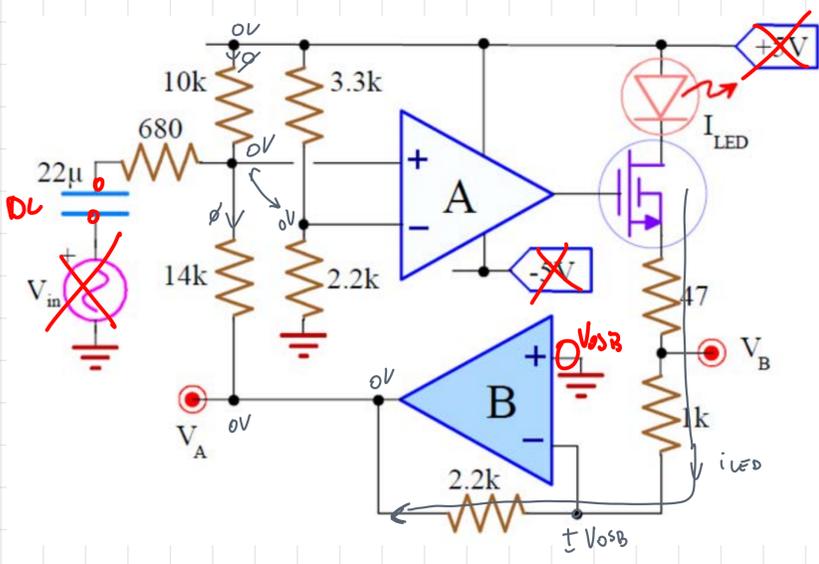
$V_{osA}$

$i = \frac{\pm V_{osA}}{10k}$

the offset is amplified  
not that big compared to VA values

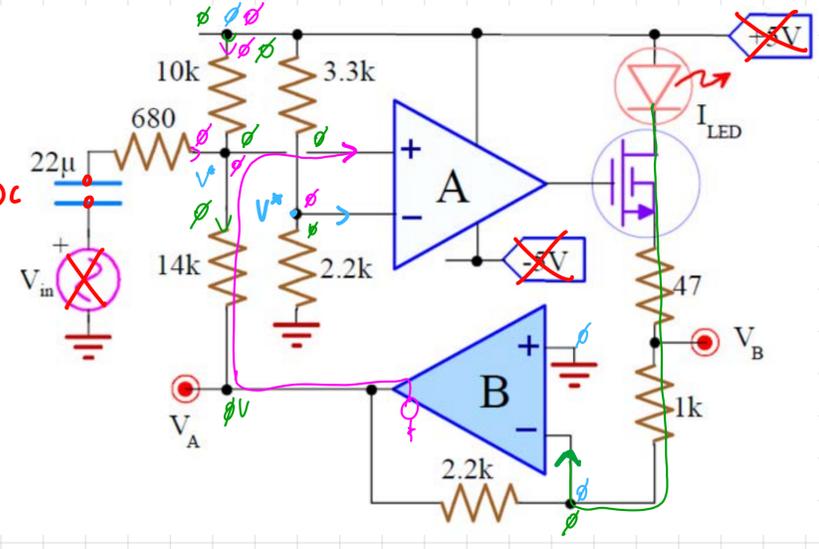
$V_A = \pm \frac{V_{osA}}{10k} (10k + 11k) = \pm V_{osA} \cdot 2.4 = \pm 12mV$

$i_{LED} \neq i \rightarrow i_{LED} = \frac{V_A}{2.2k} = \pm \frac{V_{osA} \cdot 2.4}{2.2k} = \pm 5\mu A$



$V_{osB}$

$i_{LED} = \frac{\pm V_{osB}}{2.2k} = 2.3\mu A$



$I_B$

( $I_{B+}$  no effect)

$I_{B-}$

$I_{B-}$

$V^r = I_{B-} (2.2k // 3.3k) = 264\mu V$

$i_{LED} = I_{B-} = 200pA$

$V_A = \frac{V^r}{10k} \cdot 24k$

$i_{LED} = \frac{V_A}{2.2k} = \frac{264\mu V}{10k} \cdot \frac{24k}{2.2k} = 290pA$

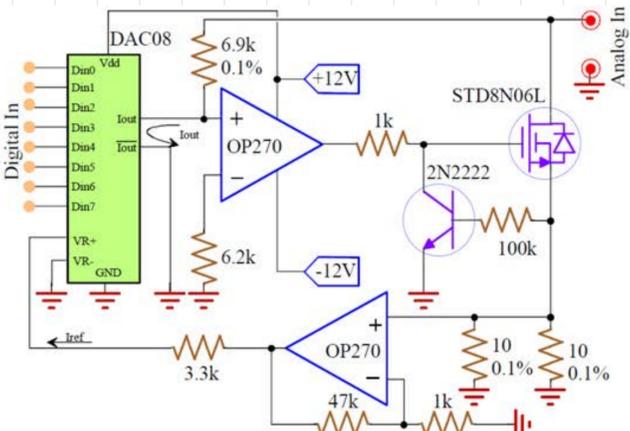
$I_{B+}$

$V_A = I_{B+} \cdot 14k$

$i_{LED} = \frac{V_A}{2.2k} = \dots$

(The sign of this contributions can be  $\pm$ )

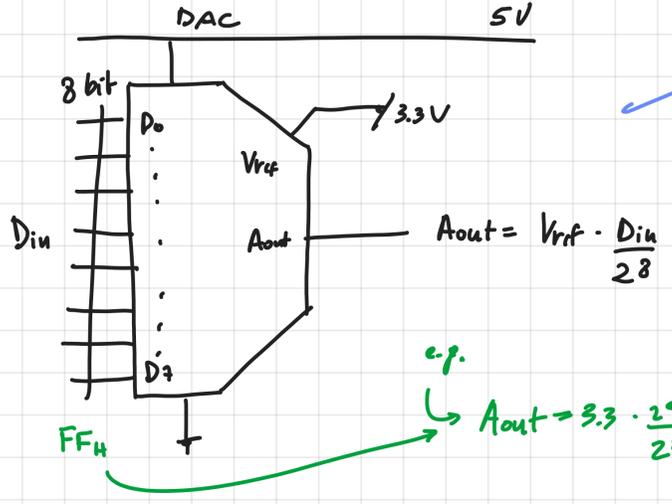
2



The DAC provides  $I_{out} = I_{ref} \cdot D_{in} / 256$  ( $D_{in}$  is the Digital In value) and **virtual ground** at its  $V_{R+}$  input. OpAmps have  $A_0 = 1500V/mV$  and  $GBWP = 5MHz$ . A voltage generator is applied at the Analog Input.

- a) Obtain the analytical relationship  $V_{in}/I_{in}$ , as a function of  $N$ .
- b) Reckon if the stage is stable or not when  $D_{in} = 255$ . Moreover, tell if stability improves by reducing  $D_{in}$ . (hint: assume  $1/g_{mMOS} = 495\Omega$  and ignore the role of 2N2222 BJT).

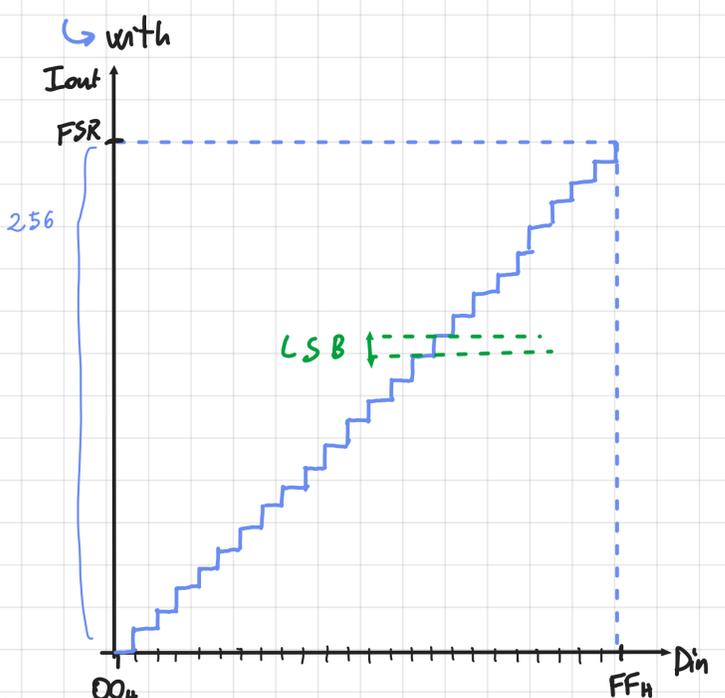
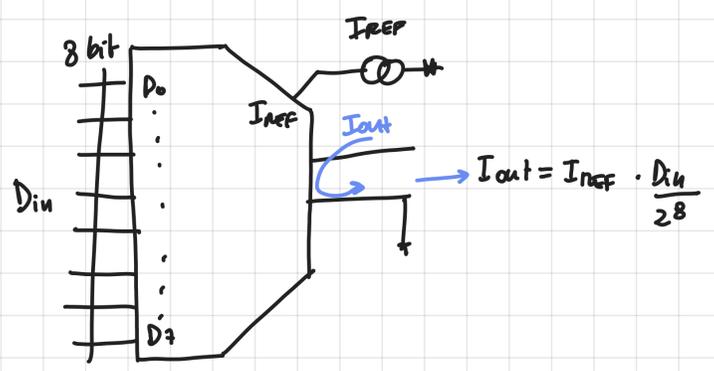
Note For DAC:



In this ex. we have an output current (not a voltage)

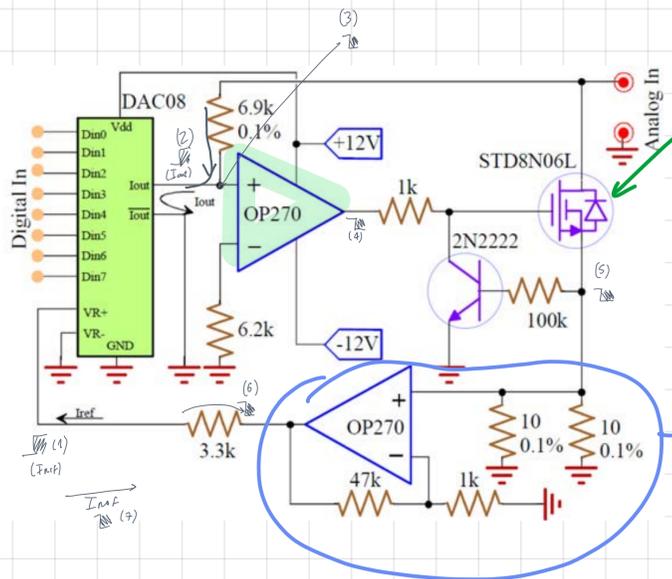
ex.  $I_{out} = 3.3 \cdot \frac{255}{28} = 3.3 \cdot \frac{255}{256} \approx 3.3V$

We consider



$LSB = \frac{FSR}{2^n} = \frac{5V}{2^8} = 19mV$

a)



From what we've seen:

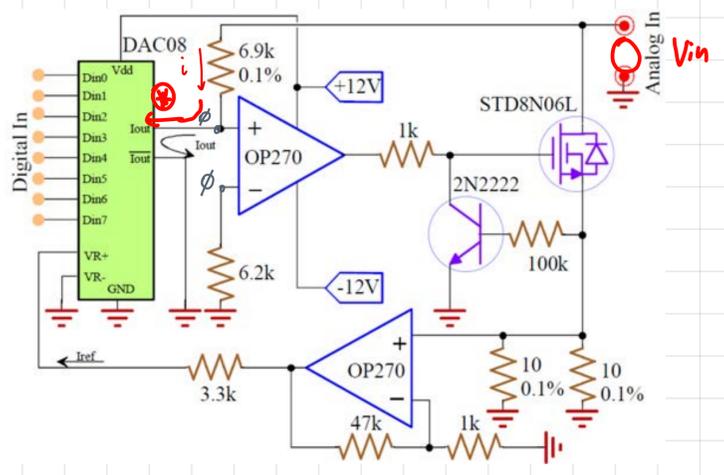
$I_{out} = I_{ref} \cdot \frac{D_{in}}{28}$

→ (negative feedback check)

$1 + \frac{47k}{1k} = 48$

this OpAmp will have high gain

So consider the signal



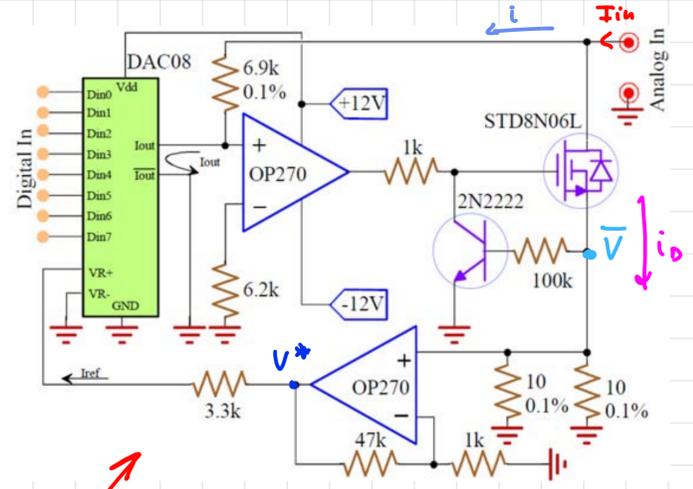
→ Usually we cannot force a current into the DAC because that  $I_{out}$  is the output current of the DAC, so it's like a current generator, BUT with the same considerations done for the negative feedback check we can see that it will increase the voltage on pin +, so the output of that OpAmp, so the input of the second OpAmp, so its output, so  $i_{ref}$ , so  $i_{out}$  → eventually, thanks to neg. feedback, the DAC output will be:

$i_{out} = i = \frac{V_{in}}{6.9k}$

Given that:

$$i_{out} = i_{ref} \frac{D_{in}}{2^8} \rightarrow i_{ref} = \frac{V_{in}}{6.9\mu} \cdot \frac{2^8}{D_{in}}$$

$$i_{out} = \frac{V_{in}}{6.9\mu}$$



$V^* = i_{ref} \cdot 3.3k$

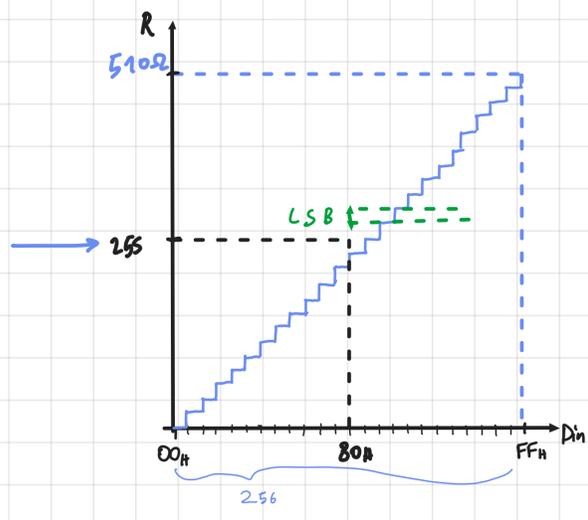
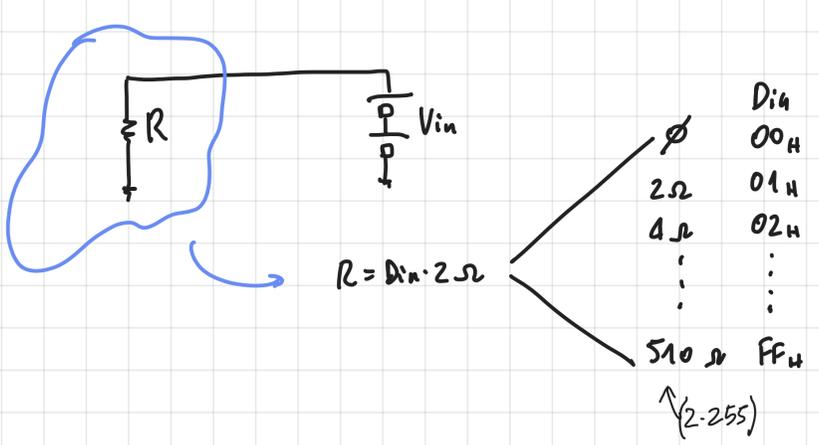
$V \cdot 48 = V^* = i_{ref} \cdot \frac{3.3k}{48}$

$i_D = \frac{V}{5\Omega} = \frac{3.3k}{5 \cdot 48} \cdot \frac{2^8}{6.9\mu} \cdot \frac{V_{in}}{D_{in}} = V_{in} \cdot \frac{0.5}{D_{in}} \frac{A}{V} = \frac{V_{in}}{2\Omega \cdot D_{in}}$

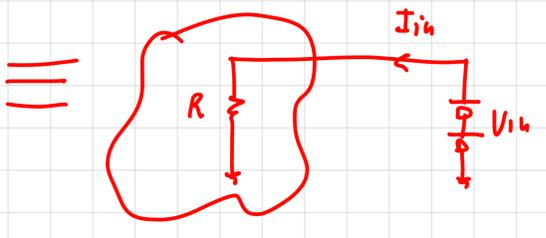
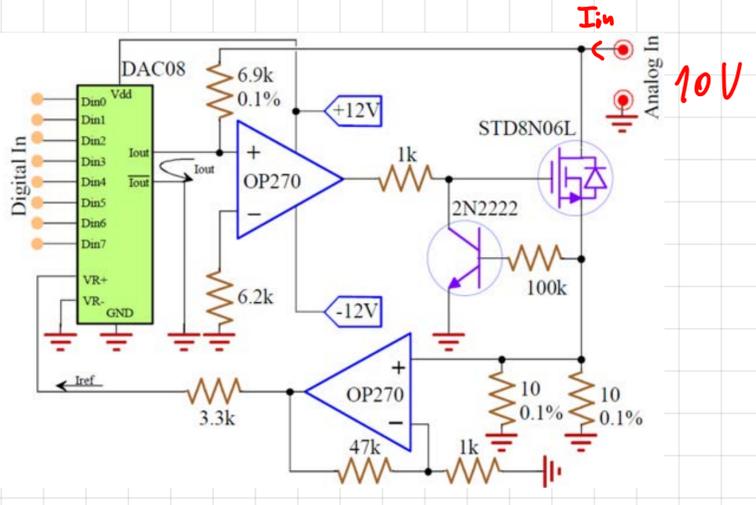
So consider that  $I_{in} = i_{in} + i_D = \frac{V_{in}}{6.9k} + \frac{V_{in}}{2\Omega \cdot D_{in}} \Rightarrow \frac{V_{in}}{I_{in}} = 2\Omega \cdot D_{in}$  through  $D_{in}$

It's like a PROGRAMMABLE RESISTOR

ex.



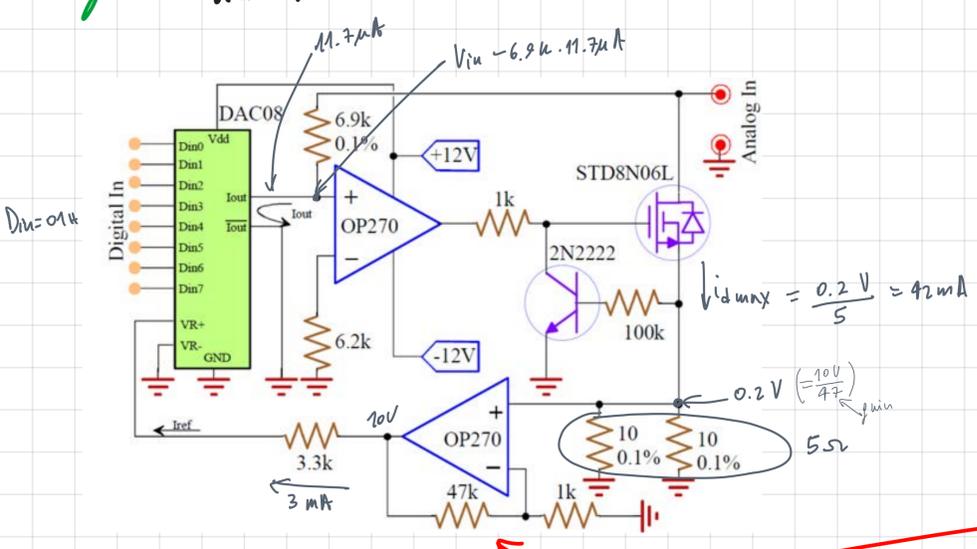
Let's check if for an input of 10V the circuit is feasible:



$I_{in} = \frac{10V}{255\Omega} = 39\mu A$  (e.g. for  $D_{in} = 80_H$ )

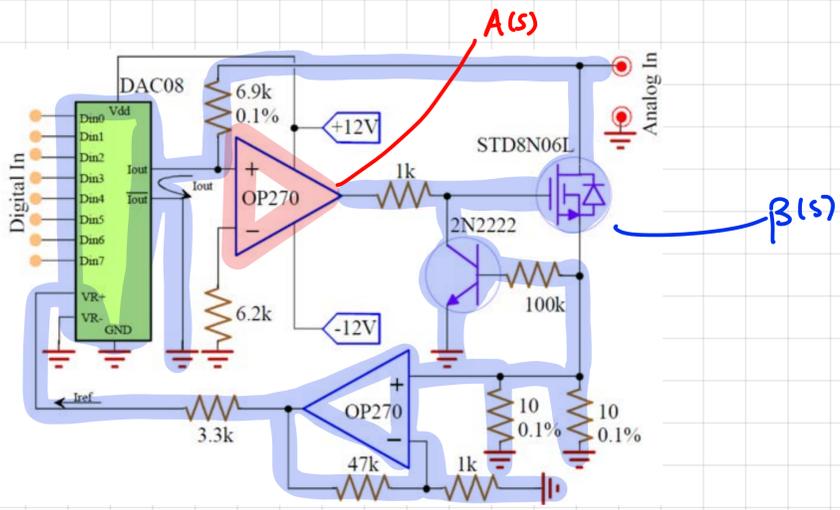
↳ we should check the  $V_{GS}$  of the MOS is reasonable and if the opAmp on the TOP can provide it within the saturation

e.g. Consider:



Low if we want a higher max we can reduce the resistors

b)



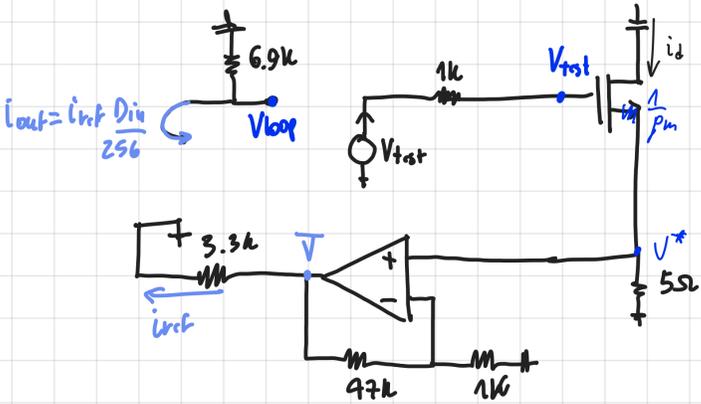
$\beta(s)$

$$\beta(s) = - \frac{5\Omega}{5s + 1} \cdot \frac{48}{3.3k} \cdot \frac{D_{in}}{256} \cdot 6.9k = -0.392 \cdot D_{in}$$

*negligible*

From:

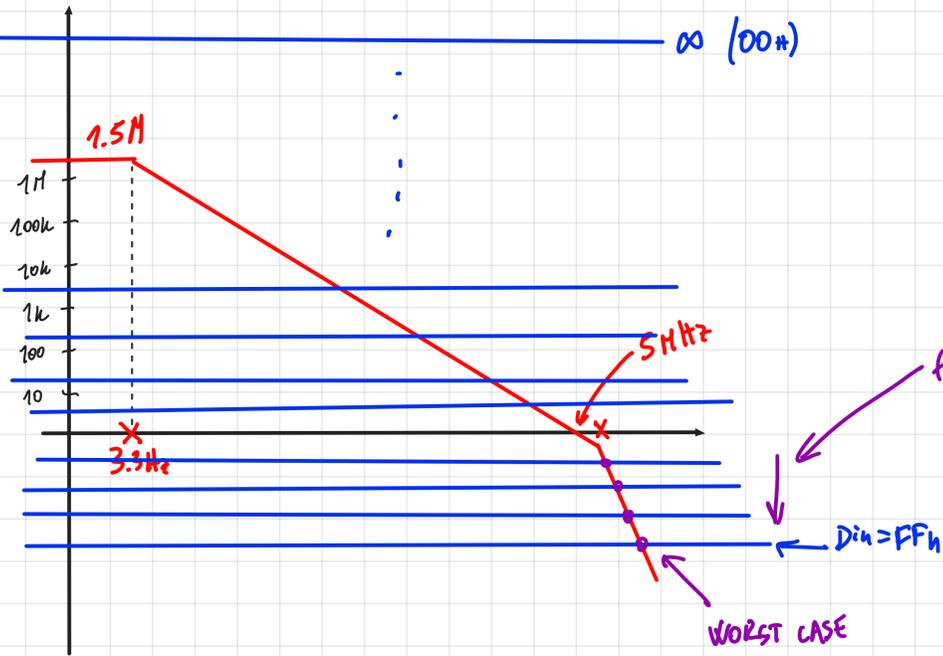
- $V^* = \frac{5\Omega}{5s + 1} V_{test}$  depends on  $i_d$
- $\bar{V} = V^* - 48$
- $i_{ref} = \frac{\bar{V}}{3.3k}$
- $i_{out} = i_{ref} \cdot \frac{D_{in}}{256}$
- $V_{loop} = i_{out} \cdot 6.9k$



$\frac{1}{\beta} = \frac{2.55}{D_{in}}$

$D_{in} = 00_{H} \rightarrow \infty$   
 $01_{H} \rightarrow 2.5$   
 $\vdots$   
 $FF_{H} \rightarrow 0.01$

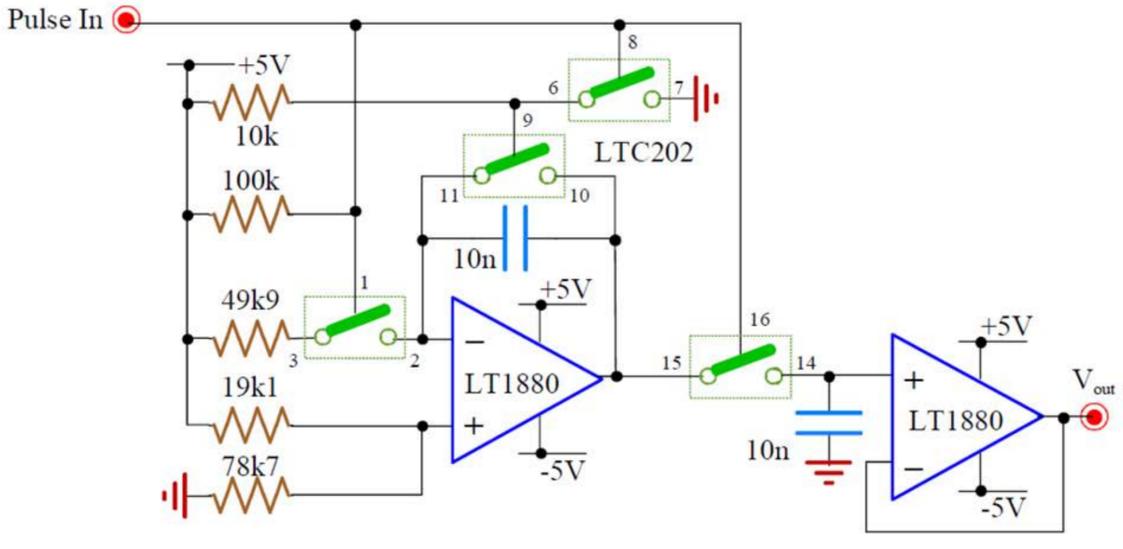
Bode:



from here the circuit becomes unstable

so we have to work with  $D_{in}$  small to be stable

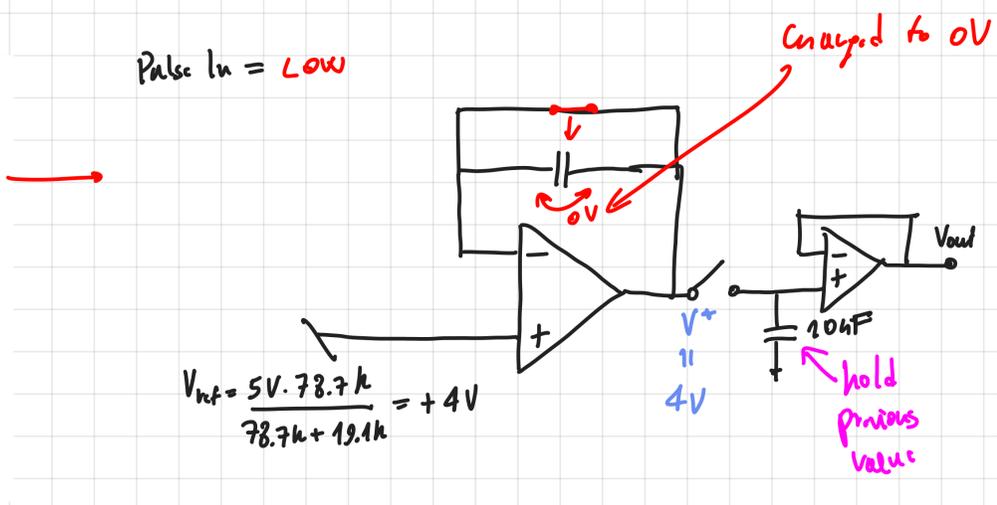
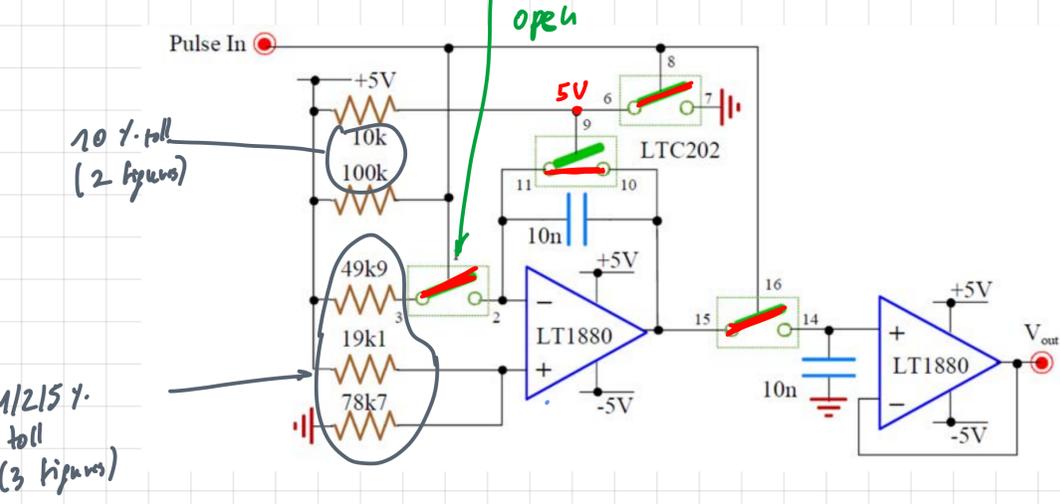
3



The LTC202 is a quad analog switch (closed when control pin is high). The input is a pulse, whose width  $T_{high}$  is in  $1ms \div 2ms$  range. The OpAmps have  $I_B < 1.5nA$  in the  $-40^\circ C \div +85^\circ C$  range.

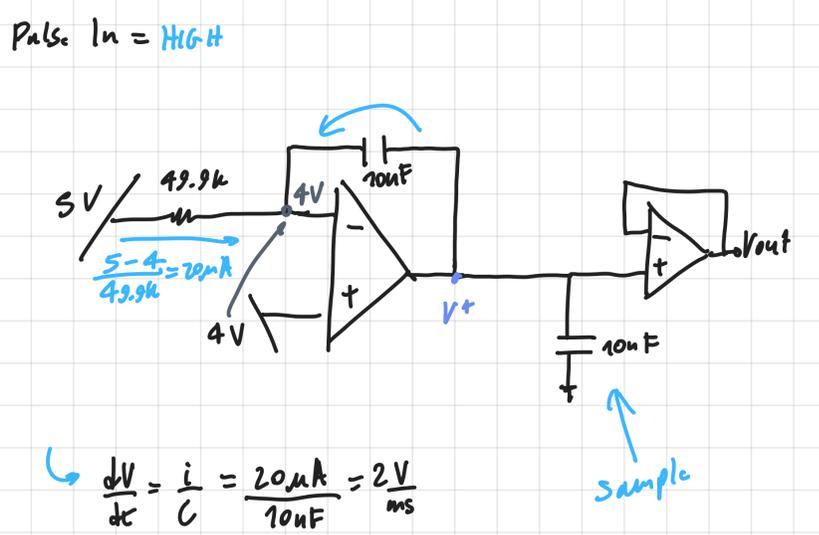
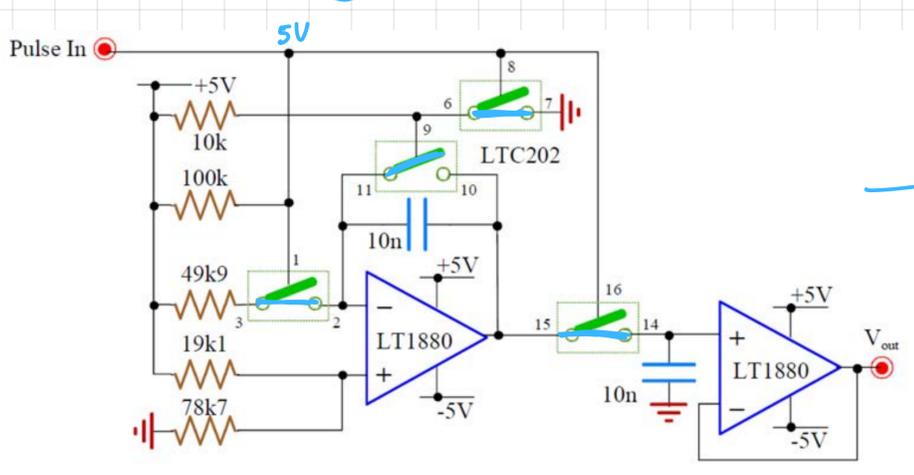
- a) Compute  $V_{out}$  as a function of  $T_{high}$ .
- b) Reckon the min  $T_{low}$  that guarantees a precision of  $1\mu s$ .

a) Consider Pulse In = low

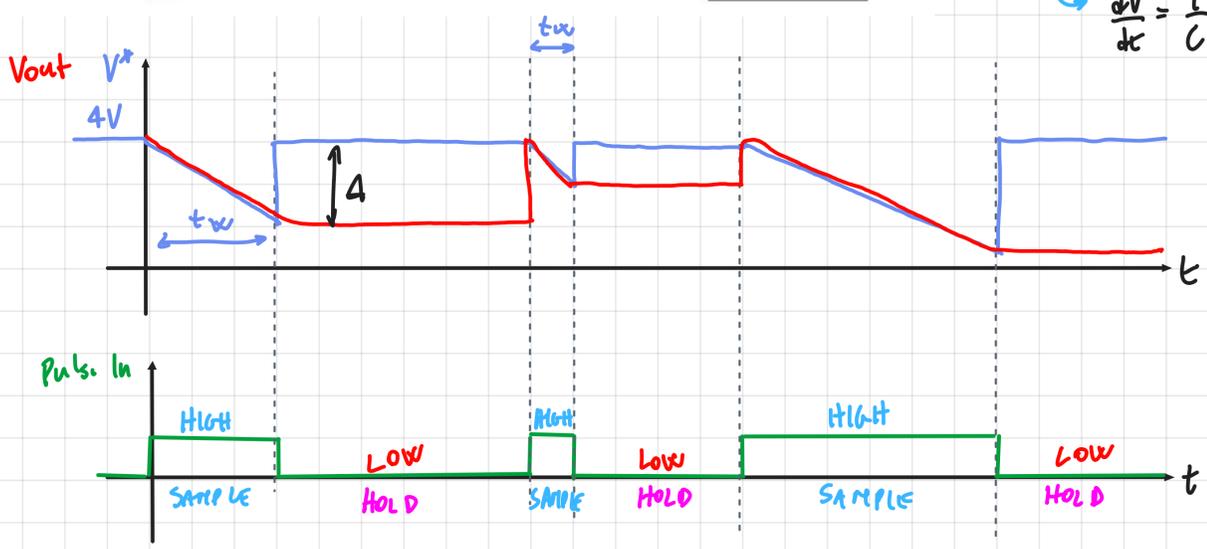


$$V_{int} = \frac{5V \cdot 78.7k}{78.7k + 19.1k} = +4V$$

Consider Pulse In = high (a.p. 5V)



$$\frac{dV}{dt} = \frac{i}{C} = \frac{20\mu A}{10nF} = 2V/ms$$



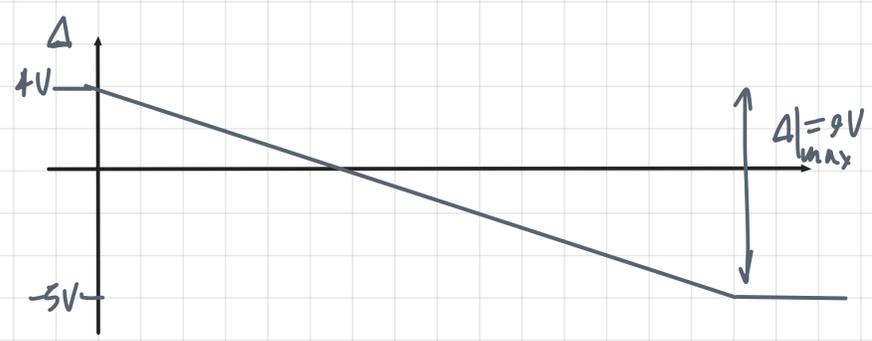
( $T_{high}$ )

$$A = \frac{dV}{dt} \cdot t_w = 2 \frac{V}{ms} \cdot t_w$$

- $t_w = 0 \rightarrow A = 0 \rightarrow V_{out} = 4V$
- $t_w = 1ms \rightarrow A = 2V \rightarrow V_{out} = 2V$
- $t_w = 2ms \rightarrow A = 4V \rightarrow V_{out} = 0V$

low pulse in  $\Rightarrow$  low Vout

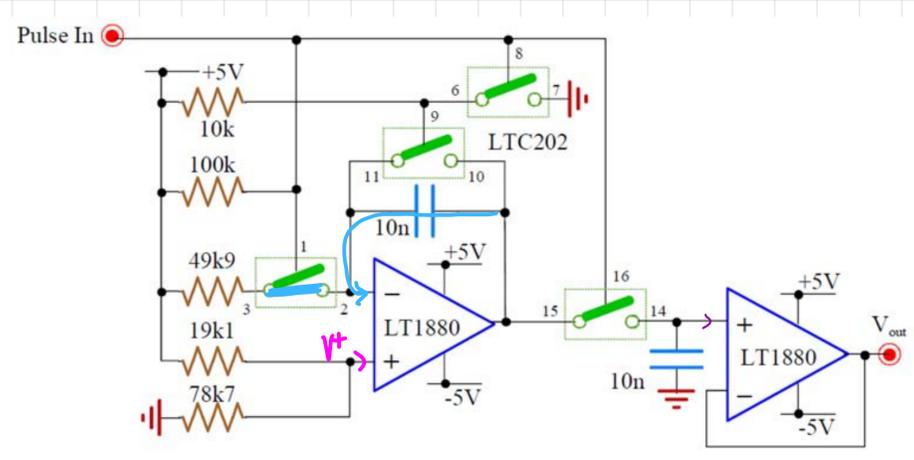
Note:  $V^+$  can go to negative value  $\rightarrow$  if the sample phase is very long  
 but can't go lower than  $-5V$



$\rightarrow t_{width} = \max \frac{9V}{\frac{dV}{dt}} = \frac{9V}{2 \frac{V}{ms}} = 4.5 ms$

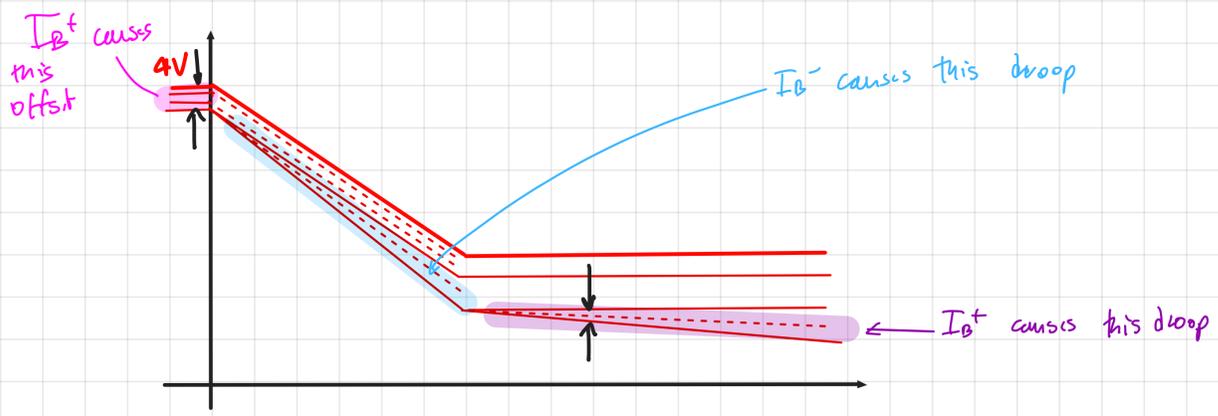


b) Consider the possible errors and tolerances



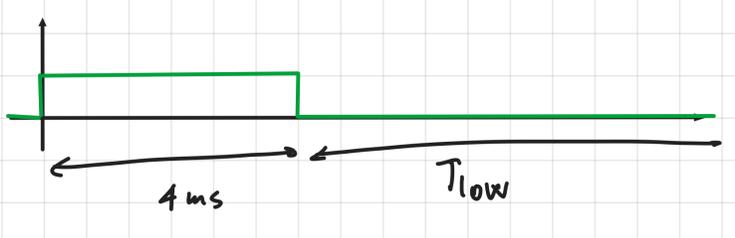
- $I_B$  const. error
- $I_{B^+}$   $\rightarrow V^+ = +4V - I_B (19.1k // 78.7k)$
- $I_B^-$   $\rightarrow$  discharges the capacitor (during SAMPLE)
- $I_{B^+}$   $\rightarrow$  discharges the capacitor (during HOLD)

So we'll have:

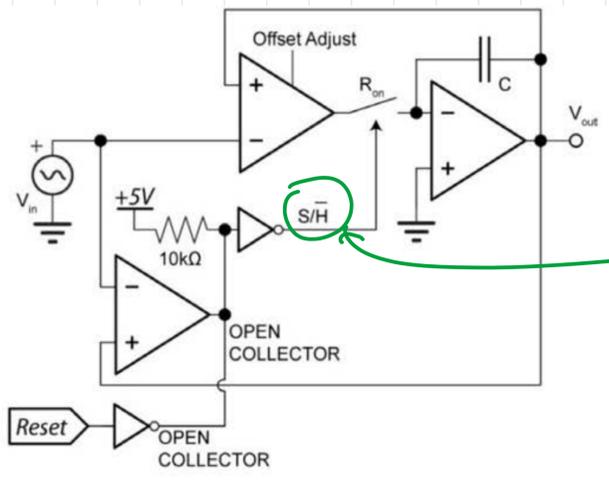


During  $T_{low}$  we have  $I_{B^+}$  and  $I_{B^-}$  errors must be limited in order to have a precision of  $1\mu V$

$\rightarrow$  so error ( $T_{low}$ )  $\leq 1\mu V$   
 To DO AT HOME



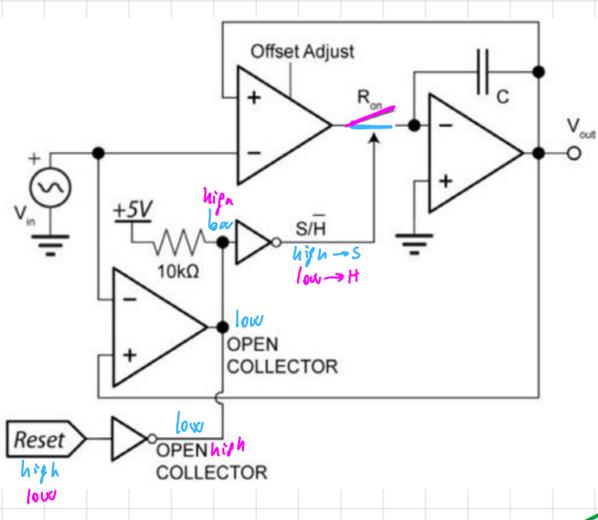
4



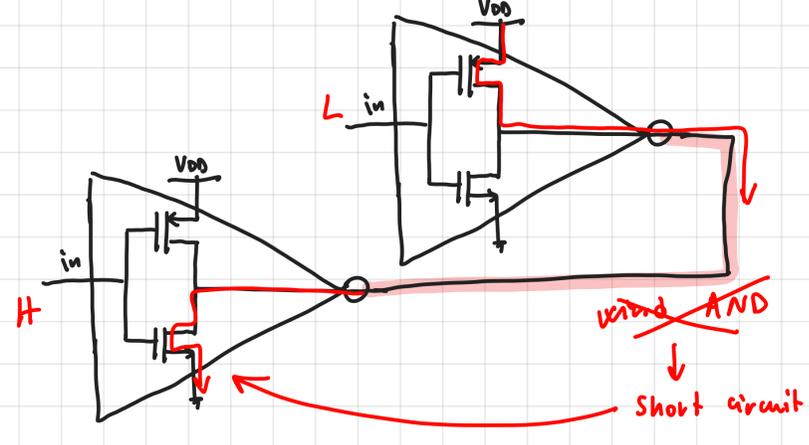
This symbol means is in S when the signal is High  
H when the signal is Low

The Offset Adjust pin is set in order to provide a +10mV output offset.  $R_{on}=10\Omega$ ,  $C_H=1nF$ ,  $GBWP=10MHz$ . The comparator has open-collector output.

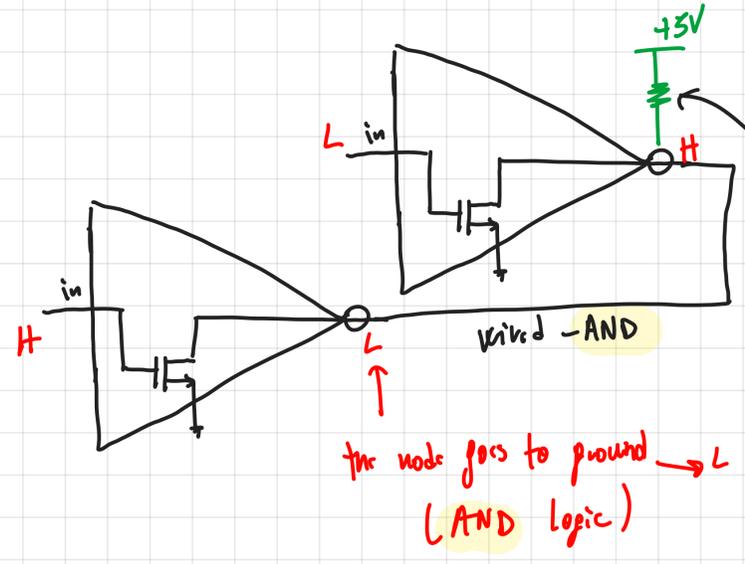
- a) Draw the quoted waveforms at all nodes when a triangular input from 0V to 2V is applied at  $V_{in}$ , with 1ms period, and a reset pulse of 50% duty-cycle is applied every 1.5ms.
- b) Explain the circuit behavior and the role of the offset adjust.
- c) Compute the max input frequency that ensures an error lower than 1LSB for a 10bit ADC with FSR=5V.



Note For the inverter  $\triangle$  we cannot have a wired-AND



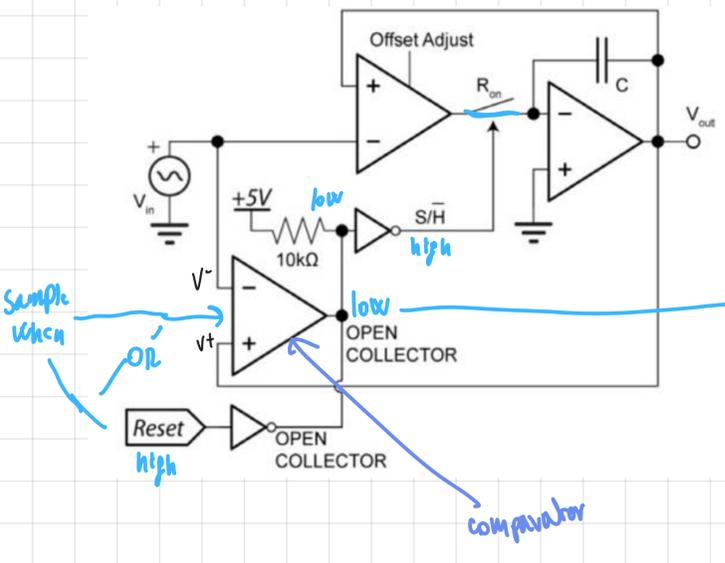
What can we do  $\rightarrow$  we can use gates without the pull-up (p-channel)



we must pull-up with a resistor ourselves

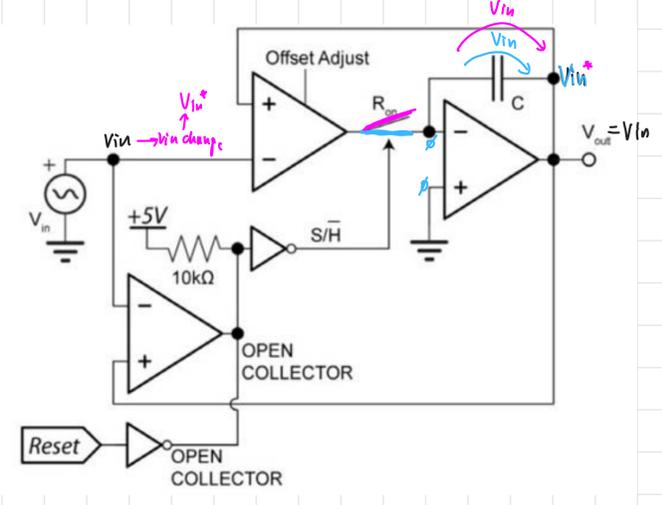
called OPEN COLLECTOR (or OPEN DRAIN)  $\rightarrow$  Symbol  $\equiv$  OPEN COLLECTOR GATE

So now let's consider the circuit:

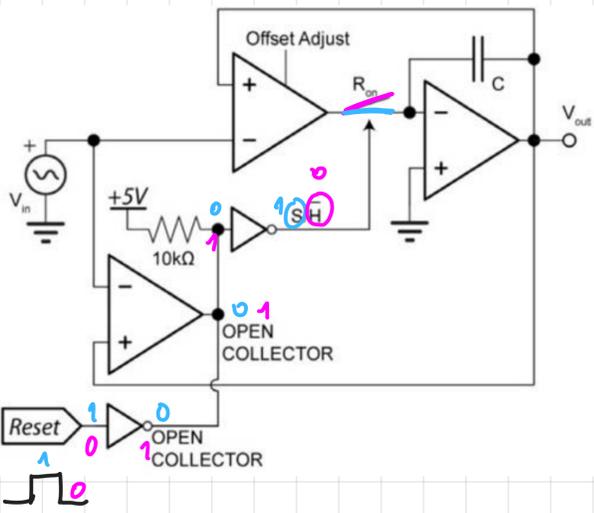


When  $V_{in} > V^+$   $\Rightarrow V_{in} > V_{out} \rightarrow S$

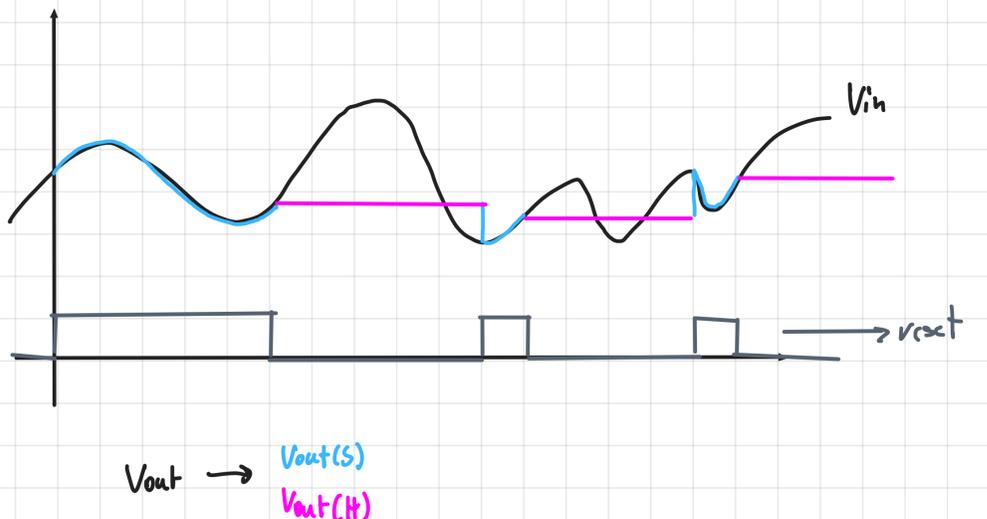
If the switch is close



a)



(Considering without the comparator)



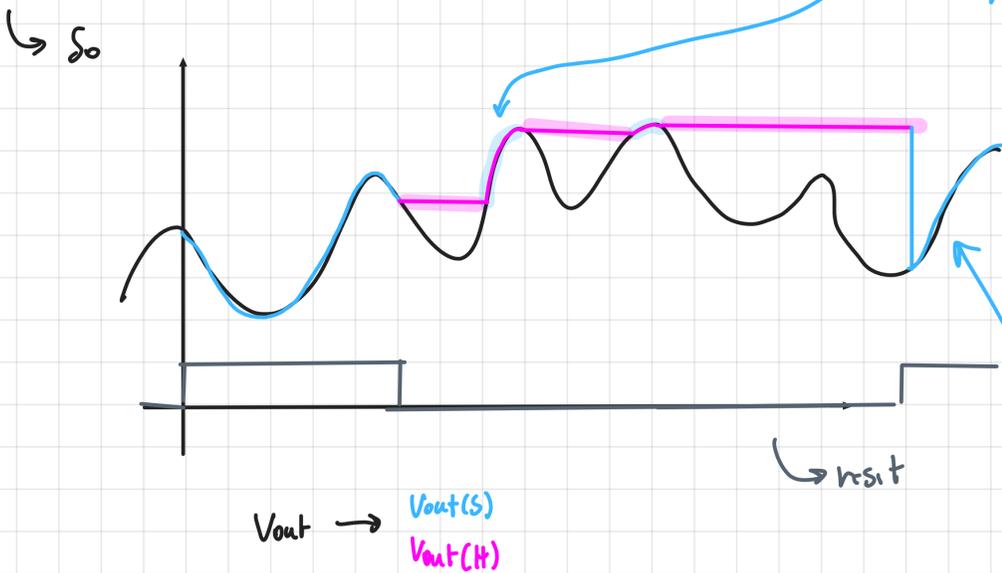
(1=high)  
(0=low)

Now consider with comparator

$$\begin{cases} V_{out} < V_{in} \rightarrow S \\ V_{out} > V_{in} \rightarrow H \end{cases}$$

AND logic that command the switch

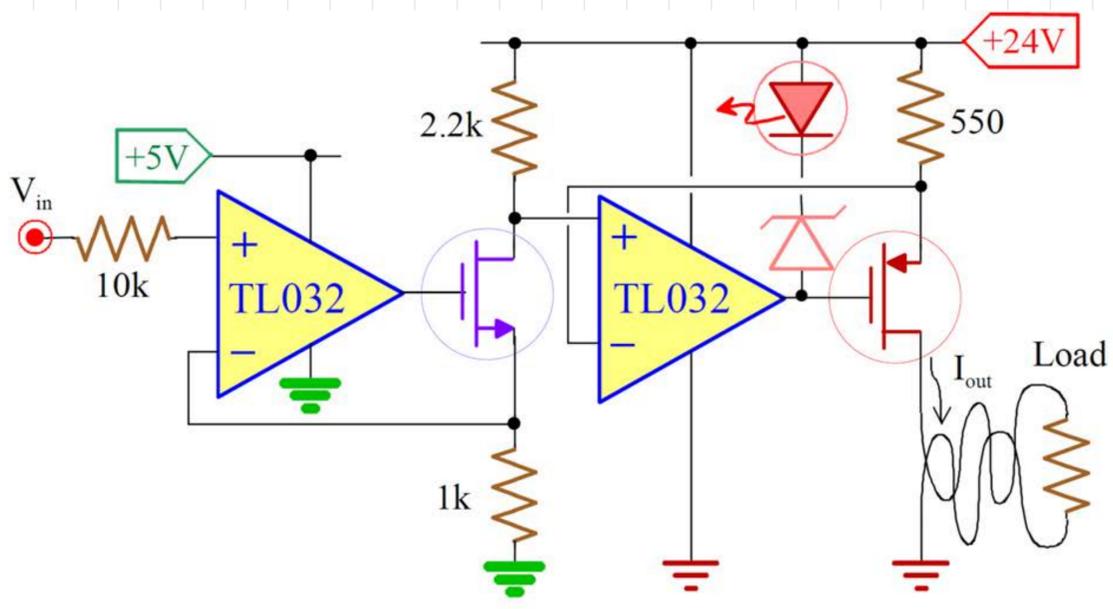
b)



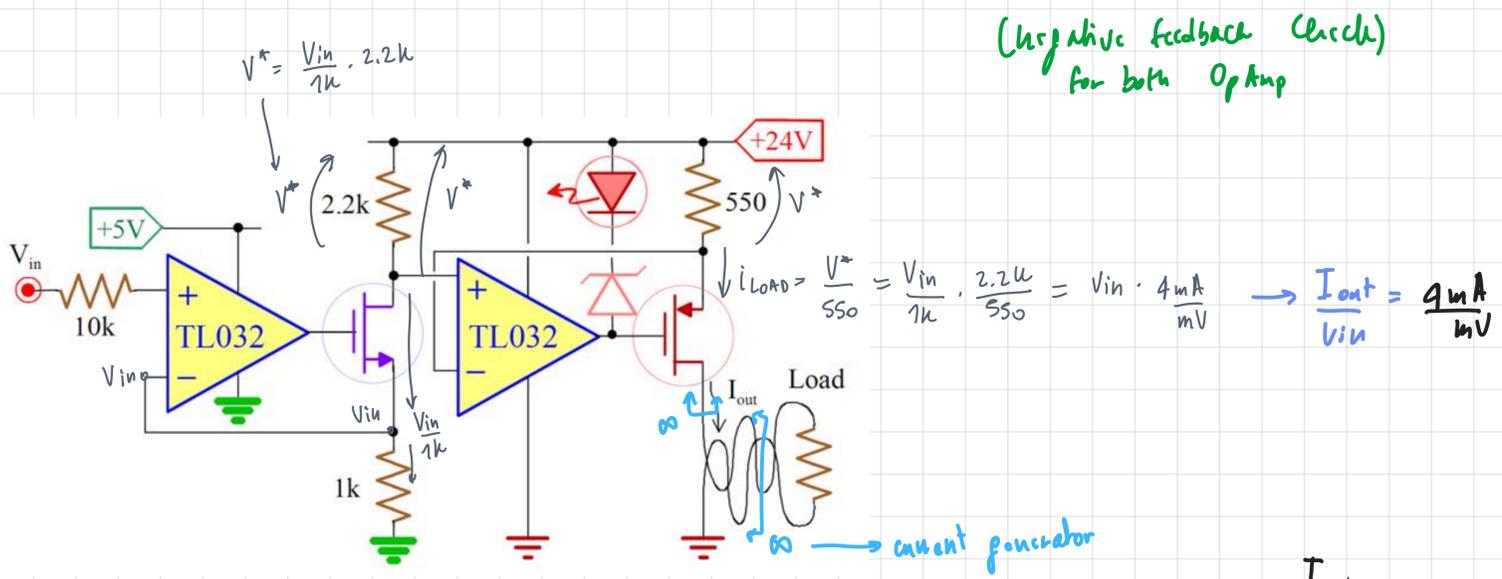
⇒ It's a peak-detector

↓  
the reset is used to  
reset to sampling values  
lower than the peak we reached

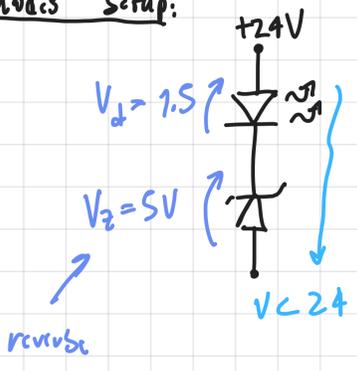
5



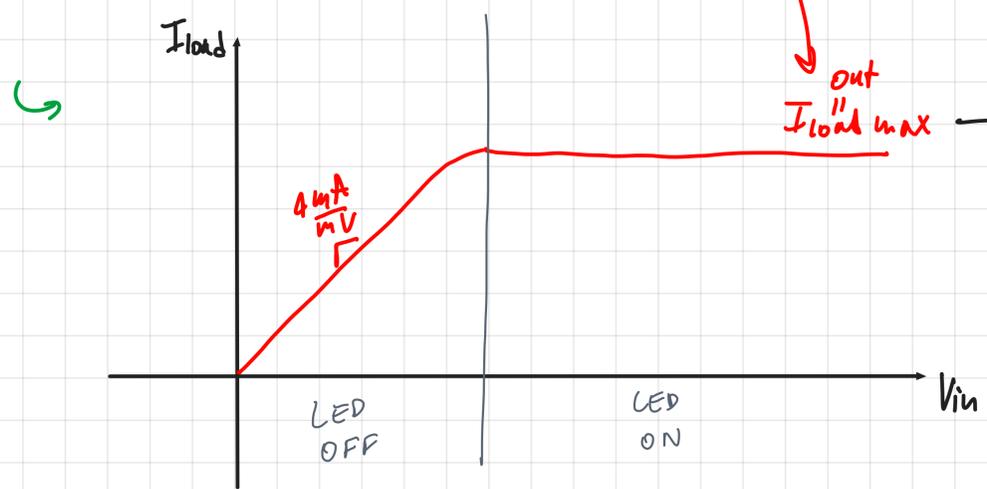
- A 5V zener is in series to a 1.5V LED. MOSFETs have  $V_T=1V$  and  $k=\frac{1}{2}\mu C_{ox}W/L=2.5mA/V^2$ .
- Find the relationship  $I_{out}/V_{in}$  and the  $V_{in,max}$  that ensures linear behavior for rail-to-rail OpAmps.
  - Change the first stage in order to employ the same +5V power supply, but providing a  $V_{in,max}=+5V$ .
  - Tell in which conditions the LED will light up and why the circuit is prone to burnings.



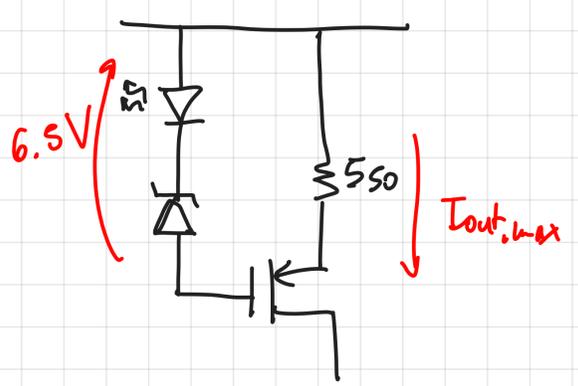
Diodes Setup:



Zener:



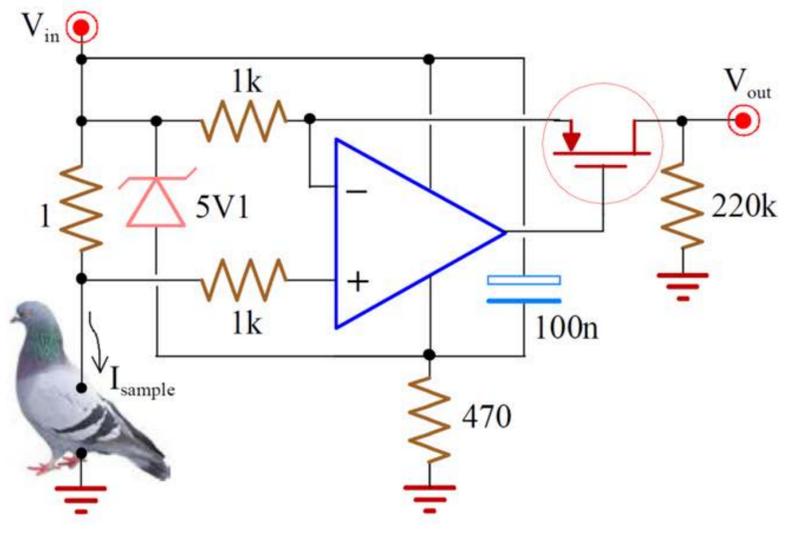
To compute it:



$$I_{out,max} = k \cdot 0^2 = k \cdot (6.5V - 550 \cdot I_D - V_t)^2$$

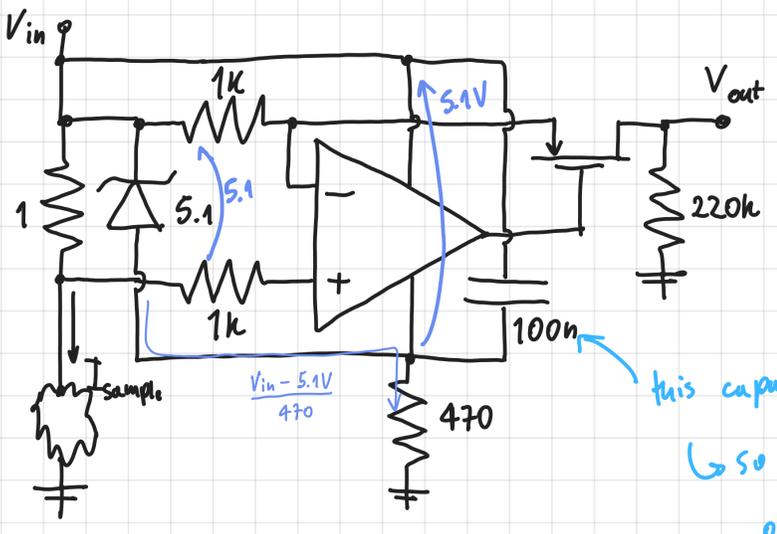
→ solve this 2nd order eq. ⇒  $I_{out,max}$

6)



- a) Compute the relationship between  $V_{out}$  and  $I_{sample}$ .
- b) Design a new stage employing OpAmps that instead forces a constant current  $I_{sample} = 10mA$  through the sample and measures the voltage developed across it with a gain of +20.

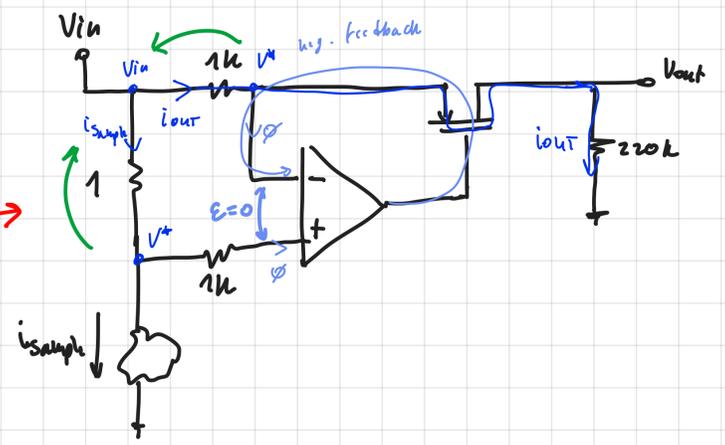
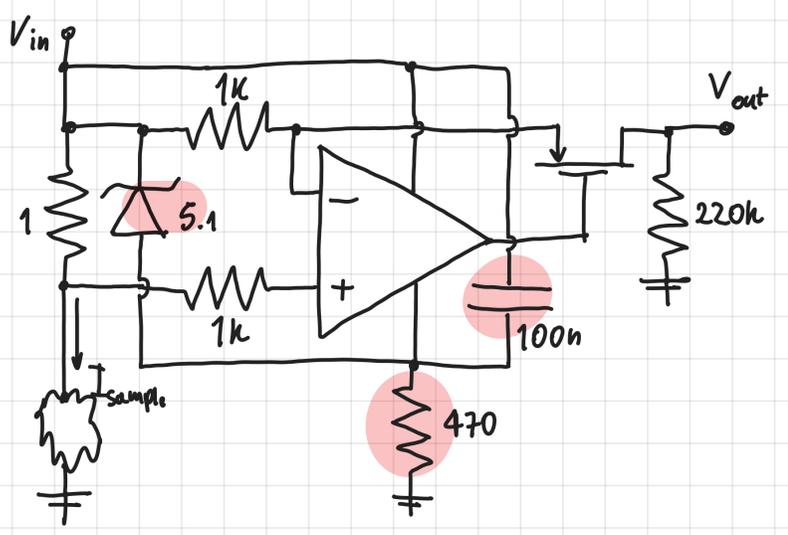
a)



→ if  $V_{in} > V_z = 5.1V$  → guarantee a 5.1 P.S. to the OpAmp

this capacitor (connected by the P.S.)  
 → so it's needed for stabilization or noise/dist. reduction

→ If we remove the Zener diode, the capacitor and the resistor



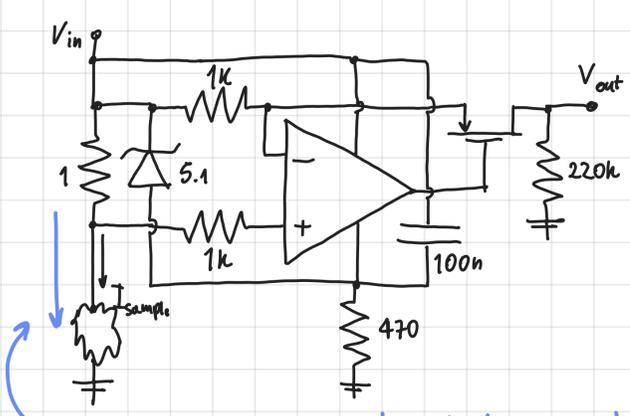
Compute the ideal Gain ( $\epsilon = 0$ ):  $I_{sample} \cdot 1\Omega = I_{out} \cdot 1k$

$$V_{out} = 220k \cdot I_{out} = 220k \frac{1\Omega}{1k} I_{sample}$$

b) Up to now the circuit was a current reader

I to V converter

$$\frac{V_{out}}{I_{sample}} = 220 \frac{mV}{mA}$$

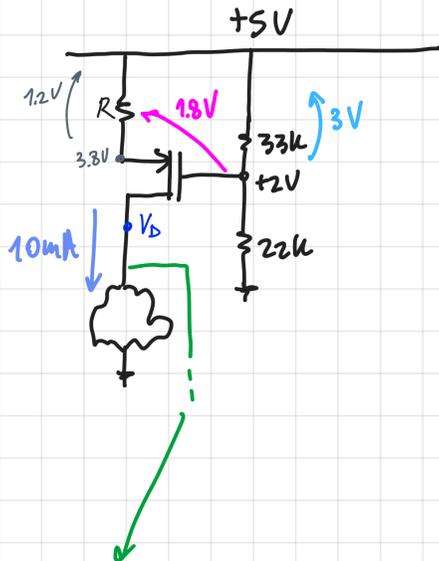


the current flows through the sample and provides a signal which is proportional to the amount

Now the signal does not vary anymore 'cause we're forcing a constant current  $i_{sample}$  through the sample

$i_{sample} = 10 \text{ mA}$

e.g. stage design:



easy current generator

$V_t = 0.8 \text{ V}$

$k = 10 \frac{\text{mA}}{\text{V}^2}$

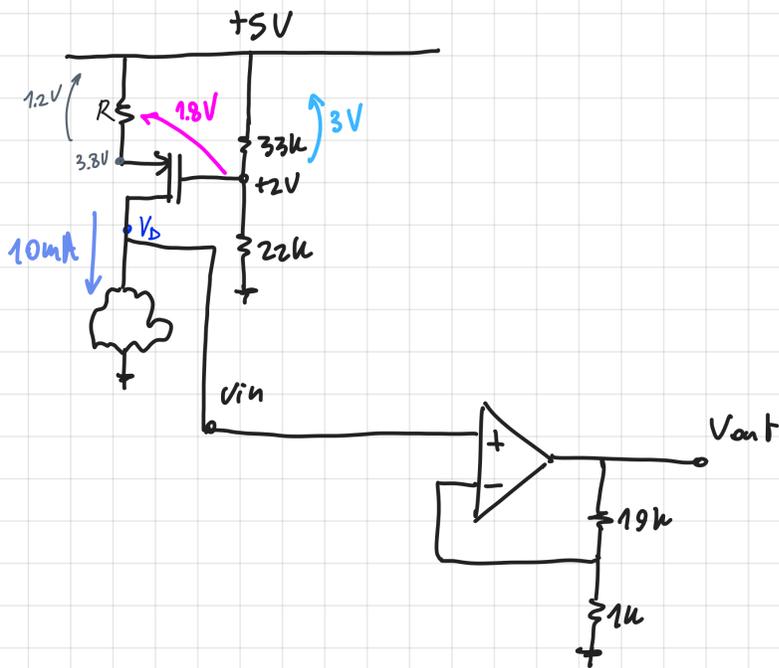
$V_{GS} = V_t + \sqrt{\frac{i_{sample}}{k}} = 0.8 + \sqrt{\frac{10 \text{ mA}}{10 \frac{\text{mA}}{\text{V}^2}}} = 1.8 \text{ V}$

$V_{DS} \geq V_{DD} = V_{GS} - V_t = 1 \text{ V}$   
overdrive

$V_D \leq 3.8 \text{ V} - 1 \text{ V} = 2.8 \text{ V}$

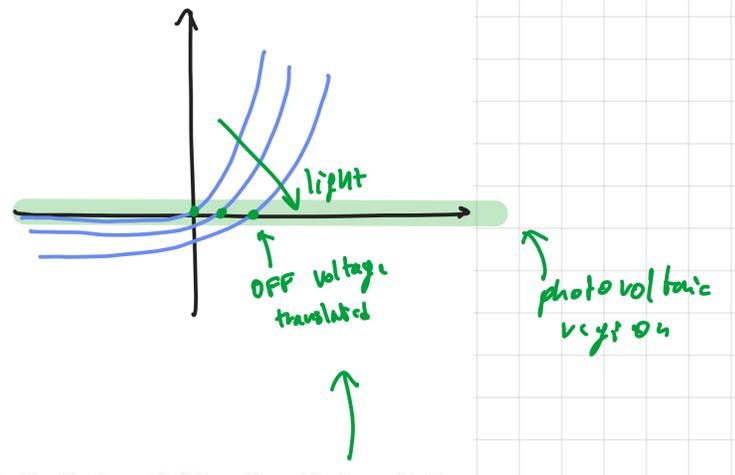
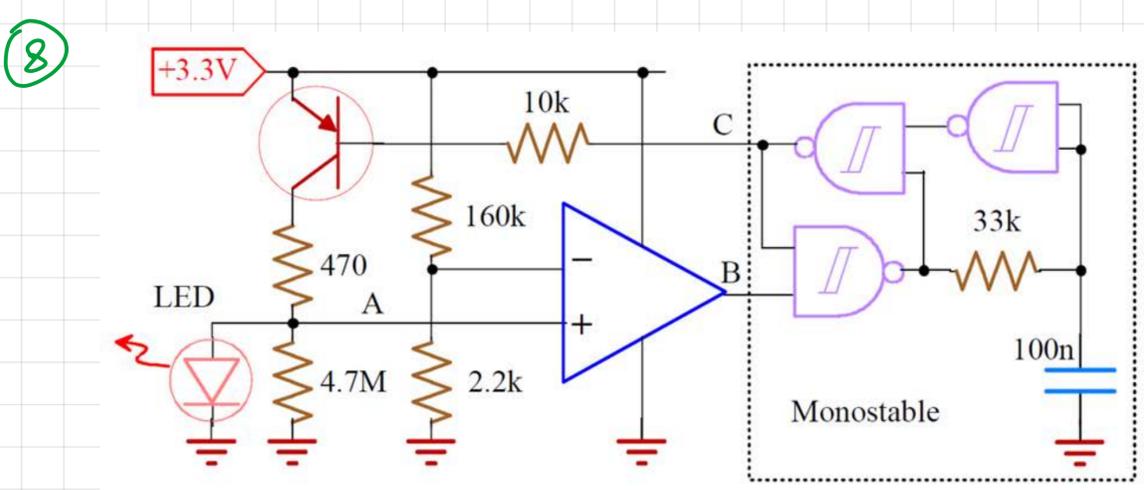
Then to measure it with a gain of 20

we should add an amplifier:



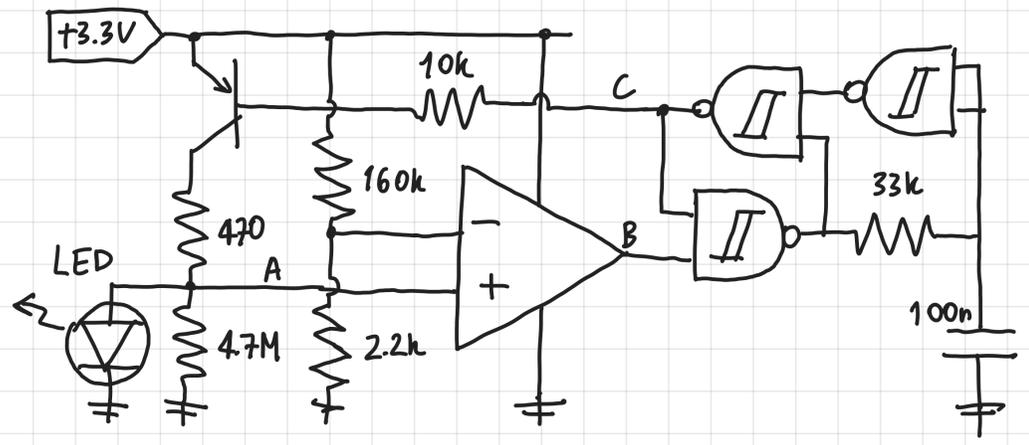
$G = \frac{v_{out}}{v_{in}} = 1 + \frac{19k}{1k} = 20 \checkmark$

7 (OF Es.16 slides → to solve at home)

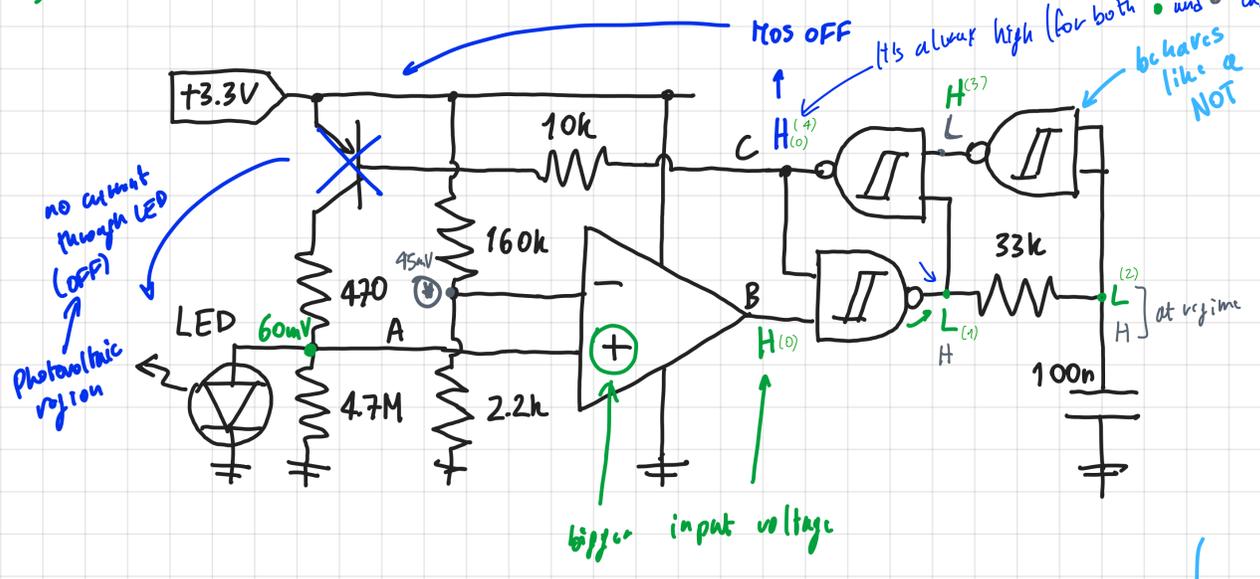


The circuit switches on/off the LED if there is dark/light, without using any other photodiode, but exploiting the photovoltaic effect of the LED itself (with no voltage applied and in light condition it produces 60÷90mV).

- a) Plot all voltage waveforms **with light**, explaining circuit behaviour.
- b) Plot all voltage waveforms **with no light**, explaining circuit behaviour.

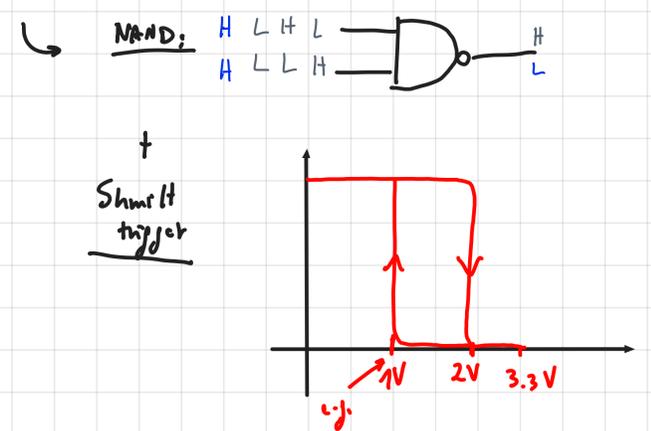


a) If there's light on the LED even if it is OFF it's in the photovoltaic region → it has voltage across it



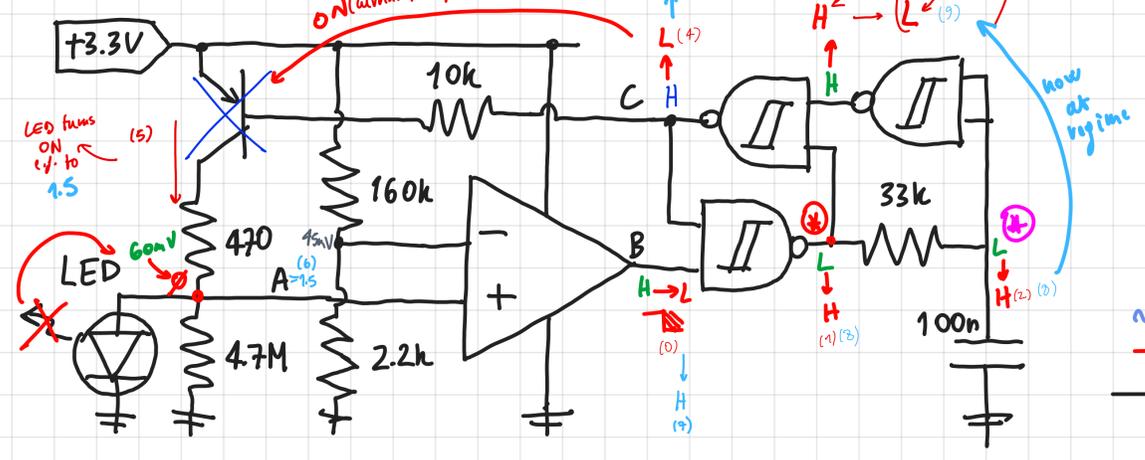
$$3.3V \cdot \frac{2.2k}{2.2k + 160k} = 45mV$$

(Remember  $\text{D} = \text{Schmitt trigger NAND gate}$ )



b)

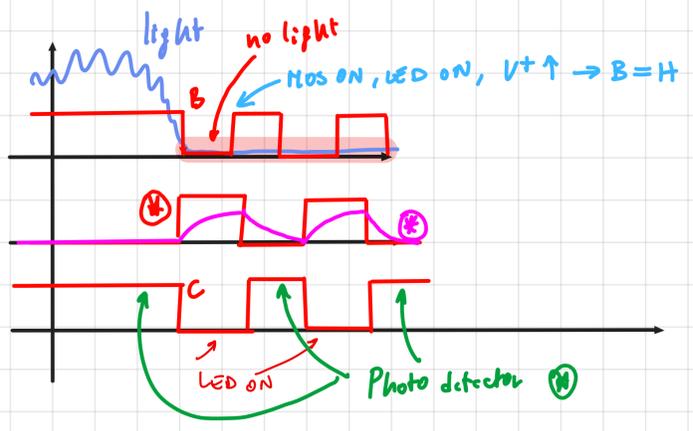
IF we TURN OFF the lights



A schmitt trigger is needed because we have an analog voltage in order to be uncertain when computing we should use Schmitt trigger digital gates

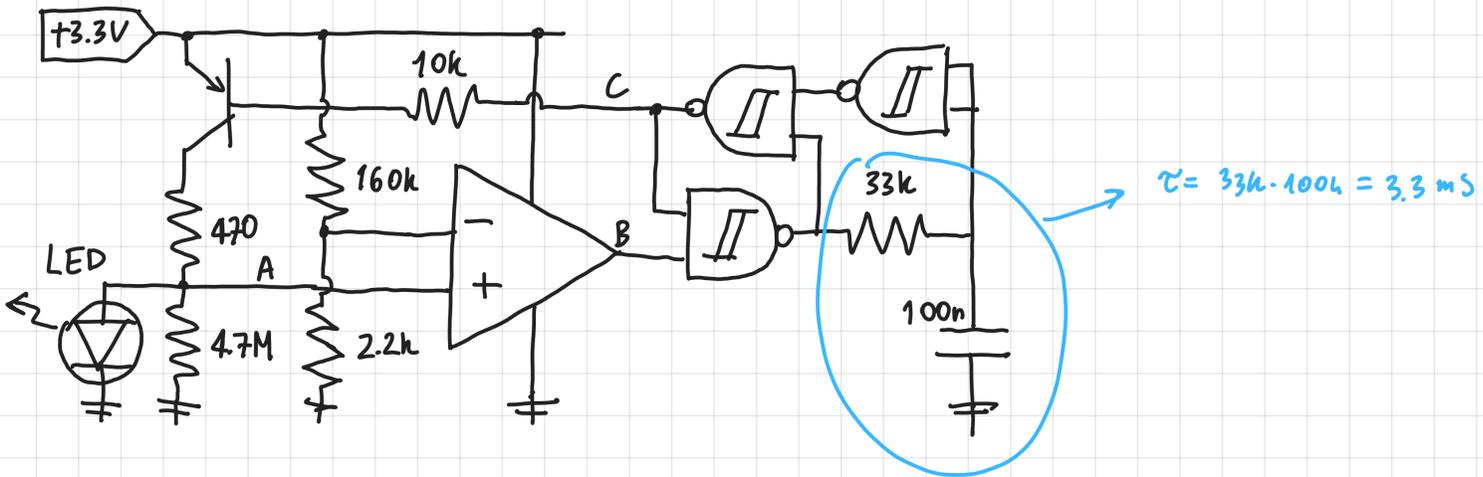
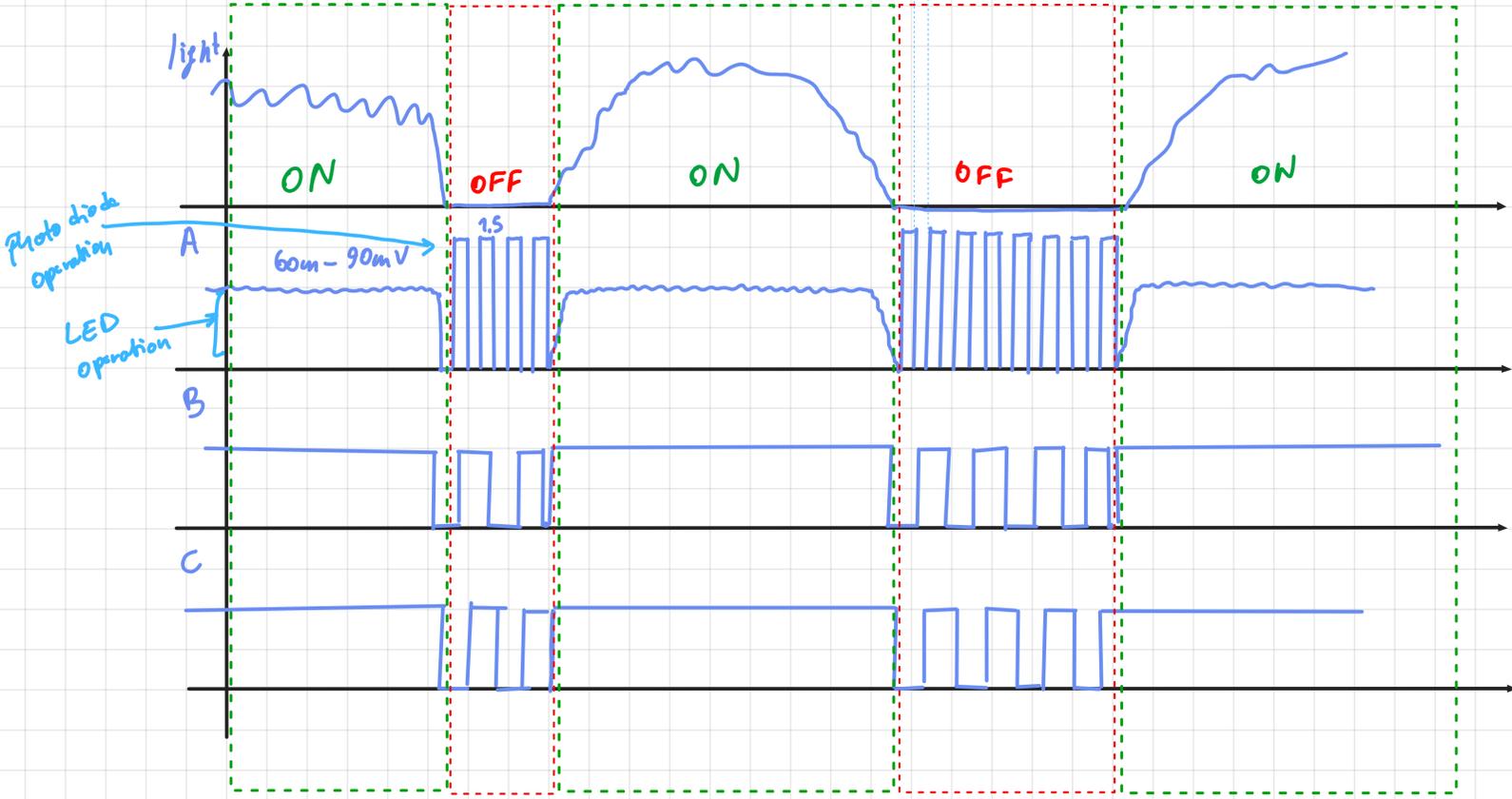
then if there's the light off again the circuit continues to compute

- If there's always light →  $C \equiv H$  (because  $B \equiv H$ )
- if there's no light →  $C \equiv L$  for  $B \equiv L$  (LED on)

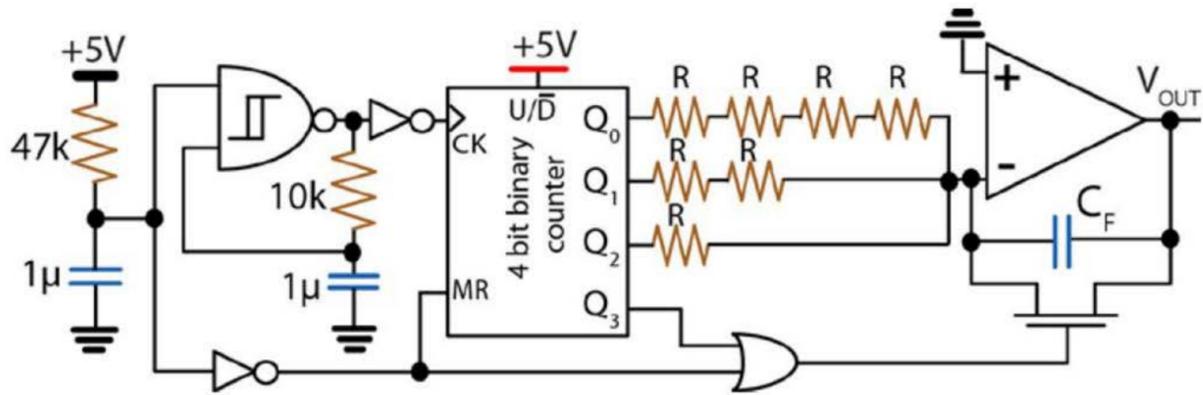


So summarizing:

$T = 6.6 \mu s$   
 $f_{osc} = \frac{1}{T} = 151 \text{ Hz}$



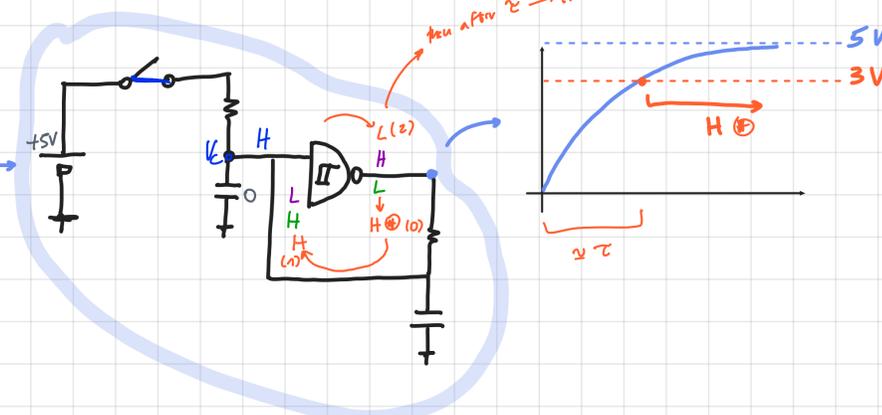
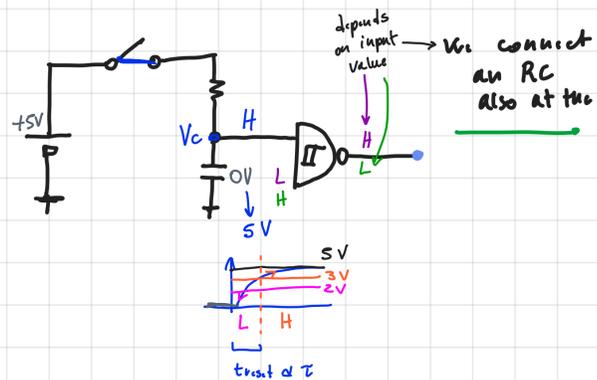
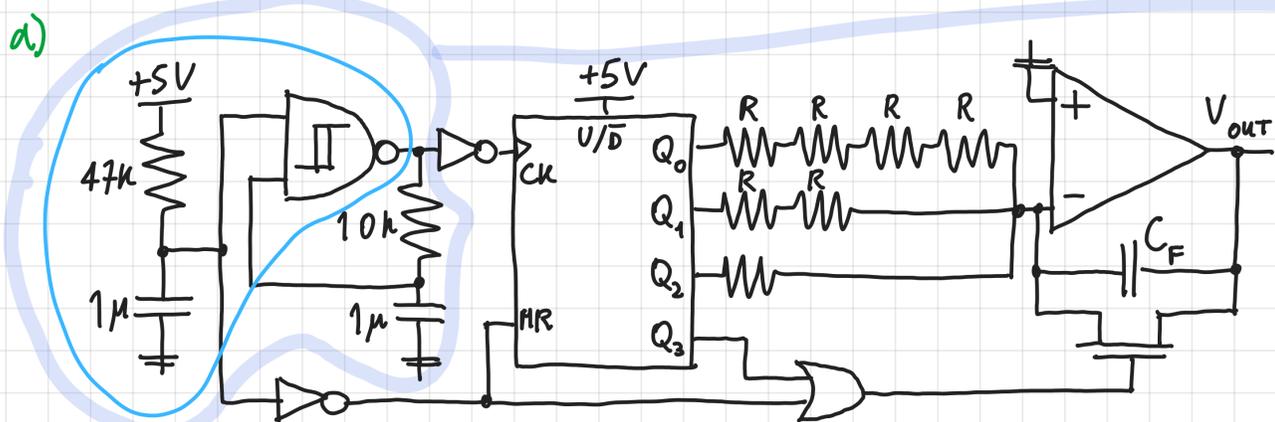
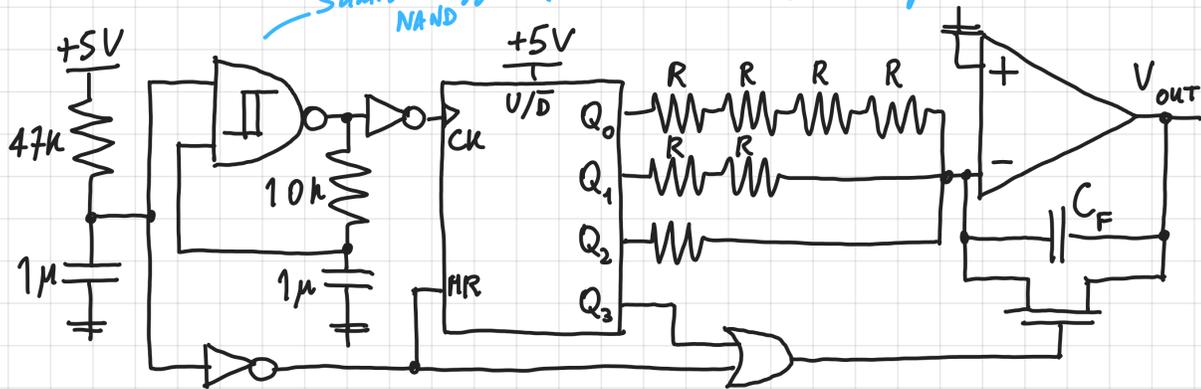
9



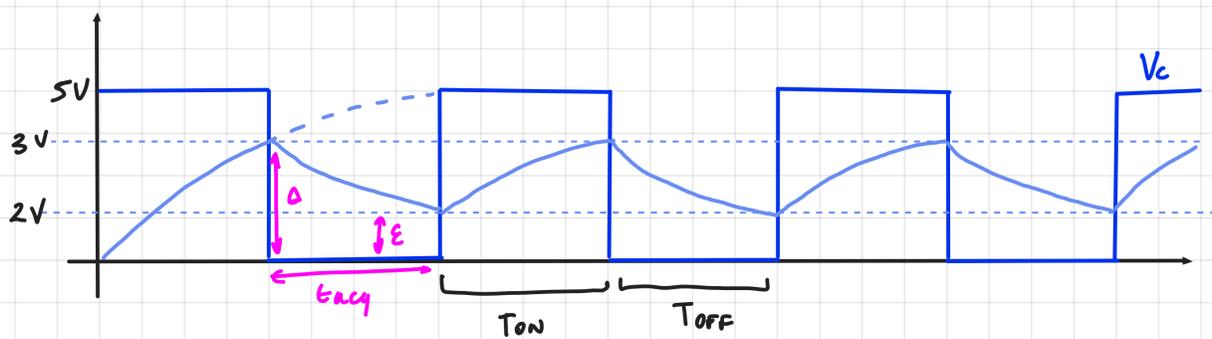
$R=100k\Omega$ ,  $C_F=10\mu F$ . The digital counter provides +5V high levels and  $Q_0$  is the least significant bit.

- a) Plot the  $V_{OUT}$  quoted waveform during the first 3 clock periods (10ms period).
- b) Plot CK and  $V_{OUT}$  waveforms during the first 200ms.

*Schmitt trigger (cause we have an analog signal that drives it)*



So for the oscillator:

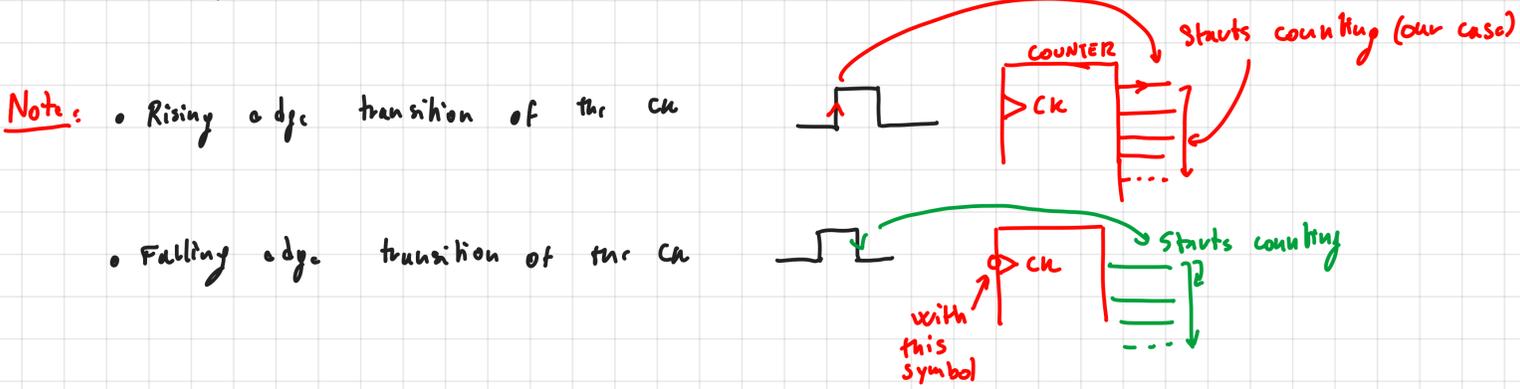
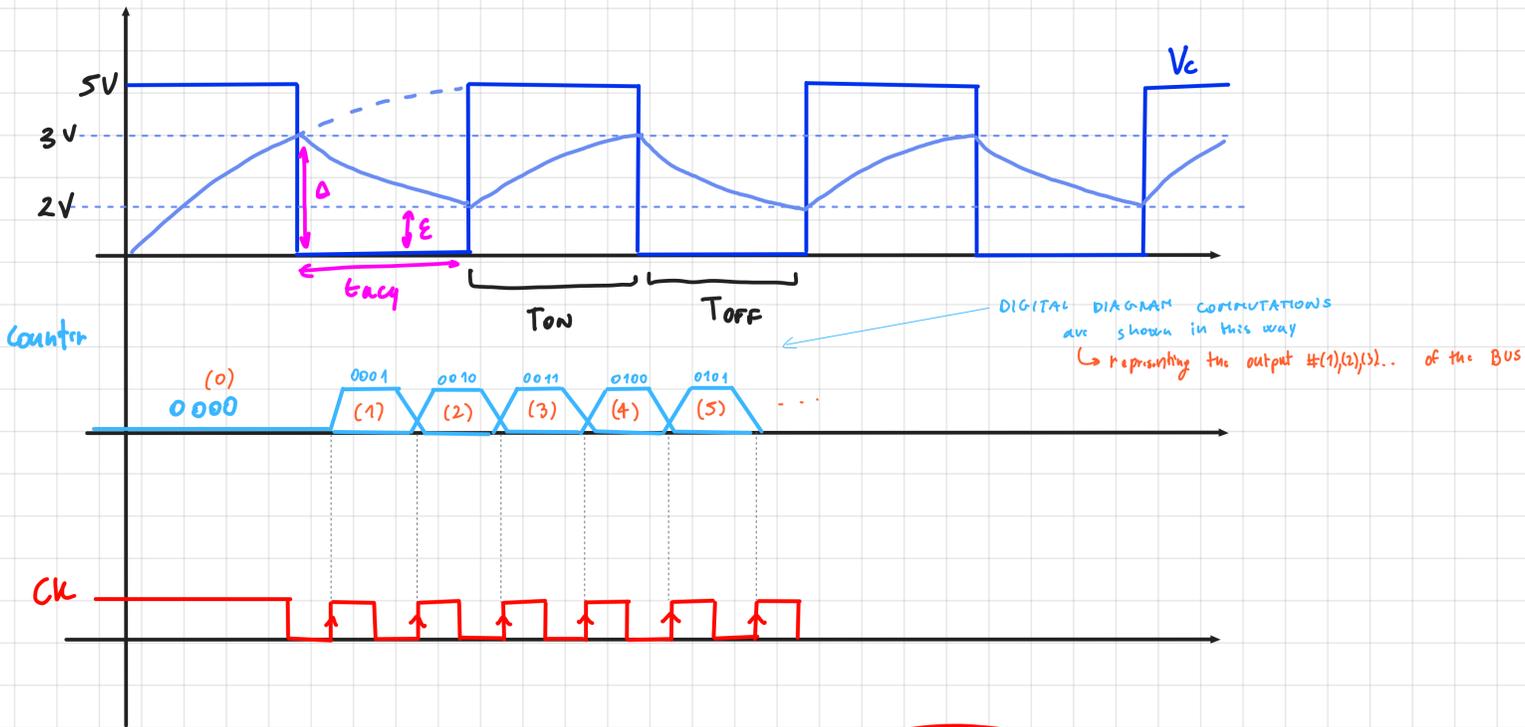
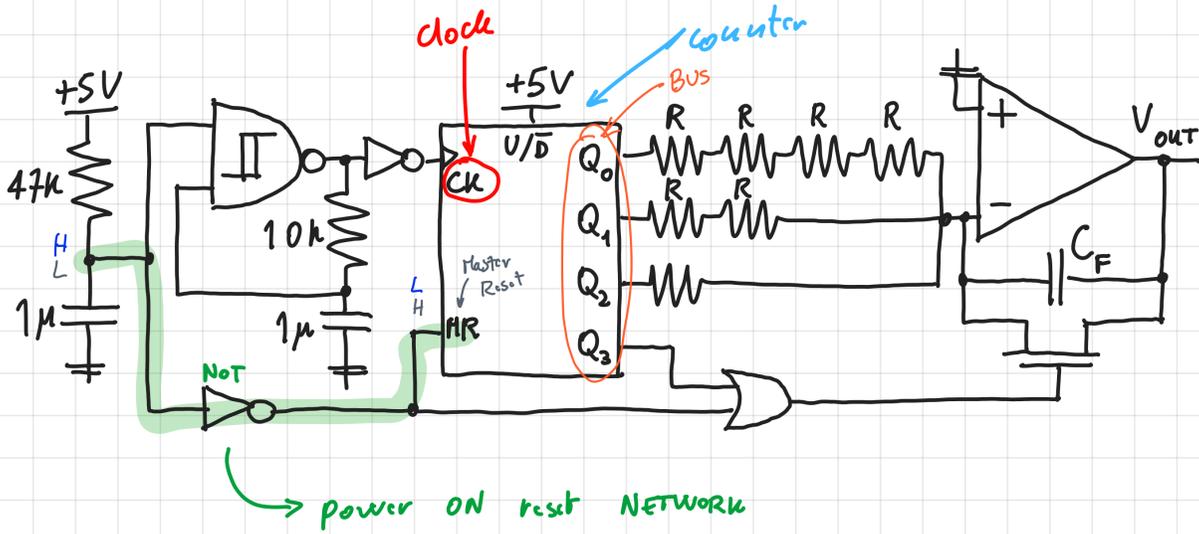


$\Delta = 3V$   
 $\epsilon = 2V$

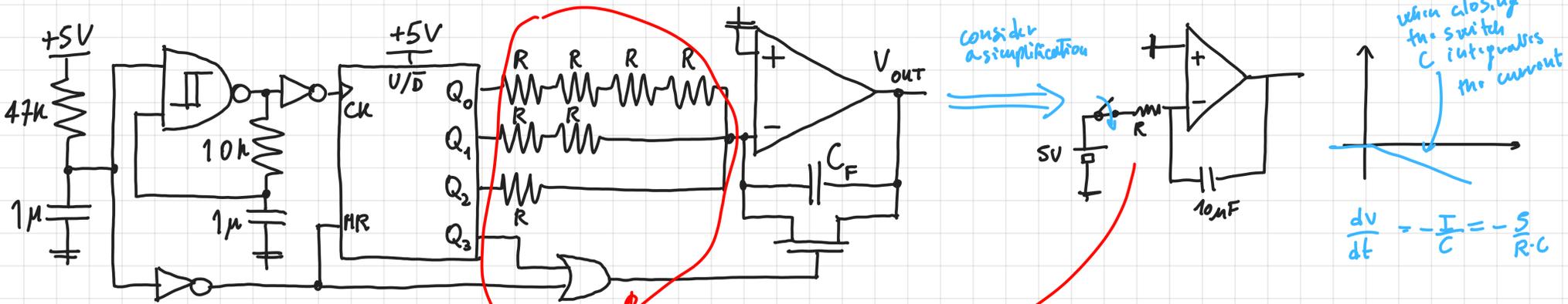
$\rightarrow T_{off} = t_{eq} = \tau \ln \frac{\Delta}{\epsilon} = \tau \ln 1.5 = \ln \cdot 40\% \approx T_{on}$

$\hookrightarrow T = T_{on} + T_{off} = 2 \cdot 40\% \cdot \tau \approx \tau \quad \rightarrow \text{for } \tau = 10\mu s \rightarrow f_{osc} = \frac{1}{T} \approx 100\text{Hz}$

Now consider

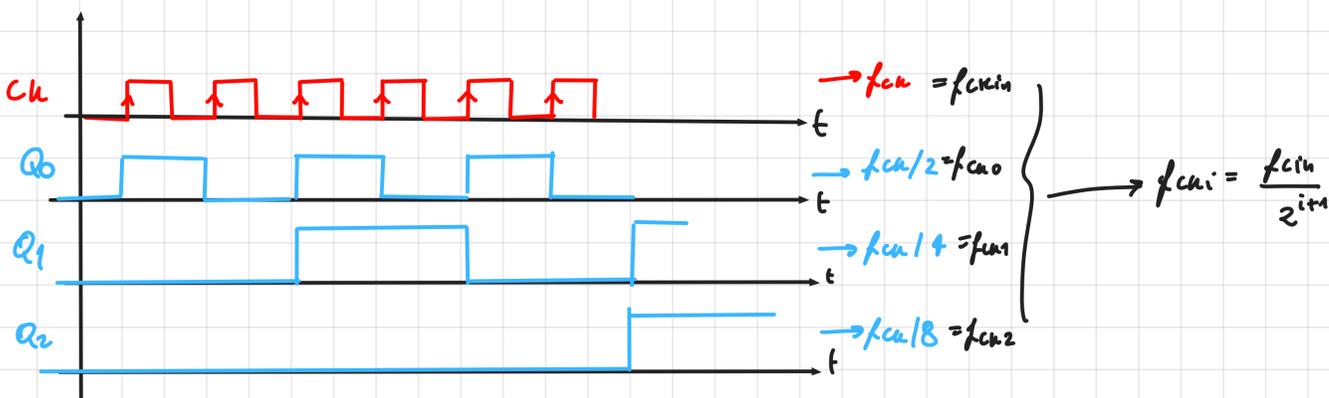


If we keep analyzing the circuit:

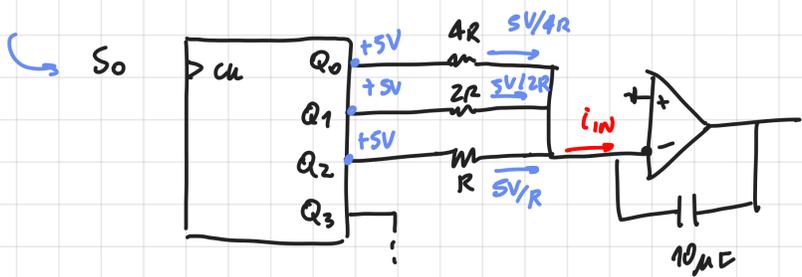
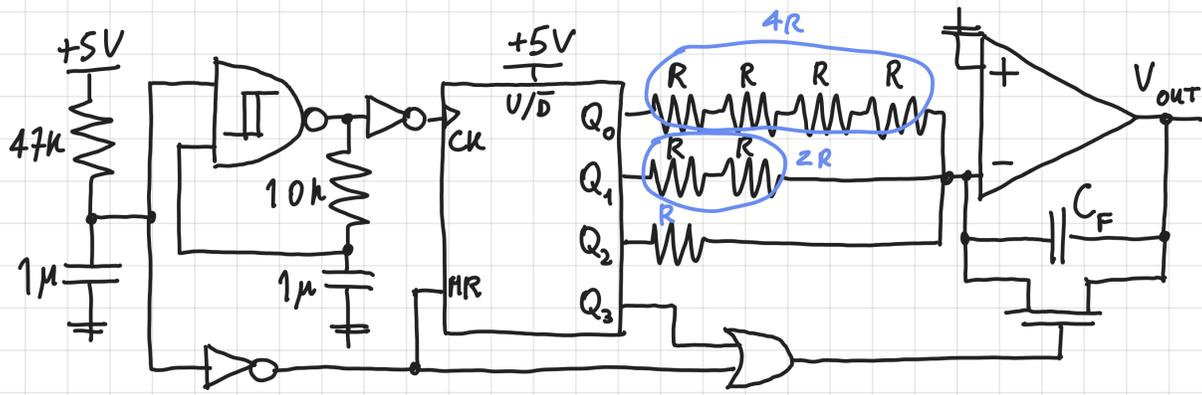


but we don't have just an R we have multiple resistors

Remember that the output of the counter is:



↳ So in the circuit, each  $Q_i$  contributes with  $R_s$  in a different way:



→ So generically:

$$i_{IN} = Q_0 \cdot \frac{5}{4R} + Q_1 \cdot \frac{5}{2R} + Q_2 \cdot \frac{5}{R} = \frac{5}{R} \left[ \frac{Q_0}{4} + \frac{Q_1}{2} + \frac{Q_2}{1} \right]$$

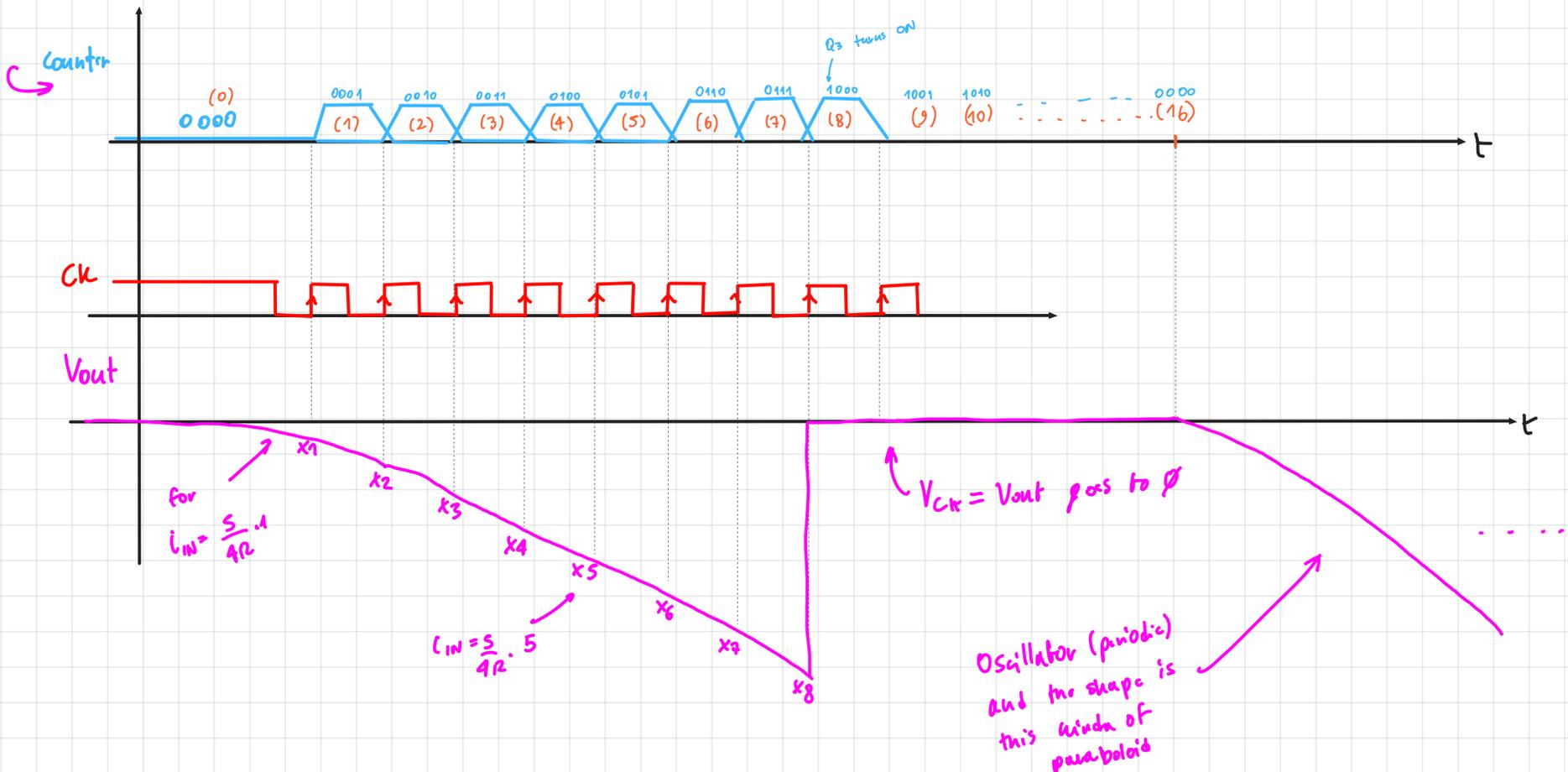
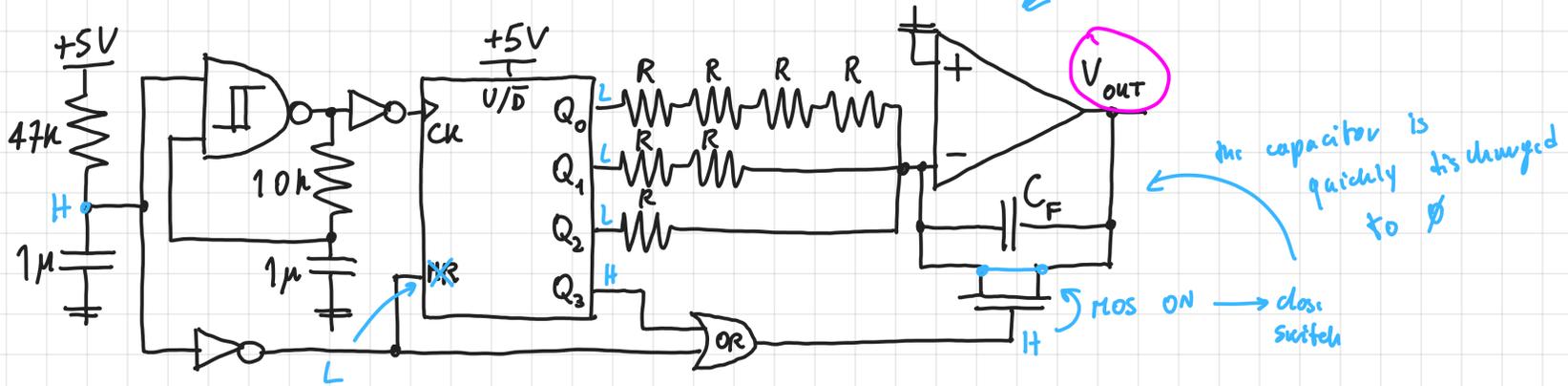
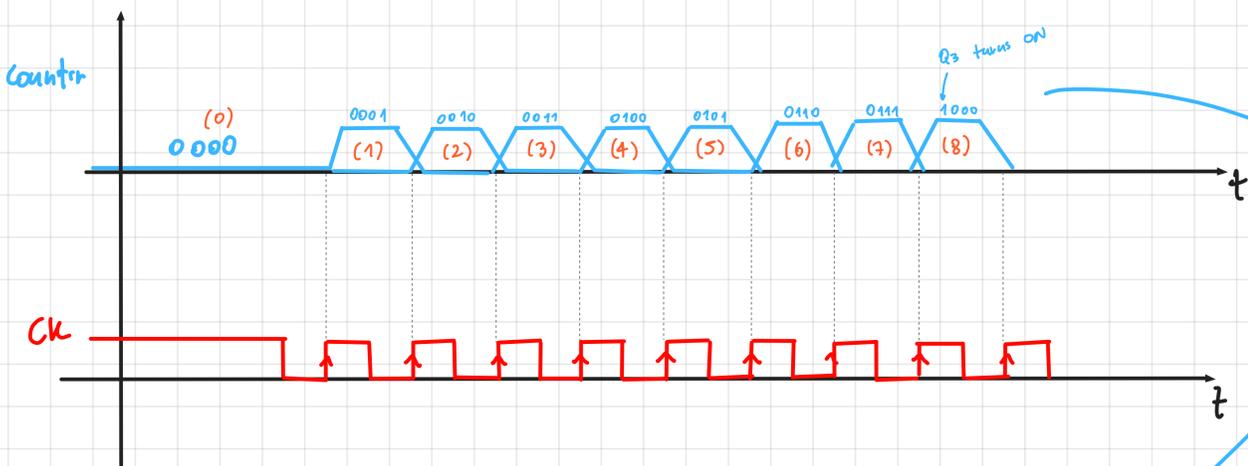
$$= \frac{5}{R} \cdot \text{Dout}$$

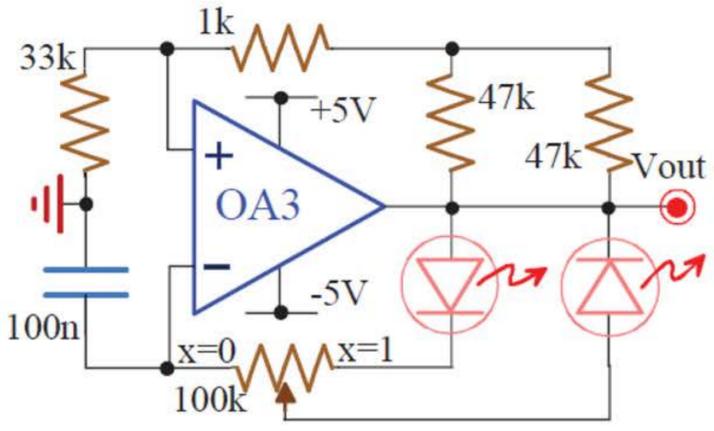
↑  
digital output of the counter

↳ So when the output (BUS) increments the current increments:

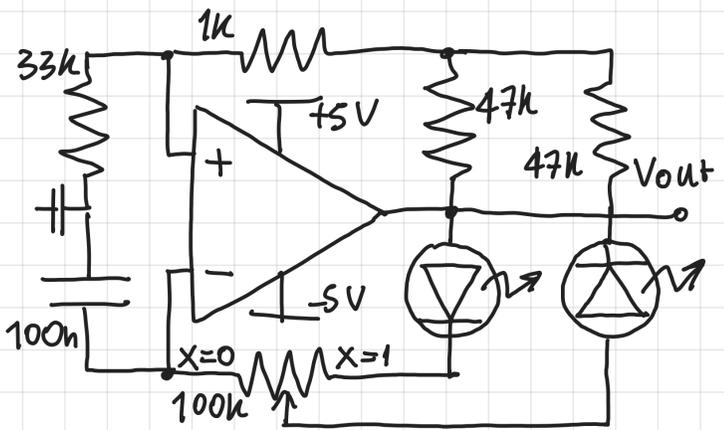
(Number format ex.)  

0	1	0	1
$Q_3$	$Q_2$	$Q_1$	$Q_0$

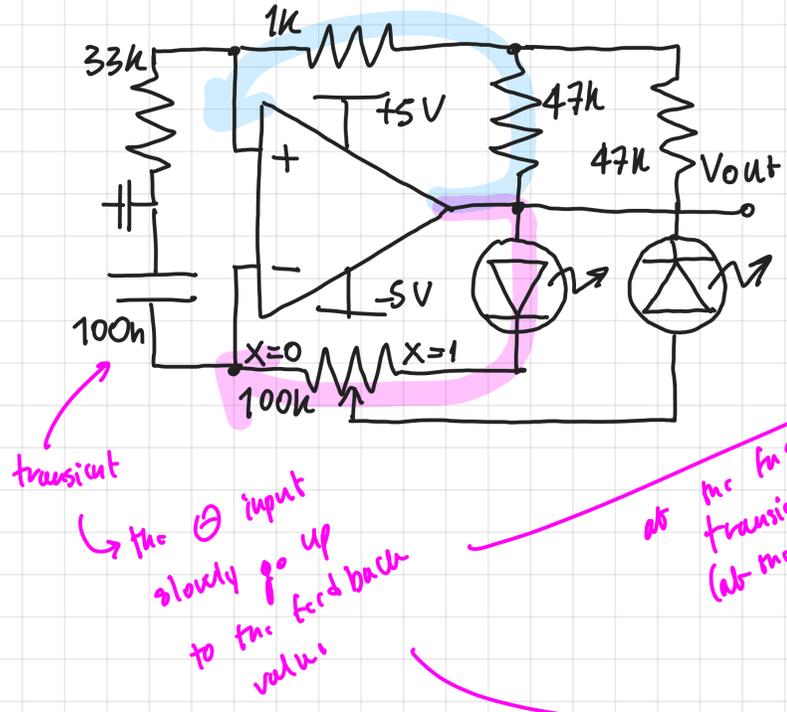




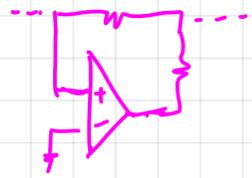
Employ 1.5V forward bias LEDs and a rail-to-rail OpAmp.  
 a) Study circuit operation when the pot is turned to 100% and draw the quoted output waveform.  
 b) Compute the analytical dependence of the output main parameters on x pot position.



a) We see two possible feedback paths → Positive / negative feedback → Which is going to prevail?

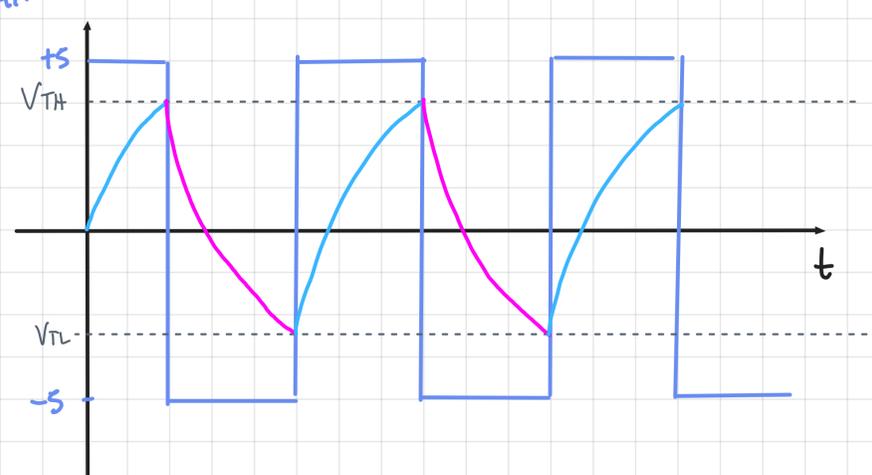
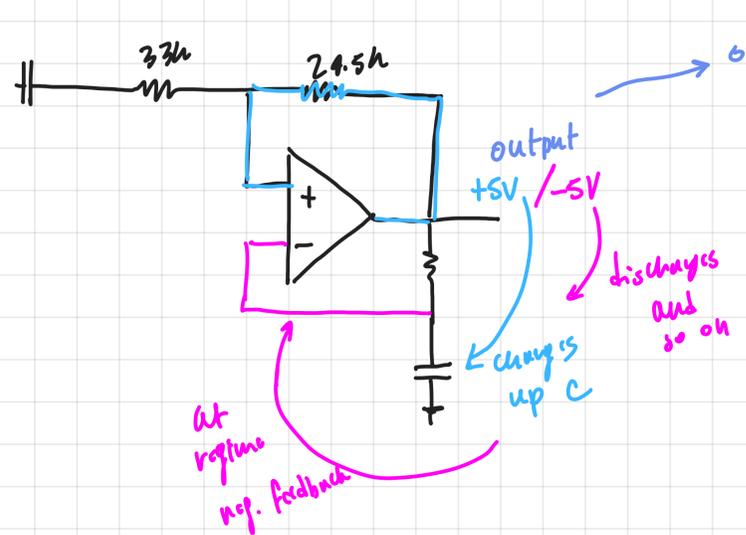


Just pos. feedback → Schmitt trigger

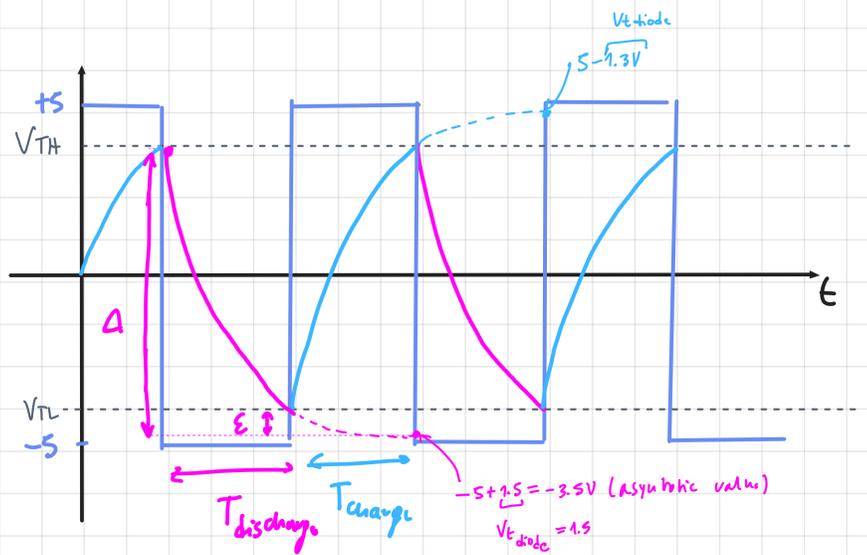
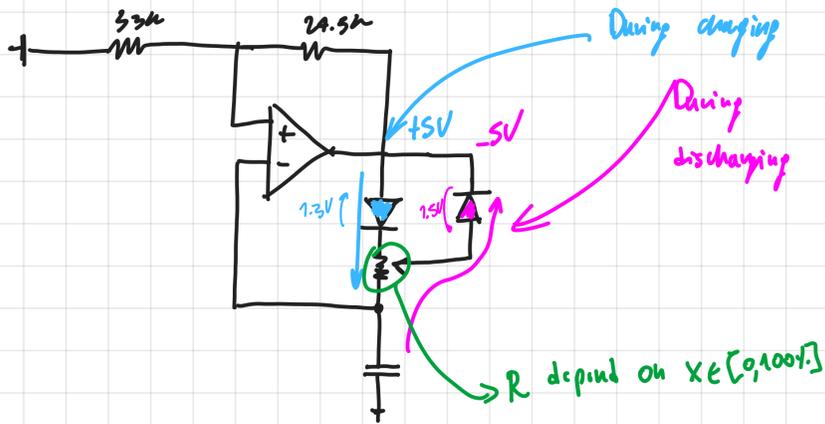


transient → pos. feedback  
 regime → neg. feedback

So analyzing the stage:



↳ our circuit is actually a little bit different:



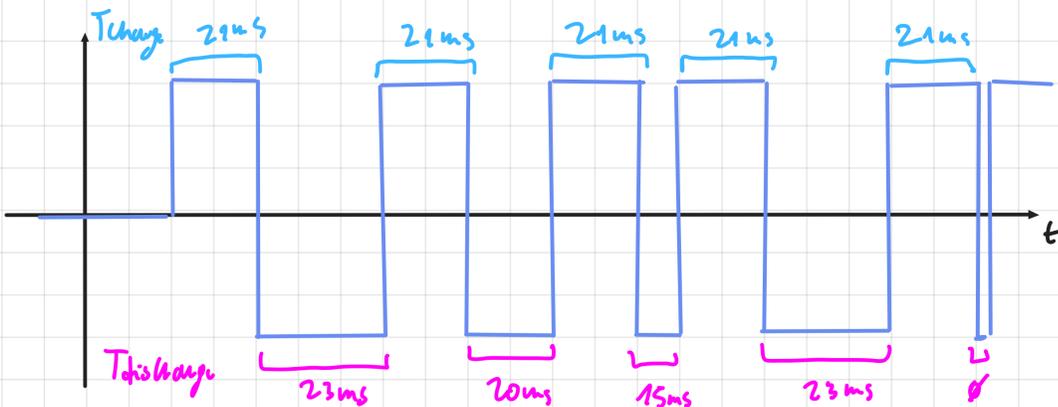
Thresholds:  $V_{THH} = \pm 5V \cdot \frac{33k}{33k + 24.5k} = \pm 2.9V$   
 symmetric

$\Delta = 2.9 + 3.5$ ,  $\epsilon = -2.9 + 3.5 \rightarrow T_{discharge} = \tau \ln \frac{\Delta}{\epsilon} = RC \ln \frac{\Delta}{\epsilon} = 100k \cdot x \cdot 100n \ln \frac{2.9 + 3.5}{-2.9 + 3.5} = 10ms \cdot x \cdot 2.3 = x \cdot 23ms$

$T_{charge} = 100k \cdot 100n \ln \frac{5 - 1.3 + 2.9}{5 - 1.3 - 2.9} = 10ms \cdot 2.1 = 21ms$  → constant duration  
 DIODE BLUE sees all R

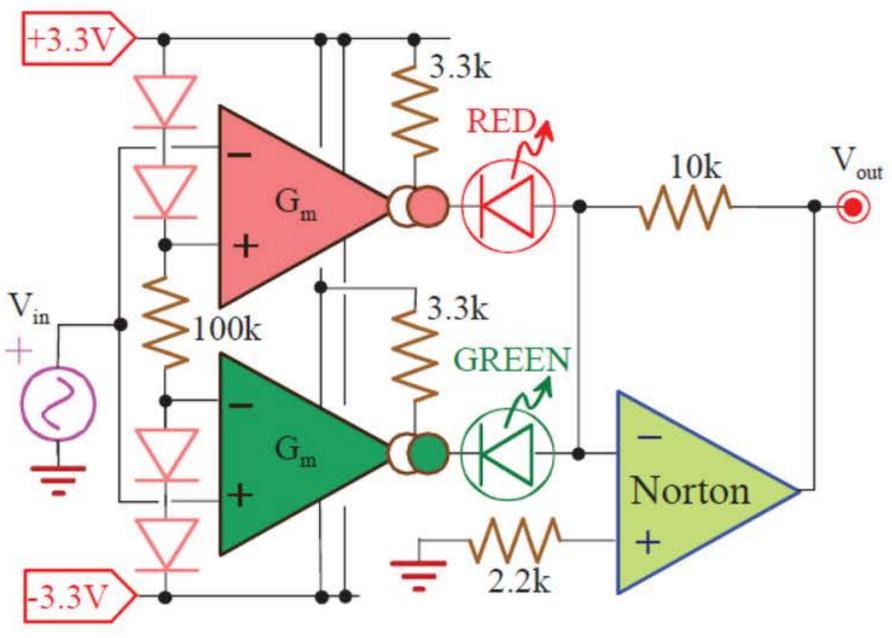
discharge duration is variable on x

↳ So in the end we have an oscillator:

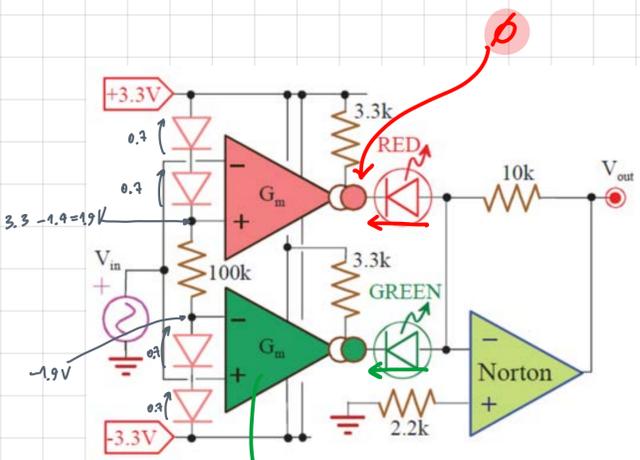


→ Variable period

$f_{max} = \frac{1}{21\mu s + \phi} = 48Hz$   
 $f_{min} = \frac{1}{21\mu s + 23ms} = 23Hz$

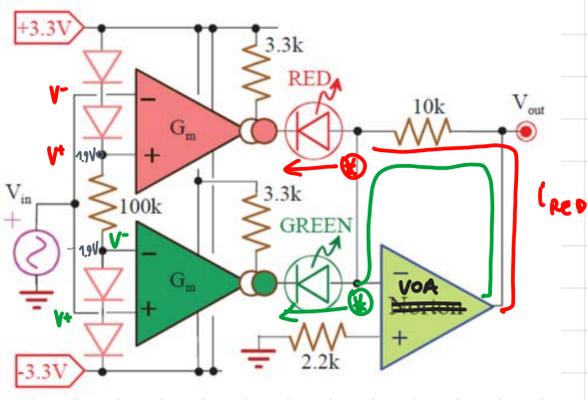


OTAs with control pin at 0V. Diodes with on-voltage of about 0.7V and LEDs with on-voltage of about 1.8V. Norton amplifier with  $A_i=10$ .  
 a) Draw the light intensity of both LEDs vs. the  $V_{in}$  input voltage across the  $-3V \div +3V$  range.  
 b) Compute the  $V_{out}$  output voltage vs.  $V_{in}$ .



$$g_m = \frac{I_{control}}{V_{th}} = \frac{I_{control}}{25mV} = \frac{3.3V - 0}{25mV} = 40 \frac{mA}{V}$$

Now start by considering instead of Norton a VoA



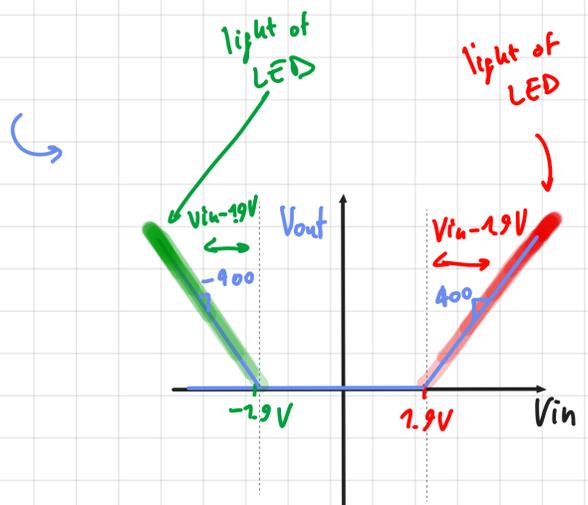
• If  $V_{in} > 1.9V$  → we have a current on RED LED

$$V_{out} = (V_{in} - 1.9V) \cdot \frac{40 \frac{mA}{V} \cdot 10k\Omega}{400 \frac{V}{V}}$$

• If  $V_{in} < 1.9V$  →  $i_{RED} = 0$

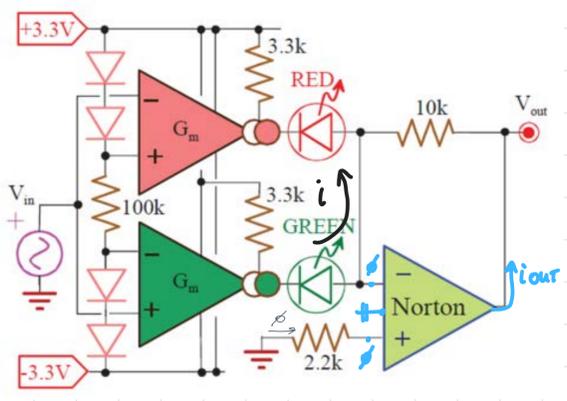
• If  $V_{in} < -1.9V$  → we have a current on GREEN LED

$$V_{out} = -|V_{in} - 1.9V| \cdot \frac{40 \frac{mA}{V} \cdot 10k\Omega}{400} = -\frac{(V_{in} - 1.9V) \cdot 400}{400}$$



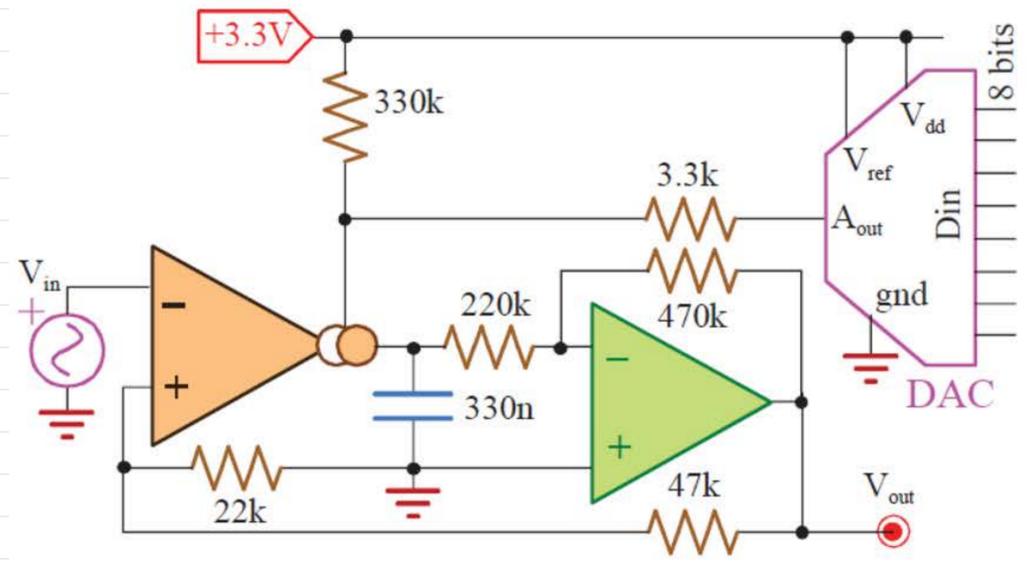
So the characteristic is very similar to this

Now consider that we actually had a Norton (Current Mode OpAmp) and not a VoA



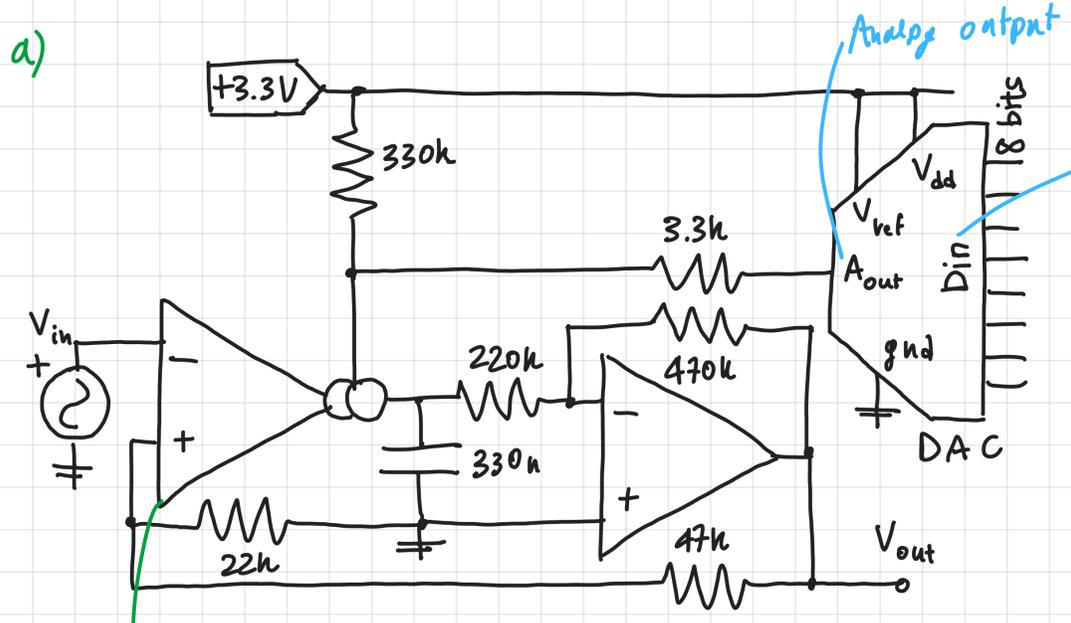
Norton:  $i_{out} = A_i(i^+ - i^-) = A_i(0 - i_{out} + i)$

$$i_{out} = \frac{A_i}{1+A_i} i_{RED} = \frac{10}{11} i_{RED} \quad V_{out} = i_{out} \cdot 10k$$

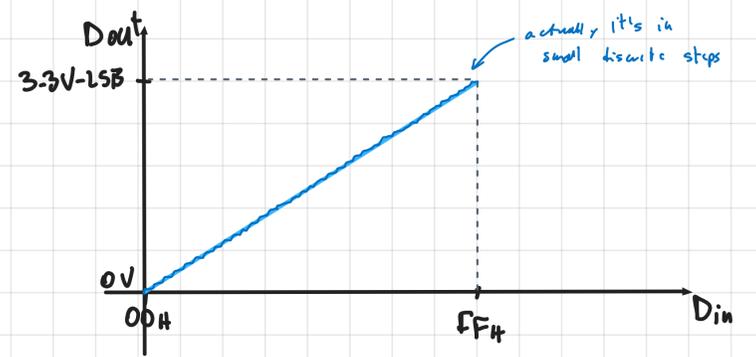


OTA with control pin at 0V and ±5V power supply.

- a) Compute the OTA's transconductance as a function of  $D_{in}$ .
- b) Compute the real  $v_{out}(f)/v_{in}(f)$  gain, bandwidth, and stability vs. the input digital code  $D_{in}$ .



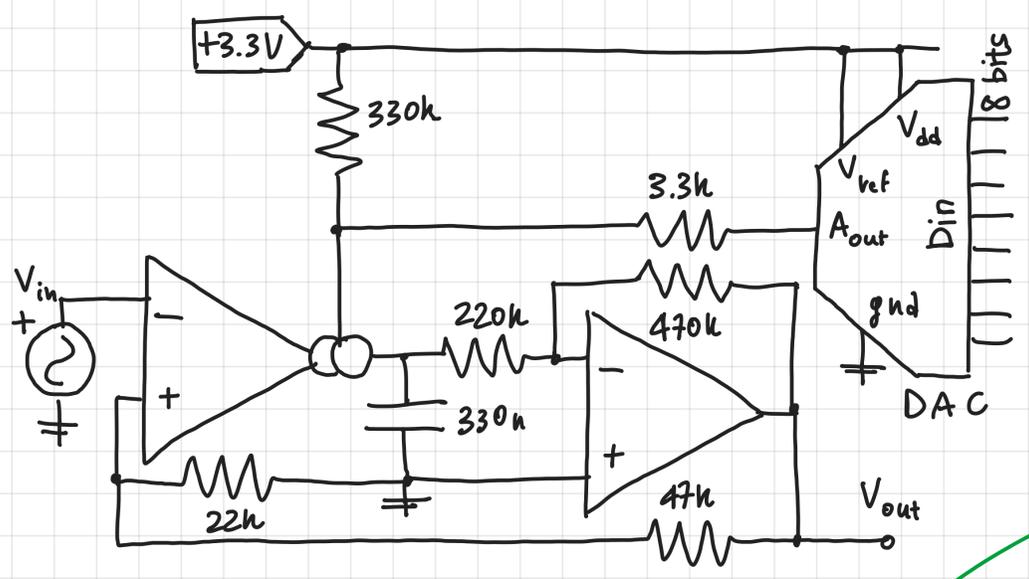
$$A_{out} = V_{ref} \cdot \frac{D_{in}}{2^8} = \frac{3.3V}{256} \cdot D_{in} = 12.9mV \cdot D_{in}$$



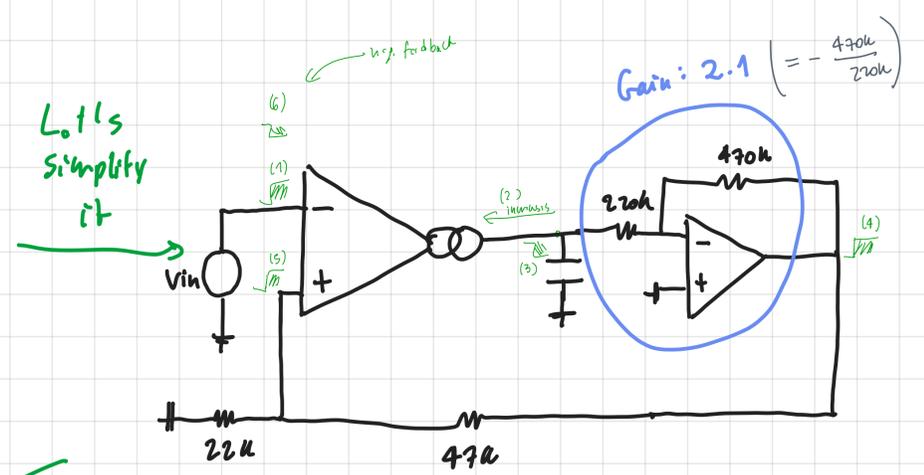
$$LSB = \frac{3.3V}{256} = 12.9mV$$

$$G_m = \frac{I_{control}}{25mV} = \frac{\frac{3.3V}{330k} + \frac{A_{out}}{3.3k}}{25mV} = 0.4 \frac{mA}{V} + 156 \frac{\mu A}{V} \cdot D_{in} = \begin{cases} G_{min} = 0.4 \frac{mA}{V} & \text{for } D_{in} = 00_H \\ G_{max} = 40 \frac{mA}{V} & \text{for } D_{in} = FF_H \end{cases}$$

b) Let's study the circuit:

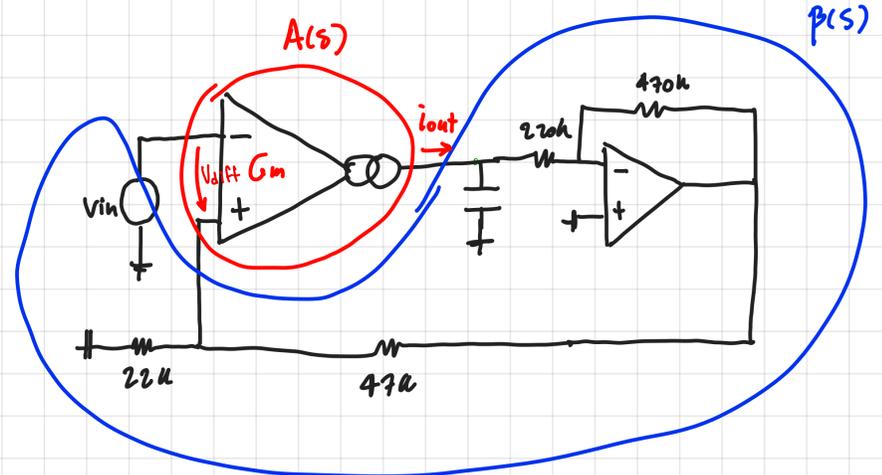


Let's study the stability:



Ideal Gain

$$V_{out} = V_{in} \left( 1 + \frac{47k}{22k} \right)^{3.1}$$

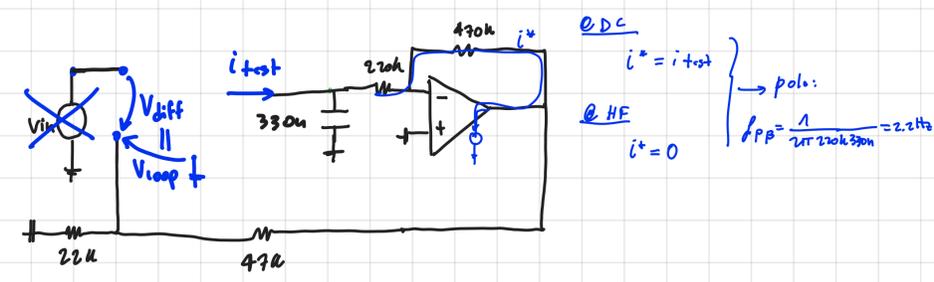


$A(s)$ :  $A(s) = G_m = \frac{i_{out}}{V_{diff}} \left[ \frac{A}{V} \right]$

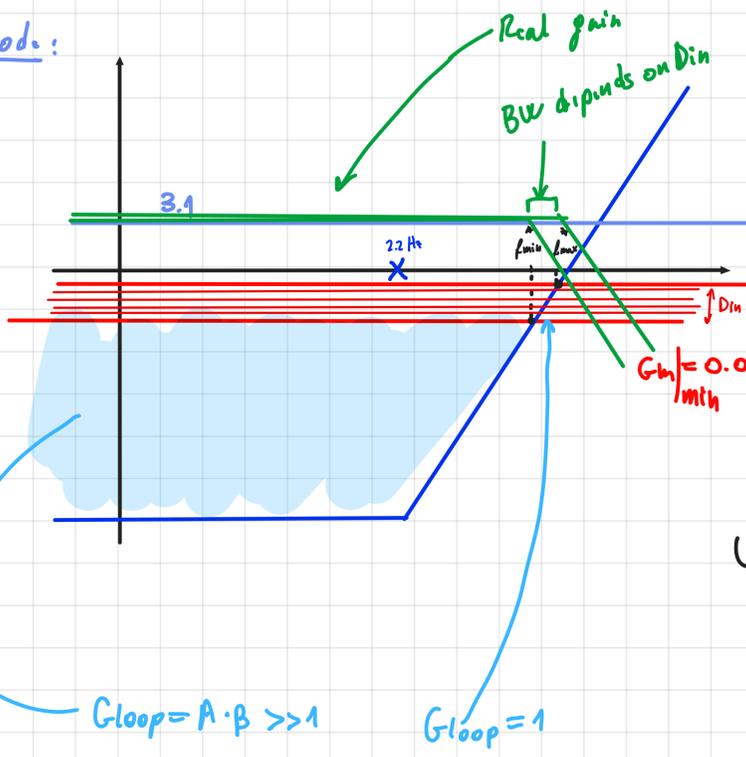
$$\beta(s) = \frac{470k \cdot 22k}{220k + 47k} = 192.900 \left[ \frac{V}{A} \right]$$

$\beta(s)$ :  $\frac{V_{diff}}{i_{out}} = \frac{V_{loop}}{i_{test}} = \beta(s)$

$P(s) = \emptyset$



Bode:



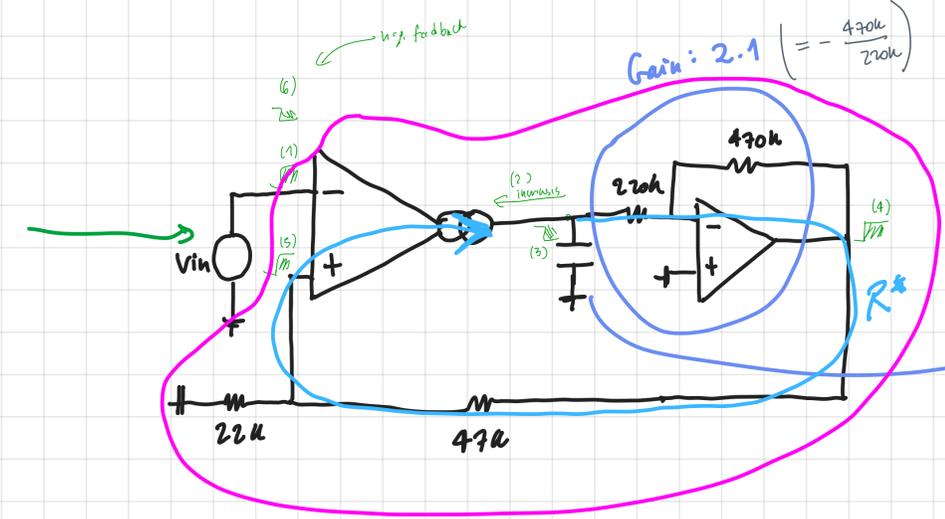
$$\frac{1}{\beta}(s) = \begin{cases} \frac{1}{\beta}(0) = 6.7 \cdot 10^{-6} \\ \frac{1}{\beta}(\infty) = \infty \end{cases}$$

$A(s)$

min BW:  $f_{min}^* = 2.2 \text{ Hz} \cdot \frac{0.4 \mu}{6.7 \cdot 10^{-6}} = 131 \text{ Hz}$

max BW:  $f_{max}^* = 2.2 \text{ Hz} \cdot \frac{40 \mu}{6.7 \cdot 10^{-6}} = 13 \text{ kHz}$

Note: Actually for the ideal gain there was a closed loop pole:



Pole:  $f_p = \frac{1}{2\pi C \cdot R^*} = \frac{(1+G_{loop})}{2\pi \cdot C \cdot 220k}$

$\frac{(220k)}{(1+G_{loop})}$

↑ must take into account the feedback

It's better to this approx to a const. ideal gain

So it depends on  $G_{loop}$

1 0910212006 - EX2

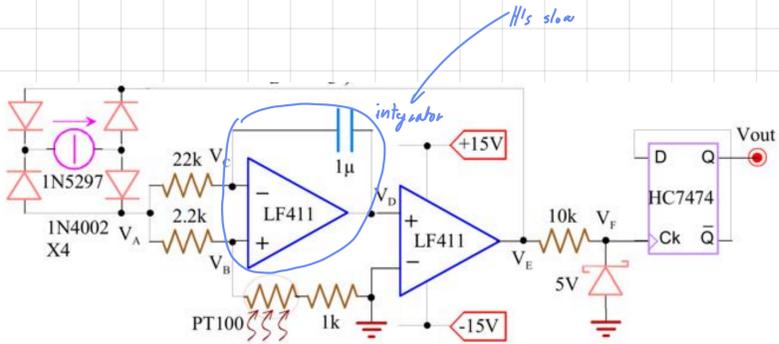
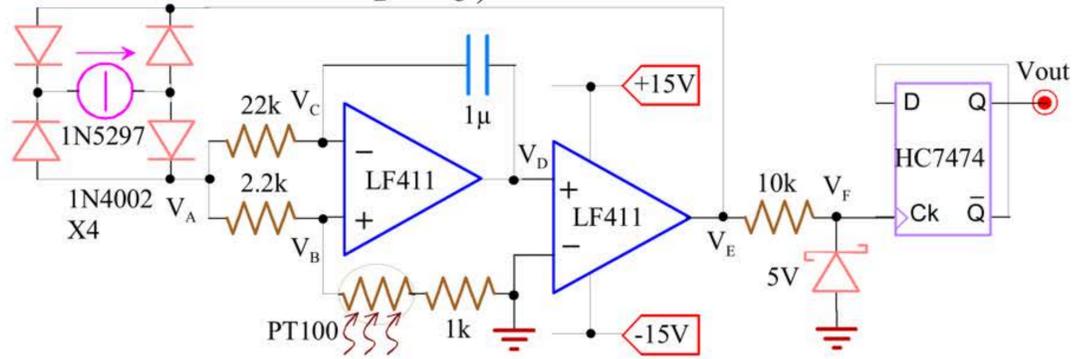
Es. 2

Il 1N5297 è un generatore di corrente integrato che genera 1mA. Il sensore termico è una PT100 che vale 100Ω a 0°C e varia di circa 0.4Ω/°C. Studiare il funzionamento del circuito a 0°C.

a) Spiegare il funzionamento del circuito. (Si suggerisce di partire dall'istante in cui  $V_E = +15V$ , per poi ricavare le correnti nelle resistenze da 22kΩ e 2.2kΩ e le tensioni  $V_B$  e  $V_C$ .)

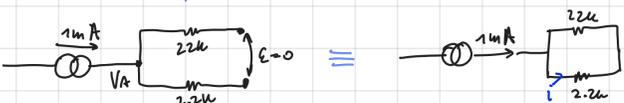
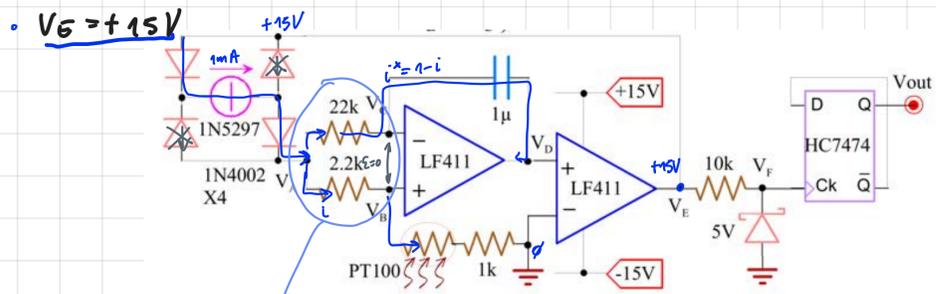
b) Disegnare le forme d'onda quotate delle tensioni ai nodi e ricavare la frequenza di commutazione di  $V_{out}$ .

c) Dire come si modificano le forme d'onda al variare della temperatura misurata dalla PT100.



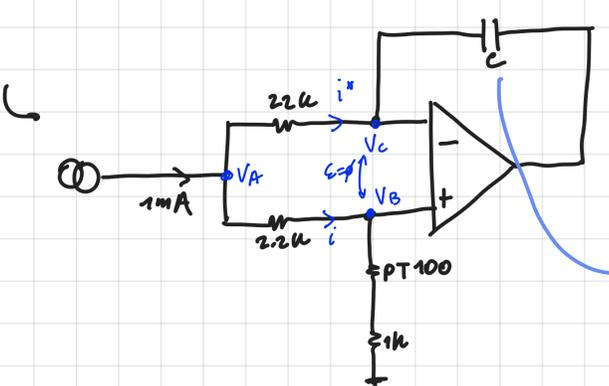
↳ From simple circuit analysis we're not sure about the working principle of the circuit

↳ start by considering the second opamp as comparator → so the output is ±15V



$$i = 1mA \cdot \frac{22k}{2.2k + 22k} = 0.91mA$$

$$i^* = 1mA - i = 0.09mA$$

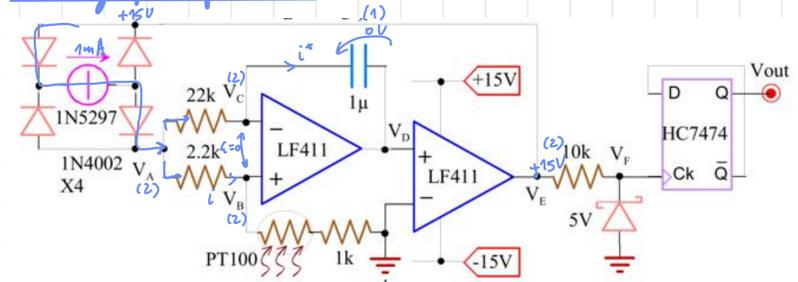


$$V_A = i(2.2k + 1k + R_{PT100}) = 0.91mA \cdot 3.2k + 0.91mA(100 + 0.4 \cdot T) = 3V + 0.36 \frac{mV}{^\circ C} \cdot T$$

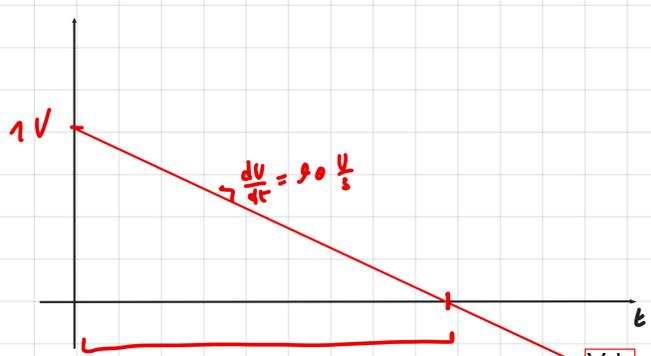
$$V_B = V_C = i(R_{PT100} + 1k) = 0.91mA \cdot 1.1k + 0.364mV \cdot T = 1V + 0.364 \frac{mV}{^\circ C} \cdot T$$

$$\frac{dV_C}{dt} = \frac{i^*}{C} = \frac{0.09mA}{1\mu F} = 90 \frac{V}{s}$$

↳ Working principle idea:

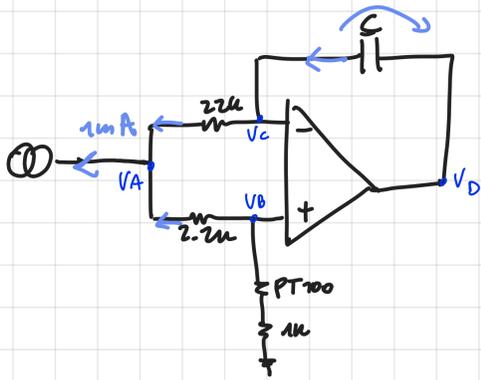
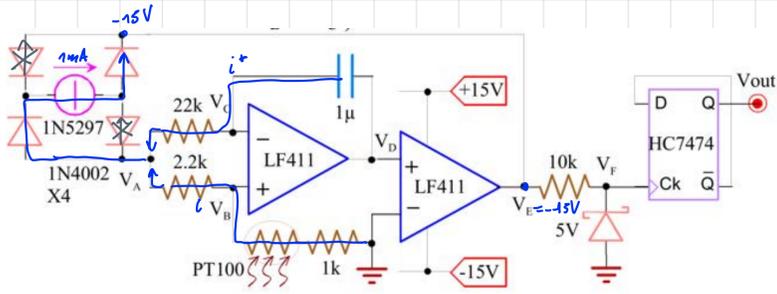


- 1 Start with discharged C,  $V_E = +15V$
- 2 Compute  $V_A, V_B, V_C$  (done before)
- 3  $V_D = V_C = V_B$  → when  $V_D < 0$  (after  $T_1$ ) comparator output  $V_E \rightarrow -15V$  (when C charges up)



$$T_1 = \frac{1V}{\frac{dV}{dt}} = \frac{1V}{90 \frac{V}{s}} = 11.1ms$$

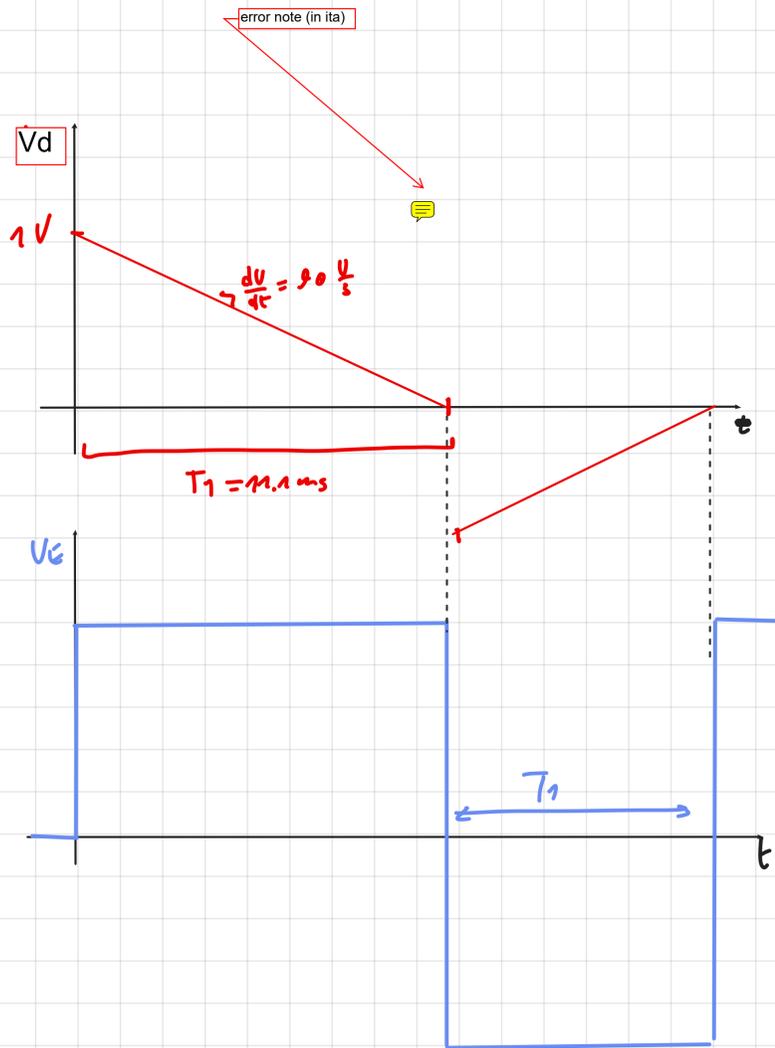
$V_E = -15V$



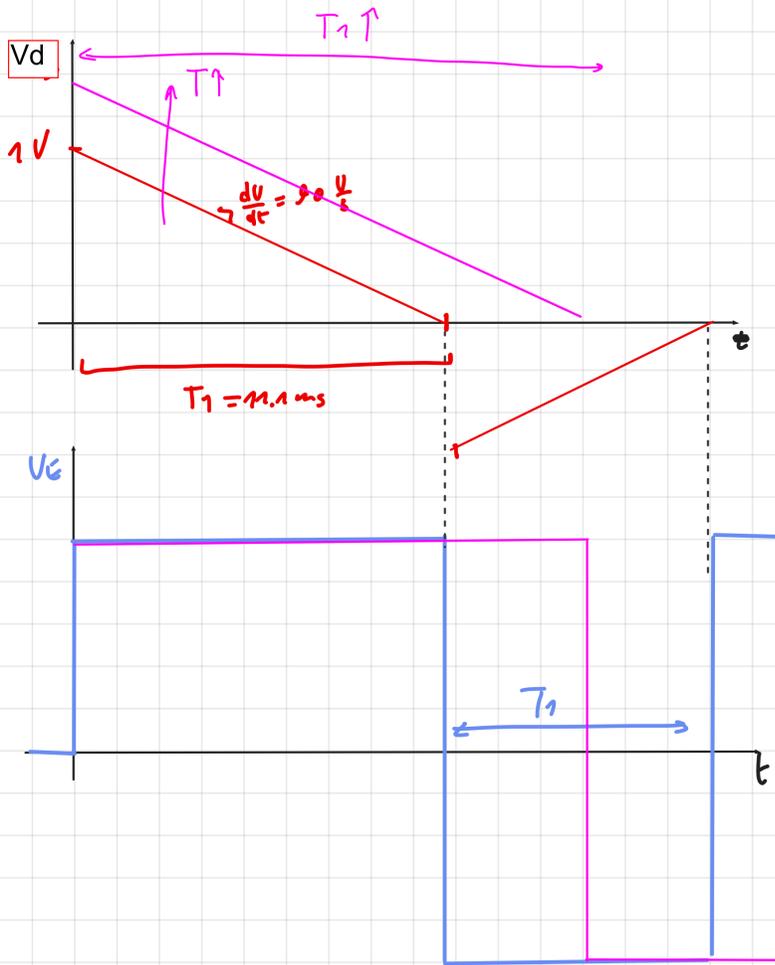
Applying the same considerations:

- $V_A = -3V - 0.364 \frac{mV}{^\circ C} T$
- $V_B = V_C = -1V - 0.364 \frac{mV}{^\circ C} T$

the comparator will charge up in the opposite direction



Note:  $V_B$  depends on  $T$  if  $T \uparrow |V_B| \uparrow \rightarrow t_{r1} \uparrow \rightarrow f \downarrow$



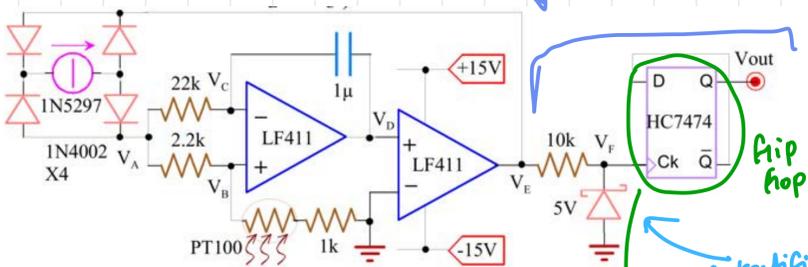
$$f_{osc}(0^\circ C) = \frac{1}{2 \cdot 2.11ms} = 45 \text{ Hz}$$

$$f_{osc}(T) = \frac{1}{2 \cdot \frac{1V + 0.364 \frac{mV}{^\circ C} T}{90 \frac{V}{s}}} = \frac{45 \frac{V}{s}}{1V + 0.364 mV \cdot T}$$

$$T_{osc}(T) = \frac{1V}{45 \frac{V}{s}} + \frac{0.364 mV}{45 \frac{V}{s}} \cdot T = 22.2 \mu s + 8.1 \mu s \cdot T$$

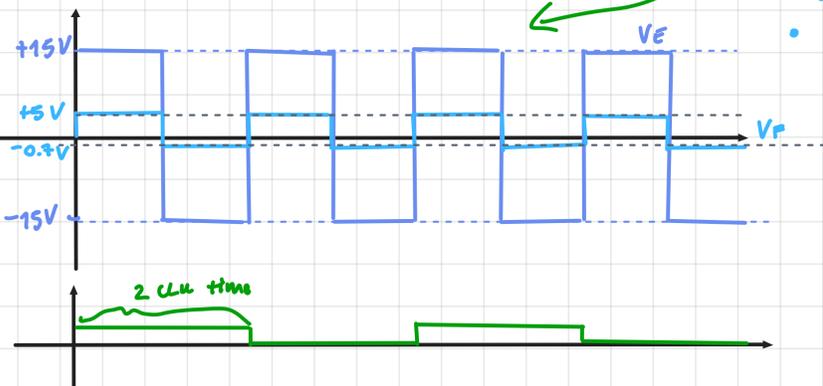
Then we consider the second part of the circuit

We have  $V_E$ :



- rectifies it at 5V when  $V_E > 0$
- zener-diode OFF  $\rightarrow -0.7V$

almost a digital wave  $\rightarrow [0.5V]$   
actually  $-0.7V$   
use it as CLK for the flip-flop

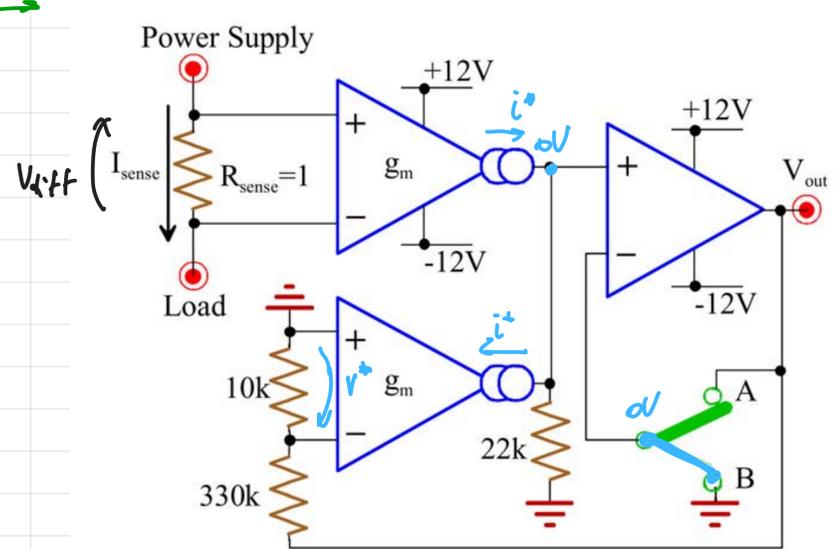
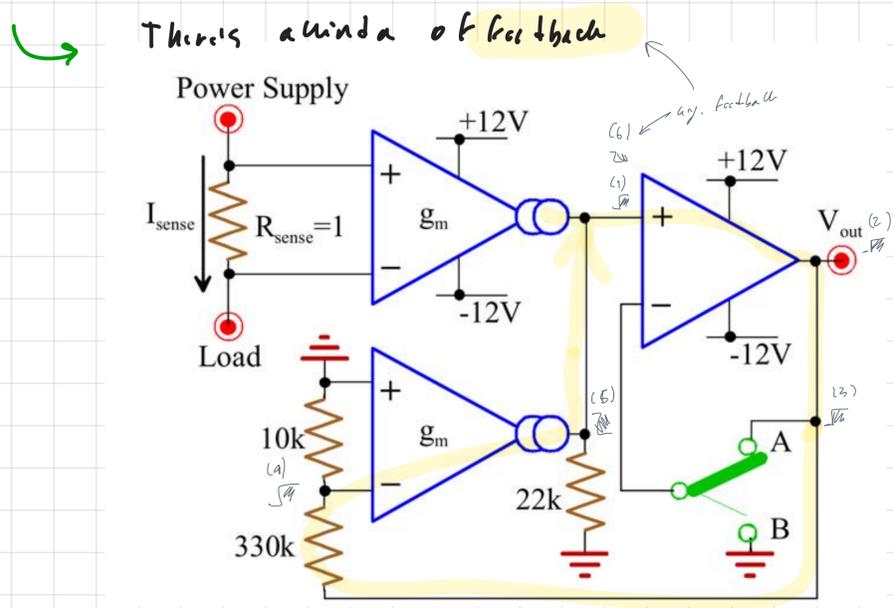
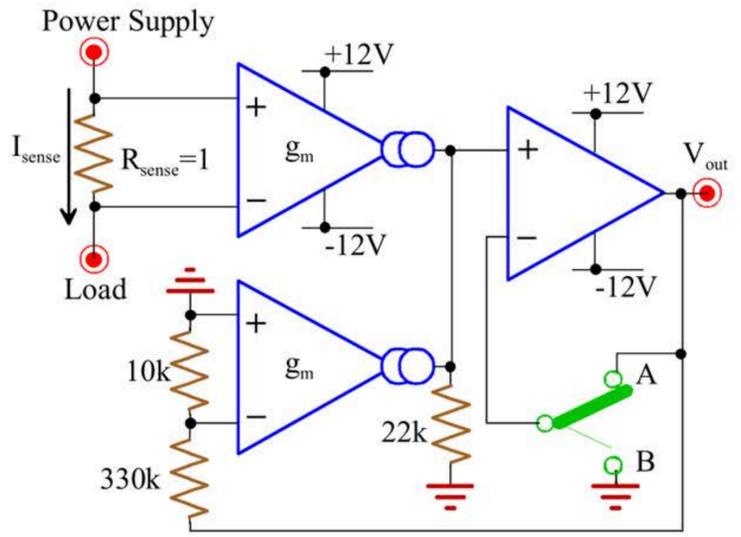


① From 28/06/2007

Es. 1

Il circuito impiega due OTA aventi transconduttanza  $g_m = 1 \text{ mA/V}$  e monitora la corrente in una  $R_{\text{sense}} = 1 \Omega$ .

- a) Quando l'interruttore è in posizione B, determinare la funzione del circuito ed il legame tra  $V_{\text{out}}$  e  $I_{\text{sense}}$ .
- b) Quando l'interruttore è in A, dire **quantitativamente** cosa cambia rispetto al caso precedente.



a)

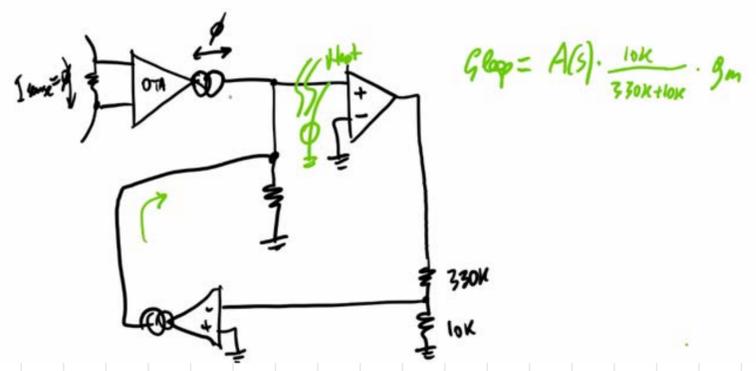
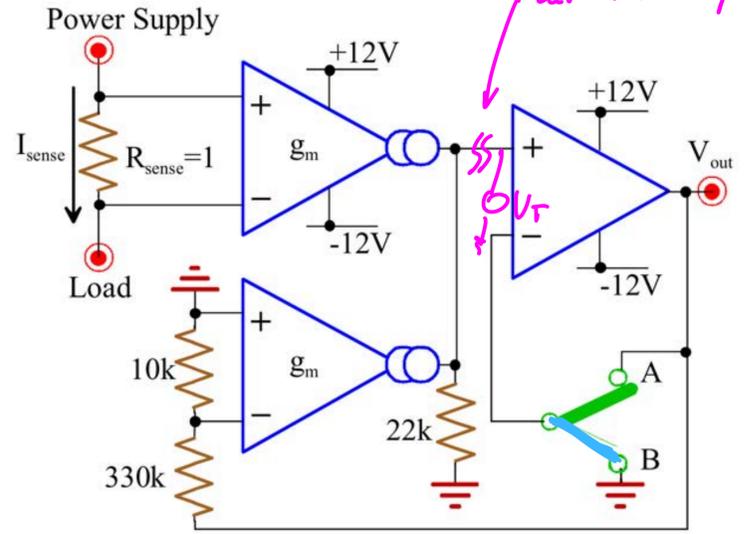
$$i^+ = V_{\text{diff}} \cdot g_m = I_{\text{sense}} \cdot R_{\text{sense}} \cdot g_m = I_{\text{sense}} \cdot 1 \cdot 1 \frac{\text{mA}}{\text{V}}$$

$$i^+ = V^+ \cdot g_m \rightarrow V^+ = I_{\text{sense}} \cdot 1 \Omega \rightarrow V_{\text{out}} = \frac{V^+}{10k} (10k + 330k)$$

$$V_{\text{out}} = I_{\text{sense}} \cdot 1 \Omega \left( 1 + \frac{330k}{10k} \right) = I_{\text{sense}} \cdot 1 \Omega \cdot 34$$

b)

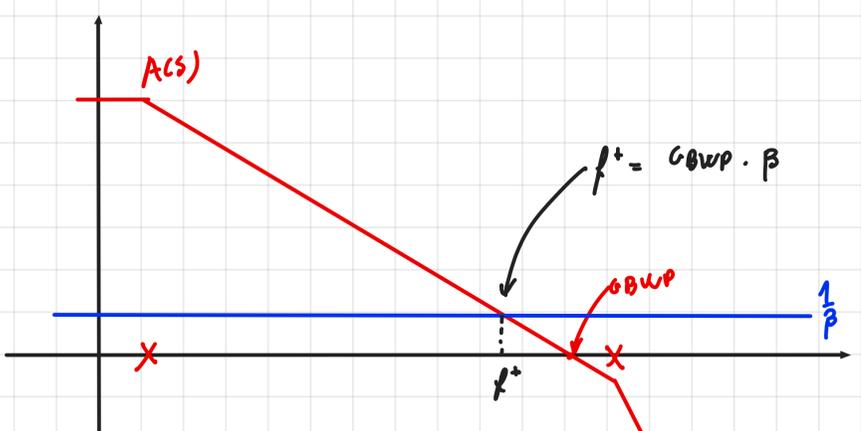
First with B



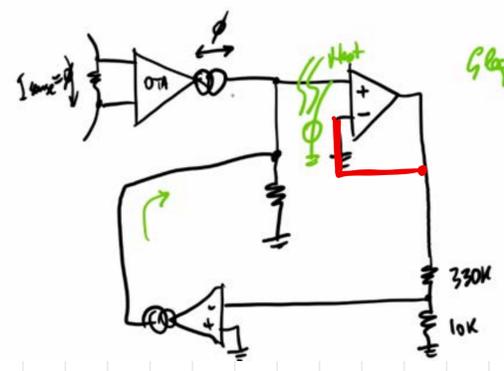
$$G_{\text{loop}} = A(s) \cdot \frac{10}{340} \cdot 22k \cdot \frac{1 \text{ mA}}{\text{V}}$$

$$0.65 \Rightarrow \frac{1}{p} = \frac{1}{0.65} = 1.5$$

Bode



• Now consider A circuit



~~$G_{loop} = A(s) \cdot \frac{10k}{330k+10k} \cdot g_m$~~

$G_{loop} = 1 \cdot \frac{10k}{330k+10k} \cdot g_m \cdot 22k = 0.65$

$V_{outid} = I_{sense} \cdot 34\Omega$

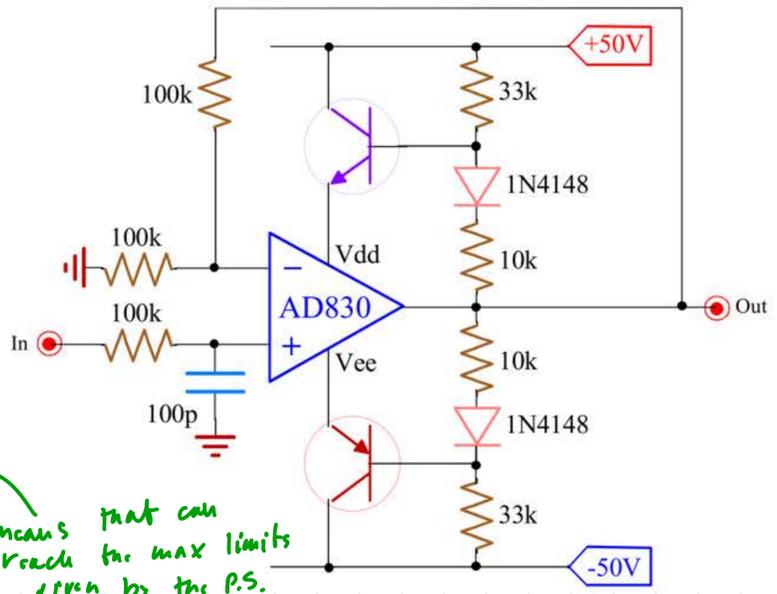
$G_{total} = \frac{V_{out}}{I_{sense}} (F) = \frac{G_{ideal}}{1 - \frac{1}{G_{loop}}} = \frac{34 \frac{V}{A}}{1 + \frac{1}{0.65}} = \frac{34 \frac{V}{A}}{1+1.5} = 13.6 \frac{V}{A}$

2

**Es. 3**

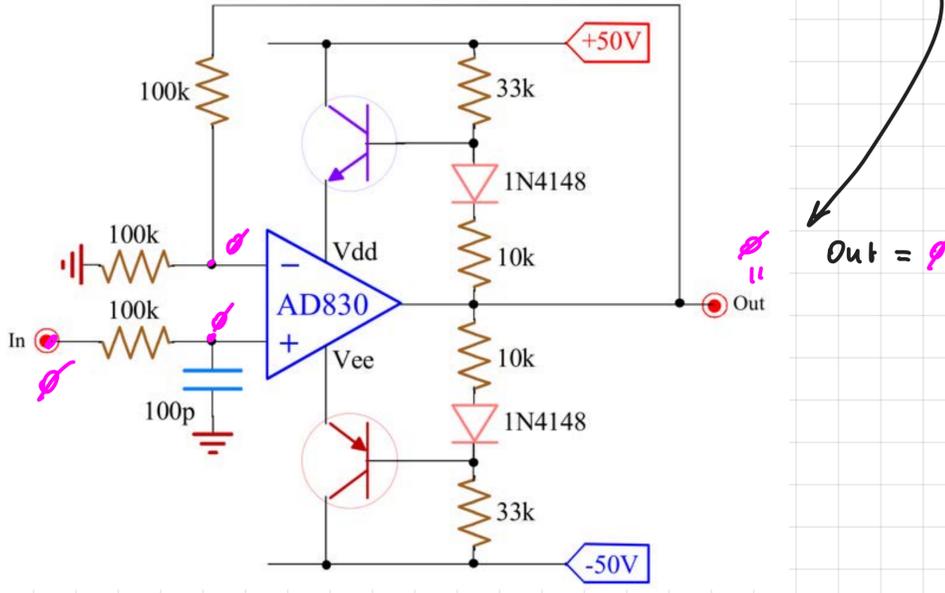
Usando un OpAmp con  $V_{supply,max} = \pm 14V$ , per potere erogare in uscita delle tensioni tra  $\pm 50V$ , ossia oltre i **maximum absolute ratings**, si impiega il bootstrap.

- a) Determinare la tensione di alimentazione  $V_{dd}-V_{ee}$  dell'OpAmp e disegnare l'andamento di  $V_{out}$ ,  $V_{dd}$  e  $V_{ee}$  quando  $V_{in} = 10V$  sinusoidali.
- b) Determinare il massimo guadagno dello stadio, oltre il quale si eccede il common mode input voltage range dell'OpAmp, sapendo che è un *rail-to-rail* sia come *input common range* che come *output voltage swing*.

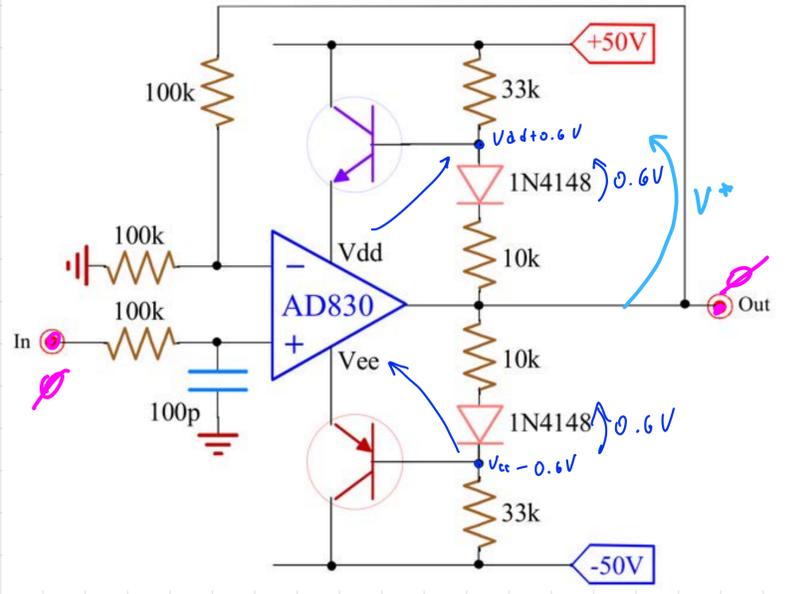


In this case both input and output means that can reach the max limits given by the P.S.

It has a negative feed. → Let's first apply  $I_n = 0$



→ Let's now consider the different components

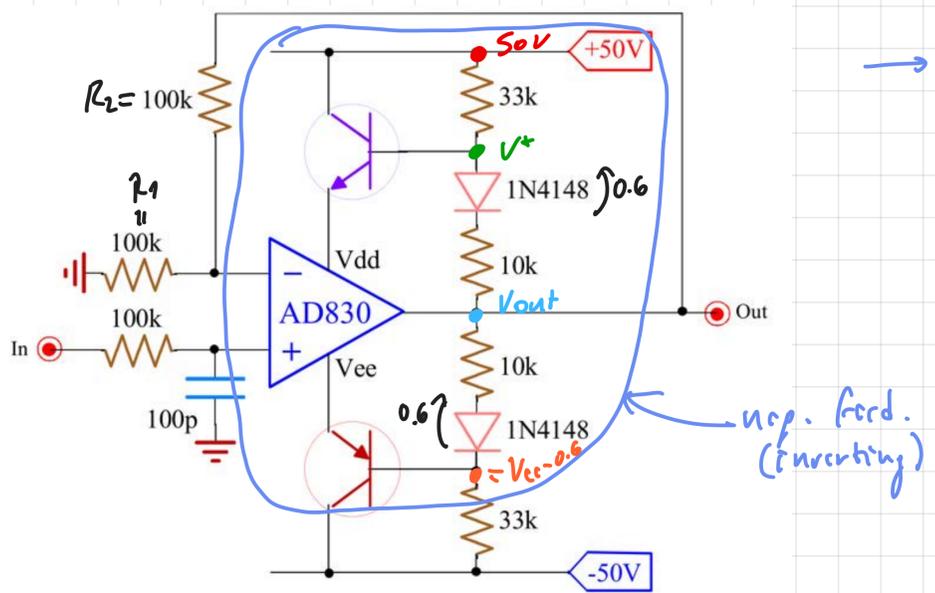


$$V^* = (50V - 0.6V) \cdot \frac{100\mu A}{10k\Omega + 33k\Omega} + 0.6V \quad \rightarrow \quad V_{dd} = V^* - 0.6V = 49.4\mu A \cdot \frac{10}{43} = 11.5V < 14 \text{ (max P.S.)}$$

same reasoning for  $V_{ee}$  →  $V_{ee} = \dots = -11.5V < 14 \text{ (max P.S.)}$

a)

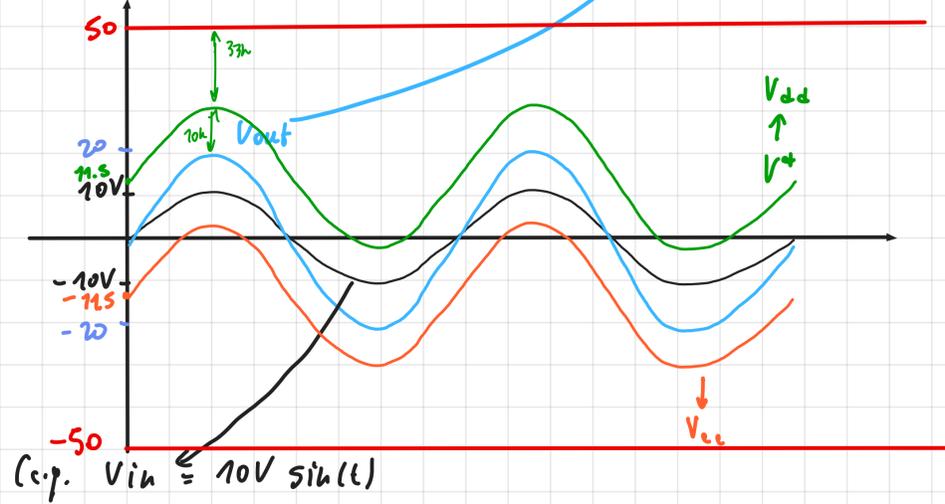
Now what happens when  $V_{in} \neq 0$



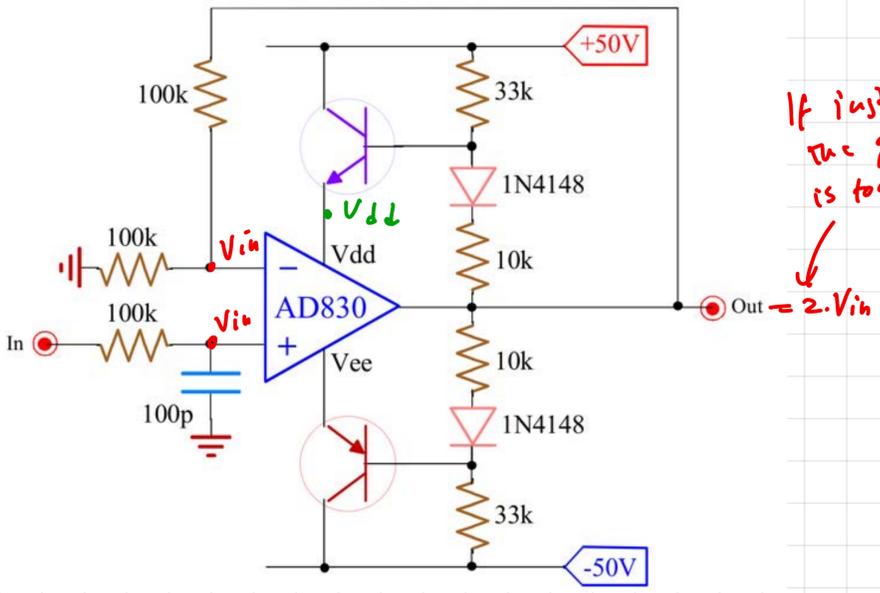
$$\rightarrow \text{Gain} = \frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1} = 1 + \frac{100k\Omega}{100k\Omega} = 2$$

$$\rightarrow V^* = V + (V - V - 0.6) \cdot \frac{10k}{43k}$$

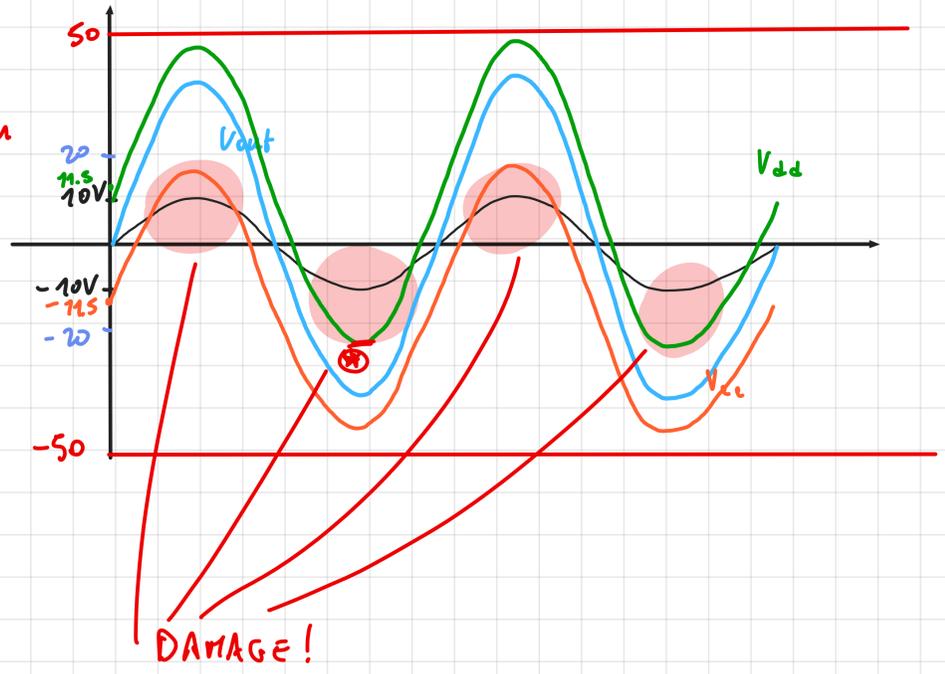
Output characteristic



b)



If instead the gain is too high



$V_{out} = G \cdot V_{in}$

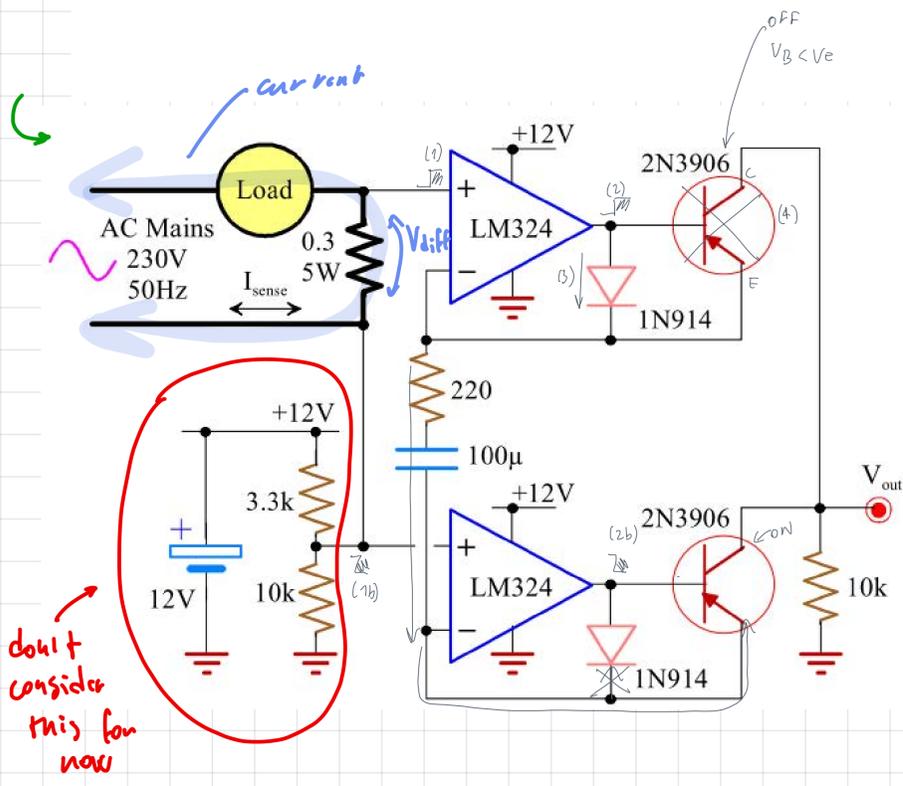
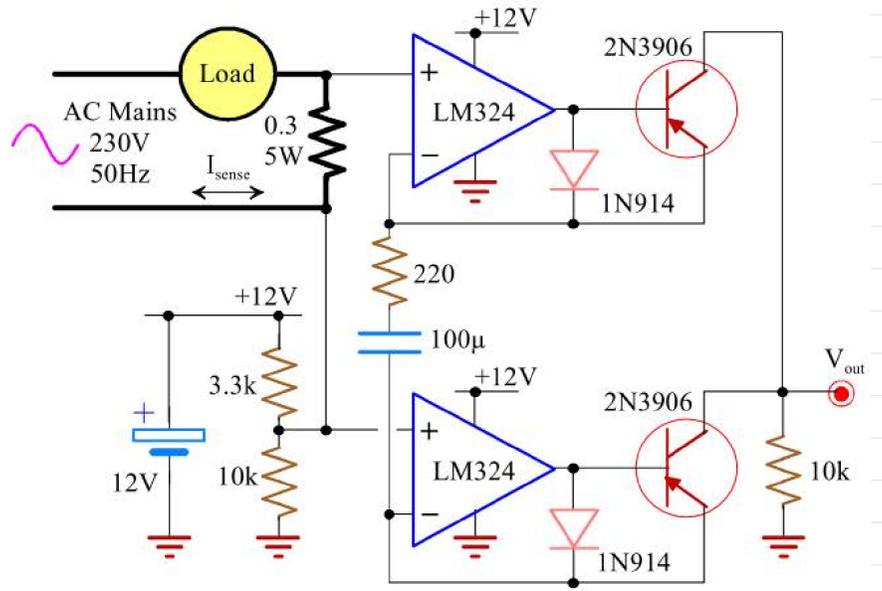
$V_{dd} = (50 - G \cdot V_{in} - 0.6) \cdot \frac{10k}{43k} + G \cdot V_{in} \geq V_{in}$

$49.4 - G \cdot V_{in} \geq V_{in} (1 - G) 4.3 \dots \rightarrow G_{max}$  for which we don't have damage

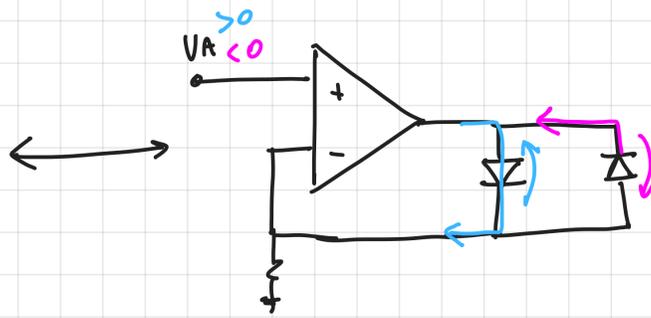
**Es. 2**

Il circuito monitora l'assorbimento di un carico in alternata a 50Hz. L'alimentazione è flottante a batteria.

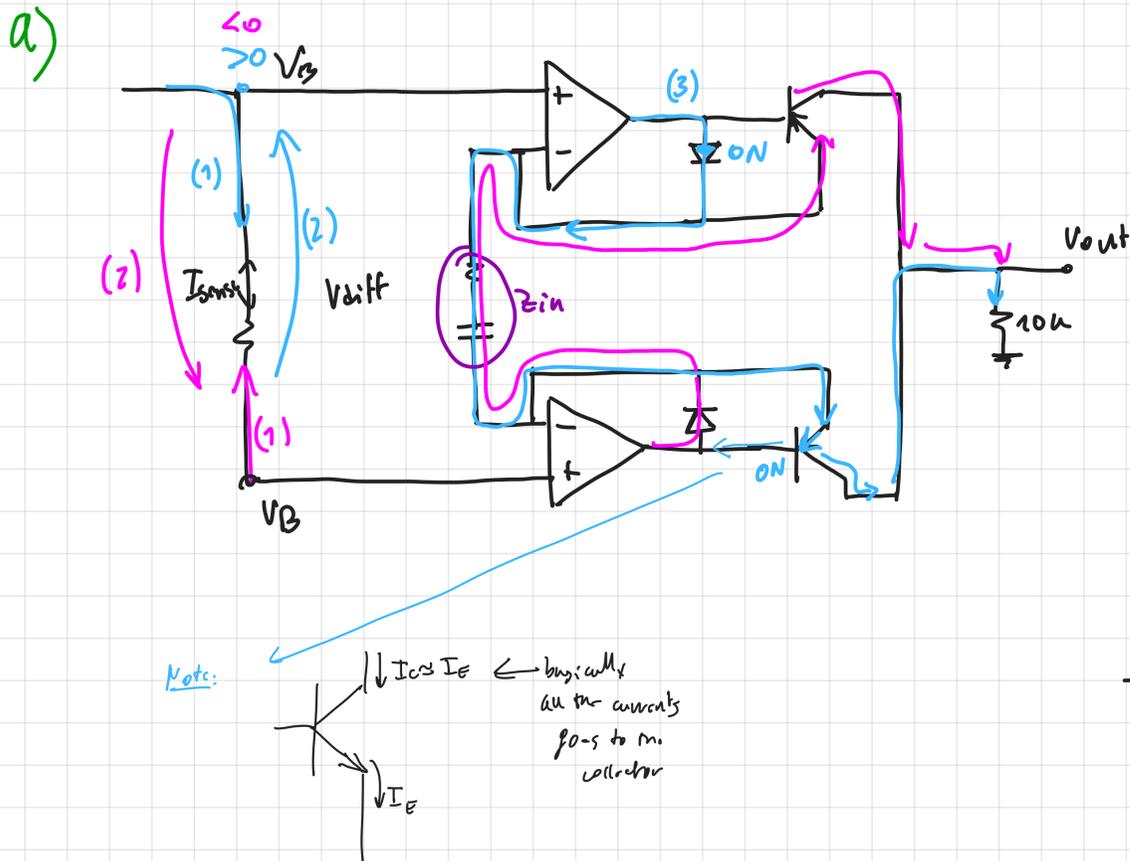
- a) Disegnare la forma d'onda quotata di  $V_{out}$  nel caso sia collegato un carico resistivo di 50W e determinare il guadagno  $V_{out}/I_{sense}$ .
- b) Realizzare l'alimentazione continua del circuito (circa 12V) partendo dalla tensione di rete, senza usare alcun trasformatore.



$$V_{diff} = i \cdot 0.3 \Omega$$



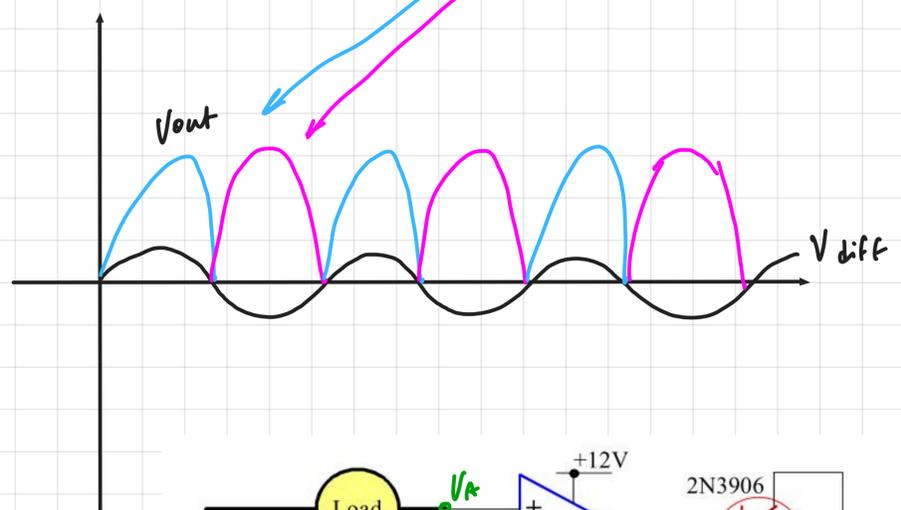
So we have



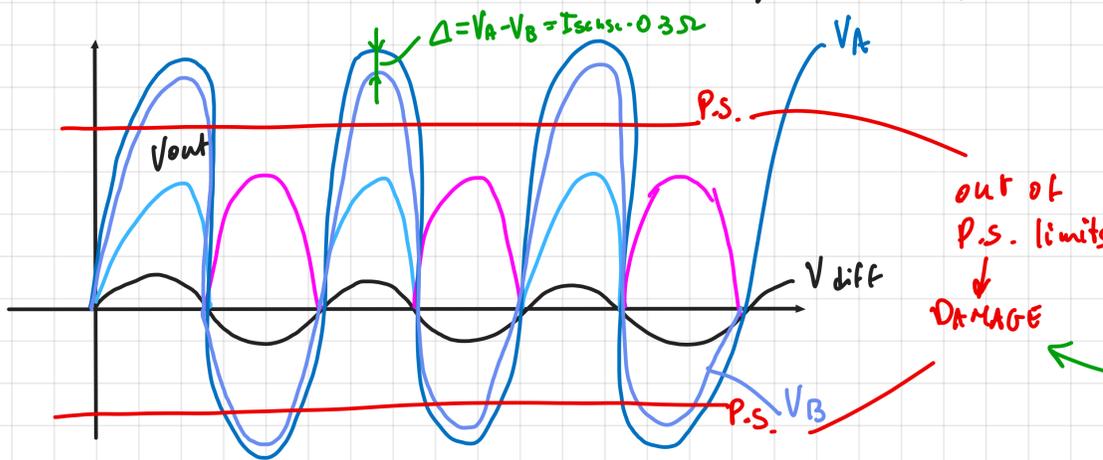
$$V_{diff} = I_{sense} \cdot 0.3 \Omega$$

$$i^* = \frac{V_{diff}}{Z_{in}} \approx \frac{V_{diff}}{R} = I_{sense} \cdot \frac{0.3 \Omega}{220 \Omega} = \frac{I_{sense}}{733}$$

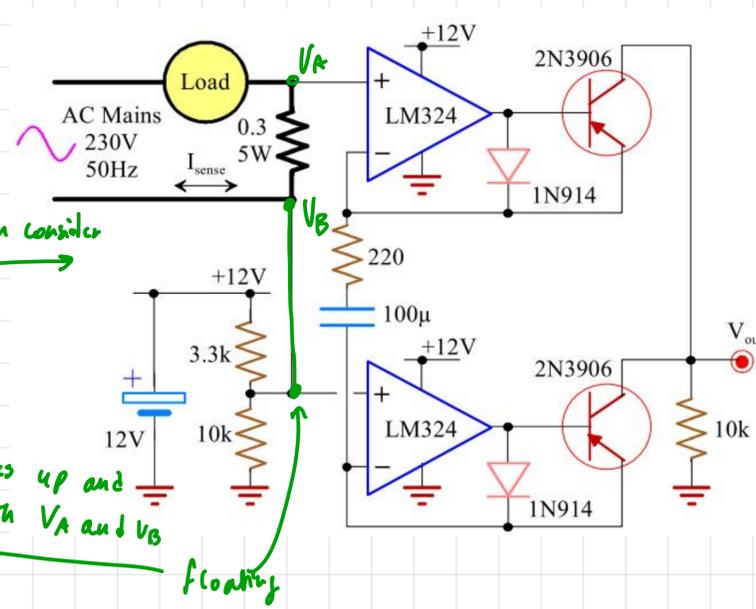
$$V_{out} = \frac{I_{sense} \cdot 10k}{733} = +I_{sense} \cdot 13.6$$



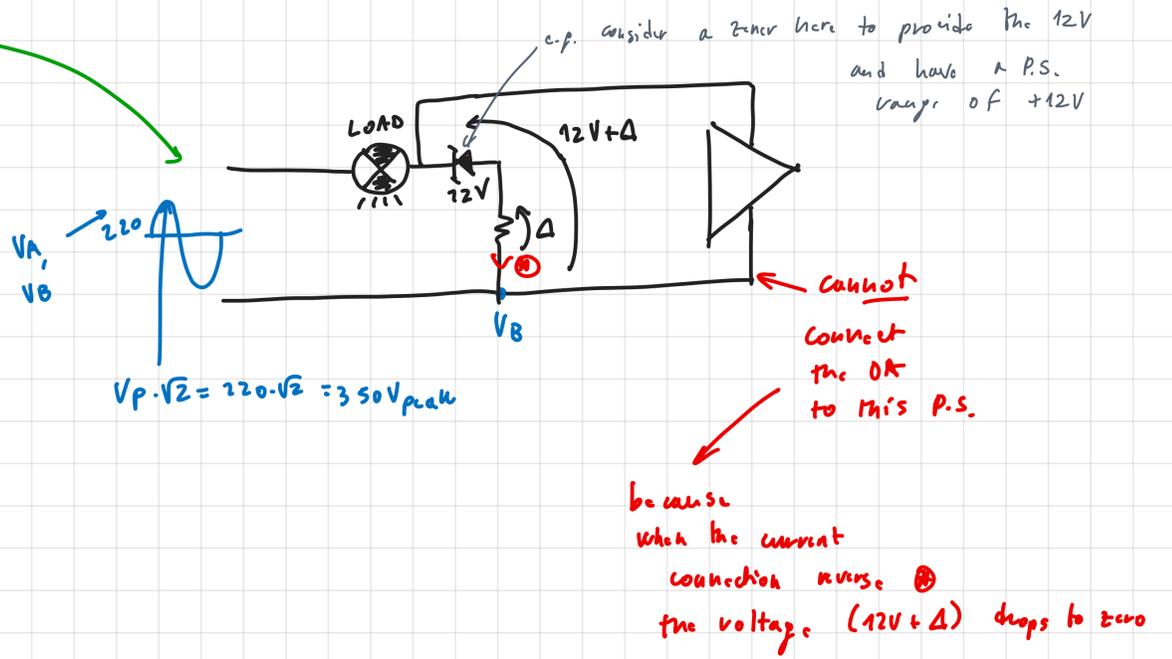
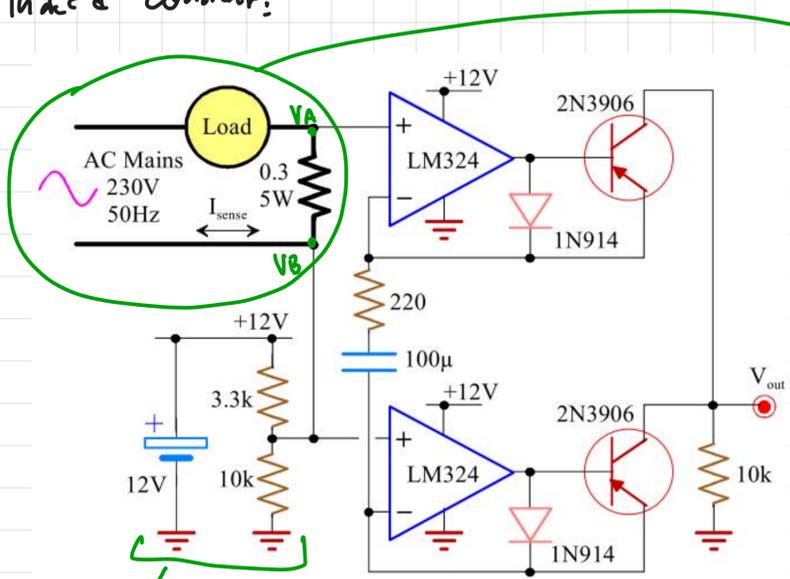
b) We have to make sure  $V_{be}$  is in the range of the P.S.



We can consider



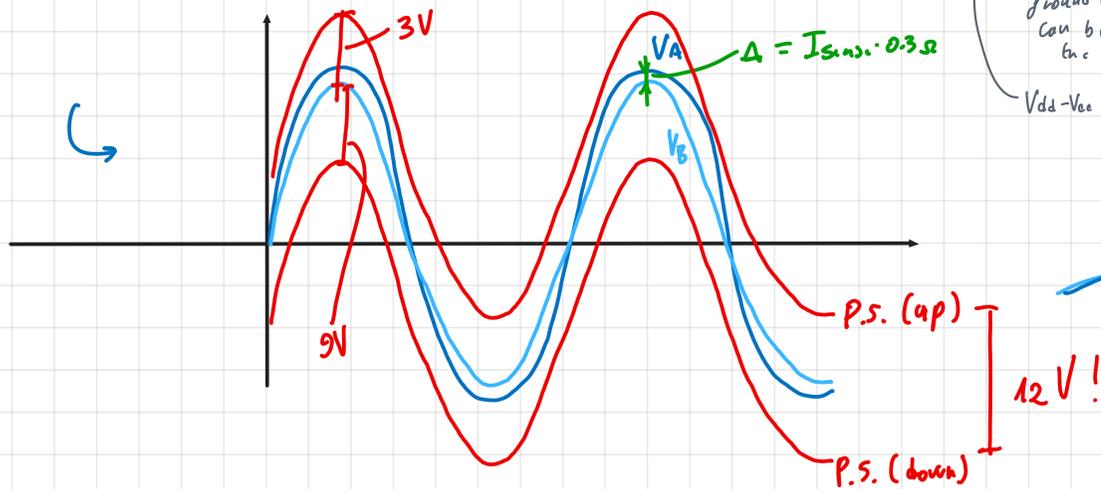
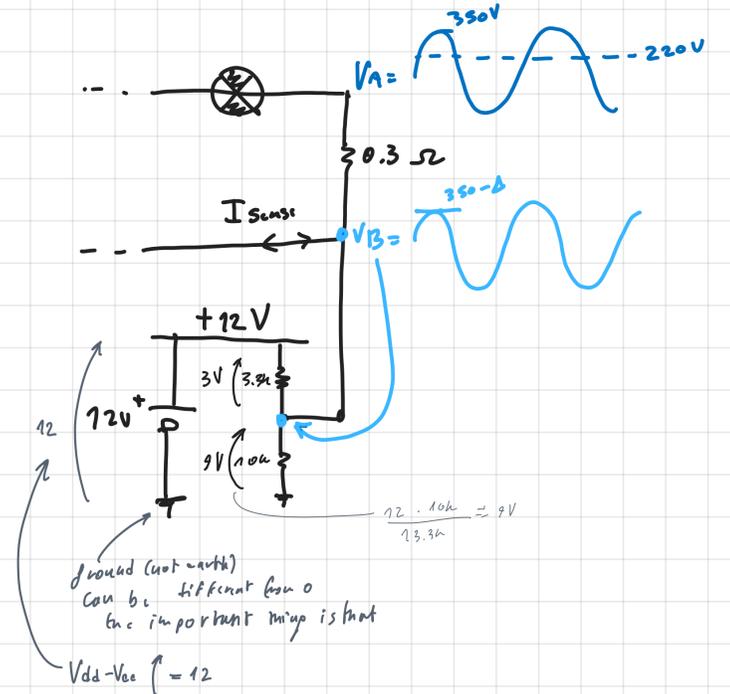
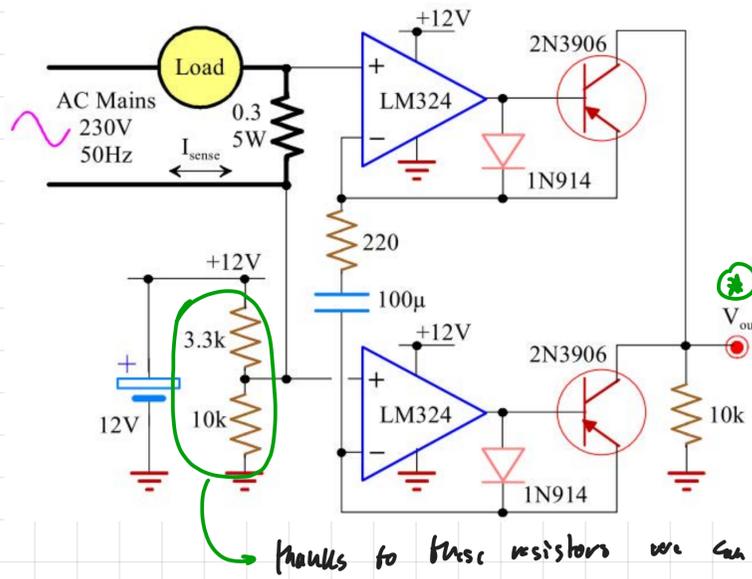
Indeed consider:



we can connect here a battery to provide 12V

to VB node

in this way the P.S. keeps moving up and down compared to that node



the signals are always in P.S. limits

BUT consider that with moving P.S. wrt the earth (0V) we'll have that Vout will move

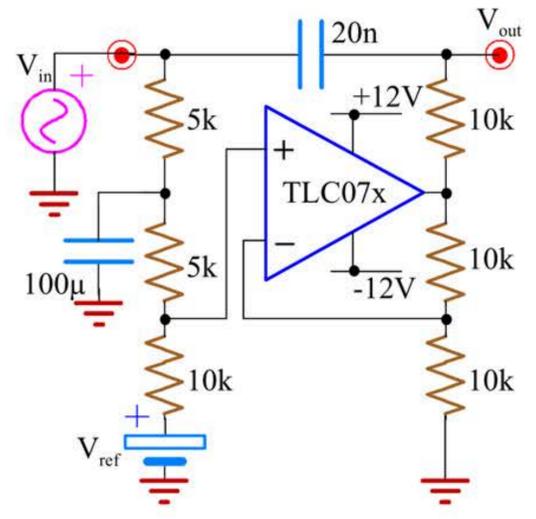
wrt the ground (P.S. down) → if Vout must be used in other circuits it keeps changing and it's not wrt earth (can be dangerous!)

4

**Es. 1**

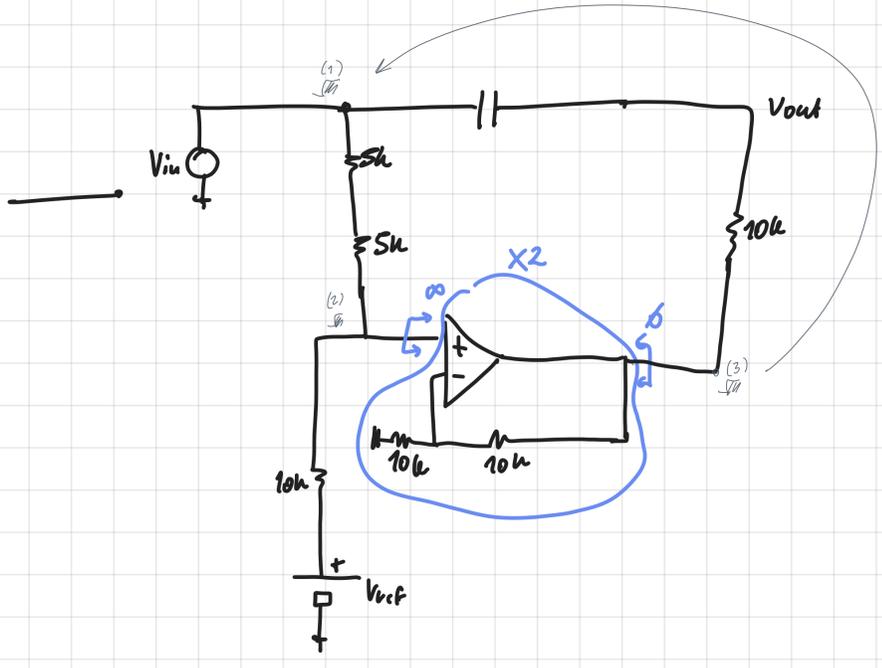
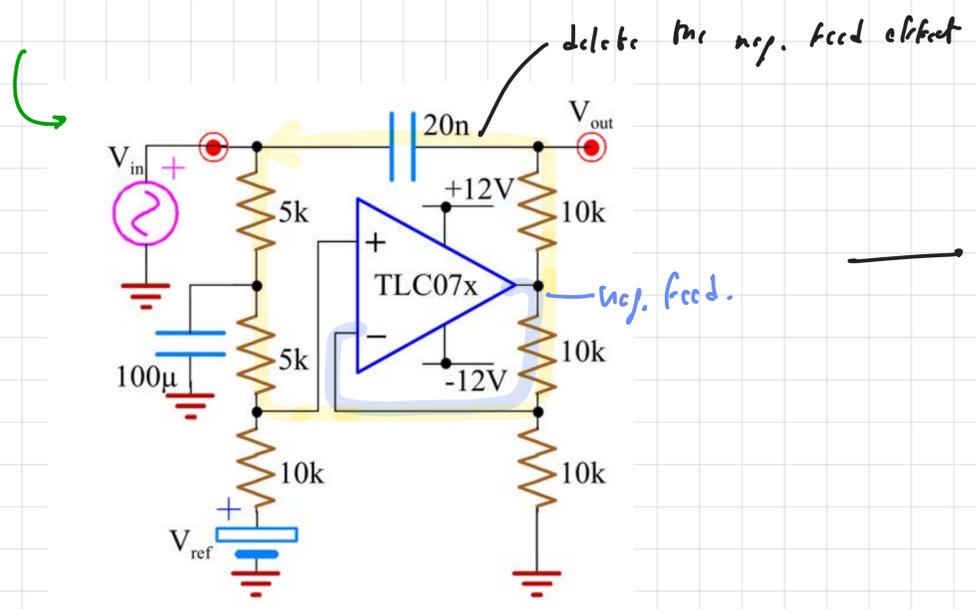
L'OpAmp ha  $A_0=120\text{dB}$ ,  $\text{GBWP}=10\text{MHz}$ ,  $I_{OS}=100\text{pA}$ ,  $V_{OS}=0.5\text{mV}$ . L'uscita è lasciata aperta.

- a) Determinare  $V_{out}/V_{in}$  in continua e ad alta frequenza ed il guadagno  $V_{out}/V_{ref}$ . Disegnare il trasferimento  $V_{out}/V_{in}(f)$  e commentare il ruolo del circuito.
- b) Calcolare l'impedenza di uscita del circuito a 10Hz quando  $V_{in}=+2\text{V}$ .
- c) Calcolare l'errore in uscita dovuto all'offset di corrente (NON alle  $I_B$ ) ed a quello di tensione.

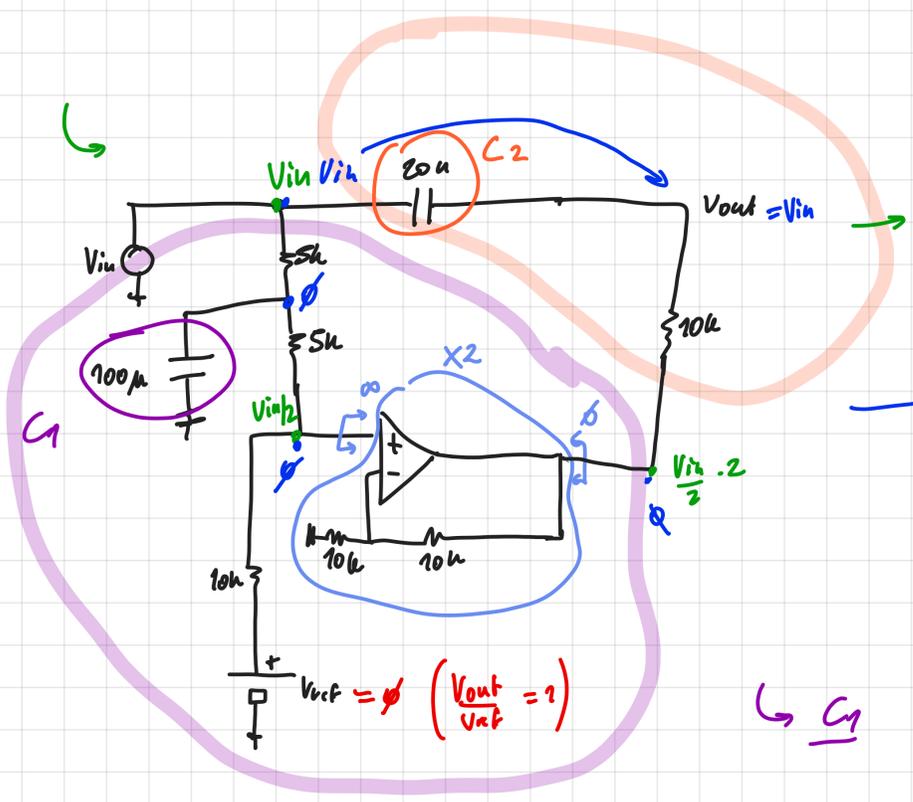


so no pos. or neg. feed.

but we cannot vary (for this case increase)

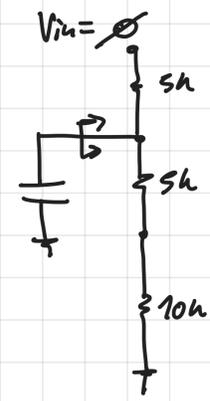


$V_{in}$  ideal volt- $\mu$ en  
 ↓  
 If there were a resistor  
 ↓  
 would have been a pos. feed.



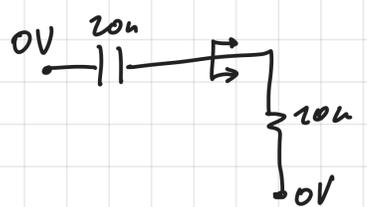
DC  
 $\frac{V_{out}(0)}{V_{in}} = 1$   
 AC  
 $\frac{V_{out}(\infty)}{V_{in}} = 1$   
 but could not be always 1  
 ↓  
 we have to study the capacitors

↳  $C_1$  pole:



$R_{C1} = 5k \parallel (5k + 10k)$   
 $f_{P1} = \frac{1}{2\pi (5k \parallel 15k) \cdot 100\mu} = 0.42\text{Hz}$

↳  $C_2$  pole:

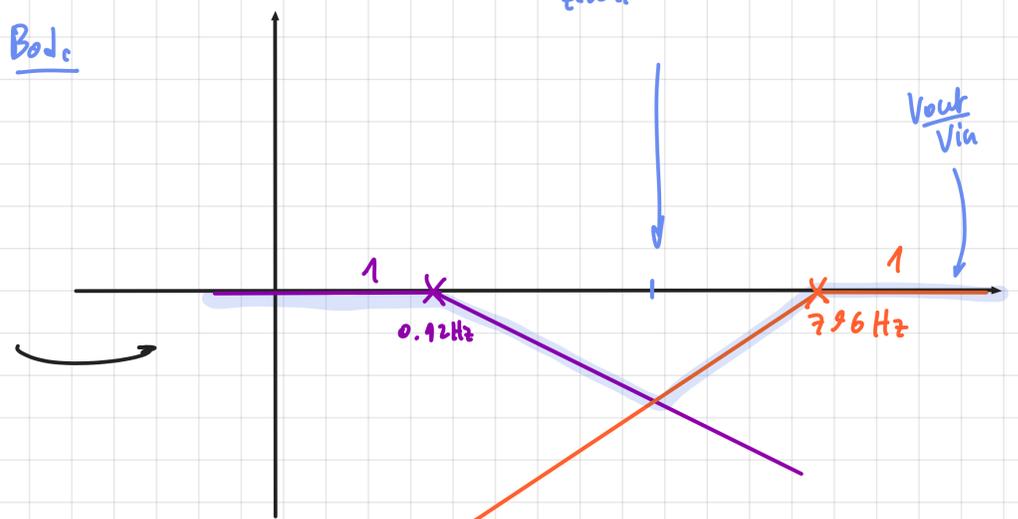


$f_{P2} = \frac{1}{2\pi \cdot 20n \cdot 10k} = 796\text{Hz}$

$z_{c1} = z_{c2} = \sqrt{f_{P1} \cdot f_{P2}} = 18.3\text{Hz}$

Bode

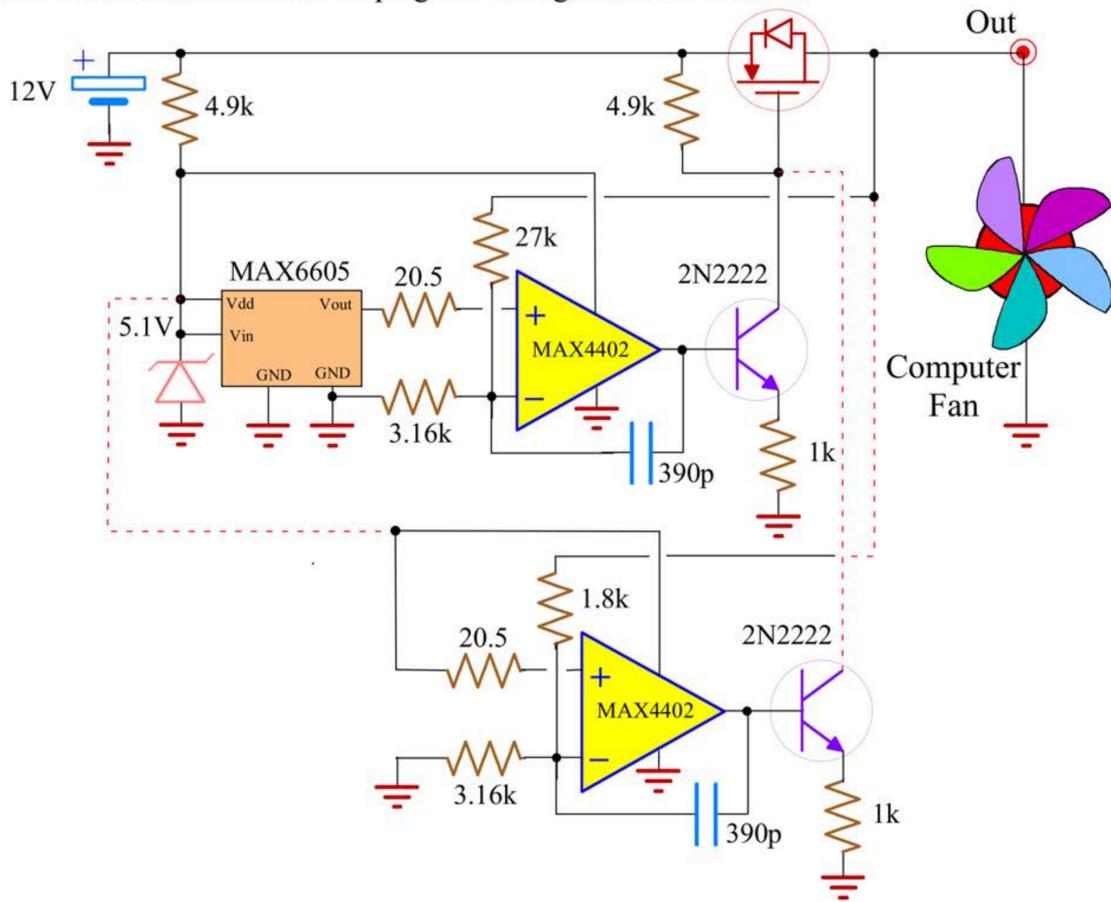
because they're parallel paths



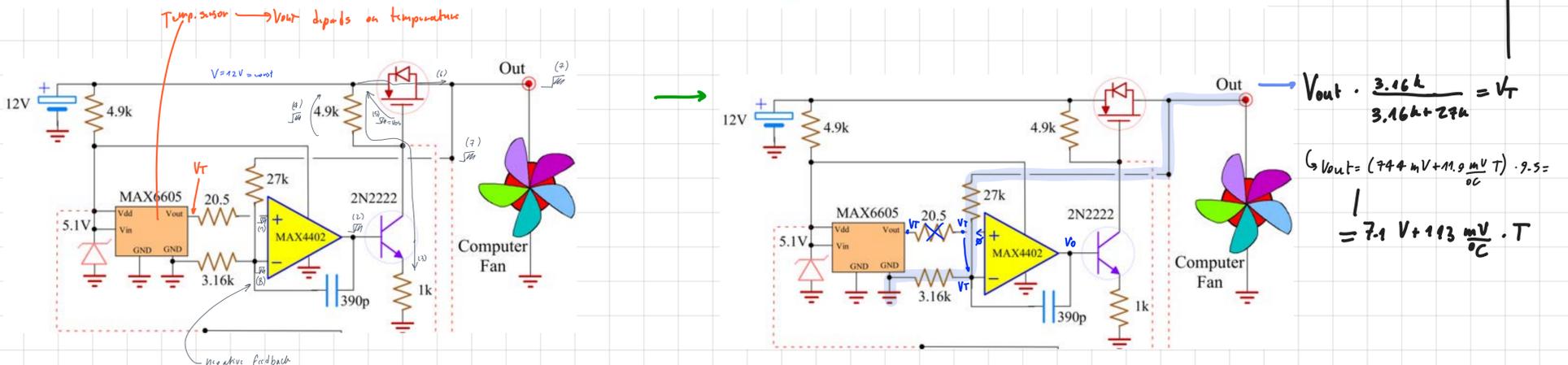
Es. 3

Il sensore di temperatura MAX6605 fornisce  $V_T(T) = 744mV + T \cdot 11.9mV/^\circ C$ .

- a) Scollegando il circuito in basso, disegnare il grafico quotato di  $V_{out}(T)$  nell'intervallo  $0^\circ C \div 50^\circ C$ .
- b) Determinare l'intervallo di temperature per cui il circuito è in zona lineare, sapendo che il MAX4402 è rail-to-rail.
- c) Includere ora il circuito in basso e spiegarne dettagliatamente il ruolo.



means that the output can reach the P.S.



Plot (out T):

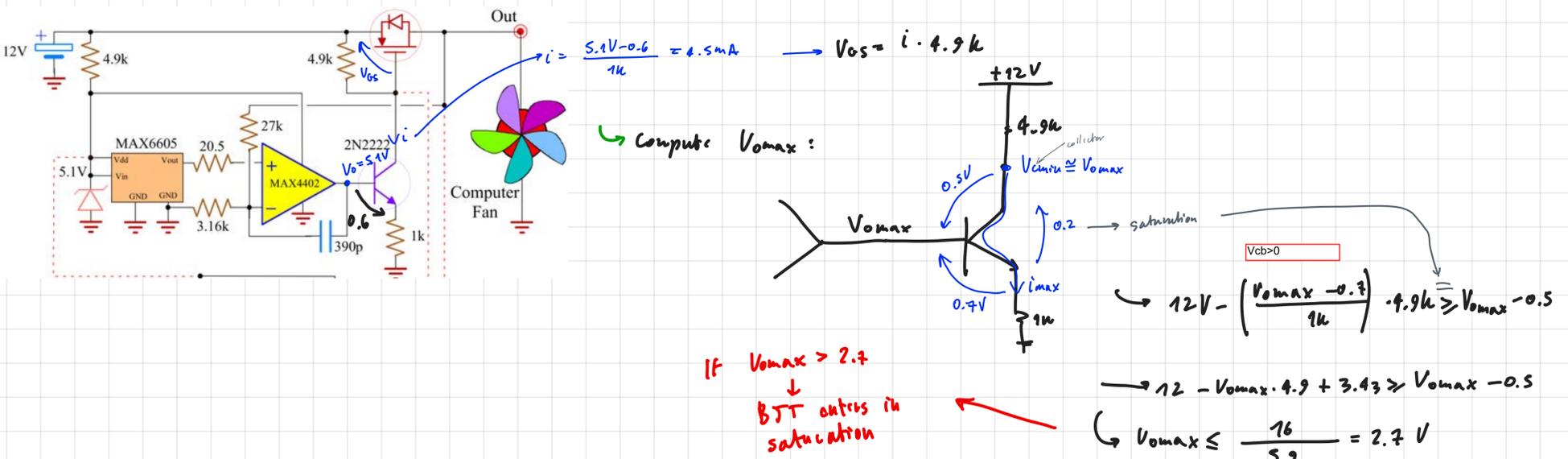


we have to check the limits

we don't have data, he could have told us to give a current of 200mA to the fan

compute Vos  
compute Voutmax and if it's in range

b) Vedere per che range è in zona lin.



If  $V_{omax} > 2.7$   
BJT enters in saturation

$$12V - \left(\frac{V_{omax} - 0.7}{1k}\right) \cdot 4.9k \geq V_{omax} - 0.5$$

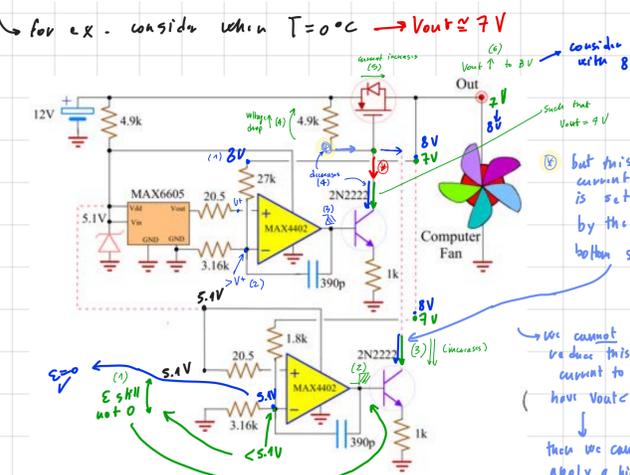
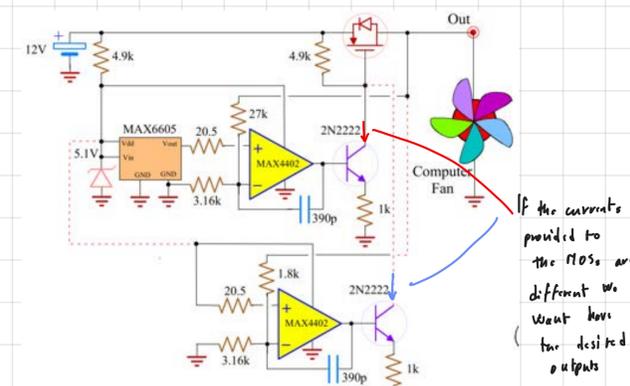
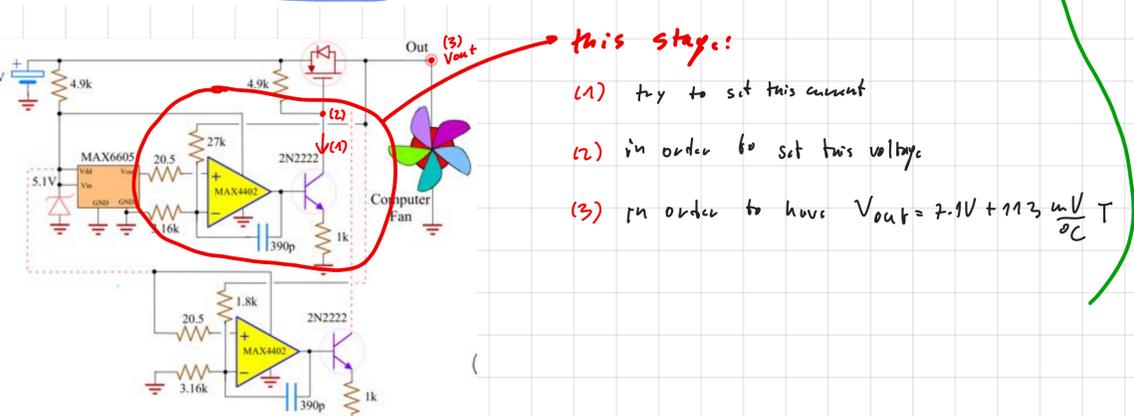
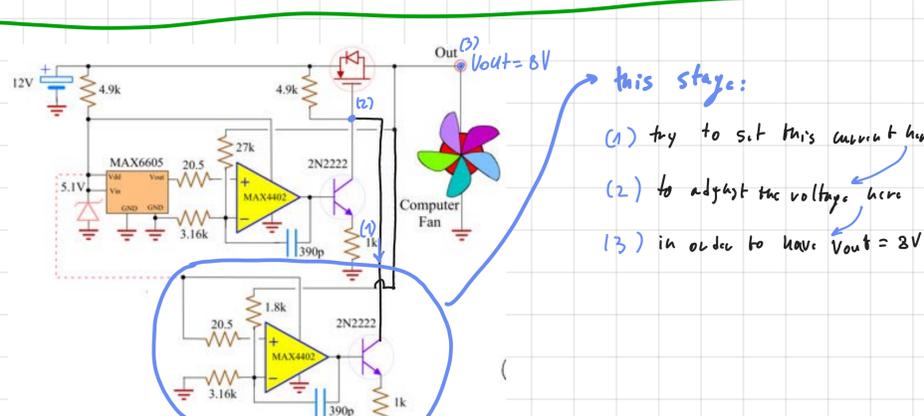
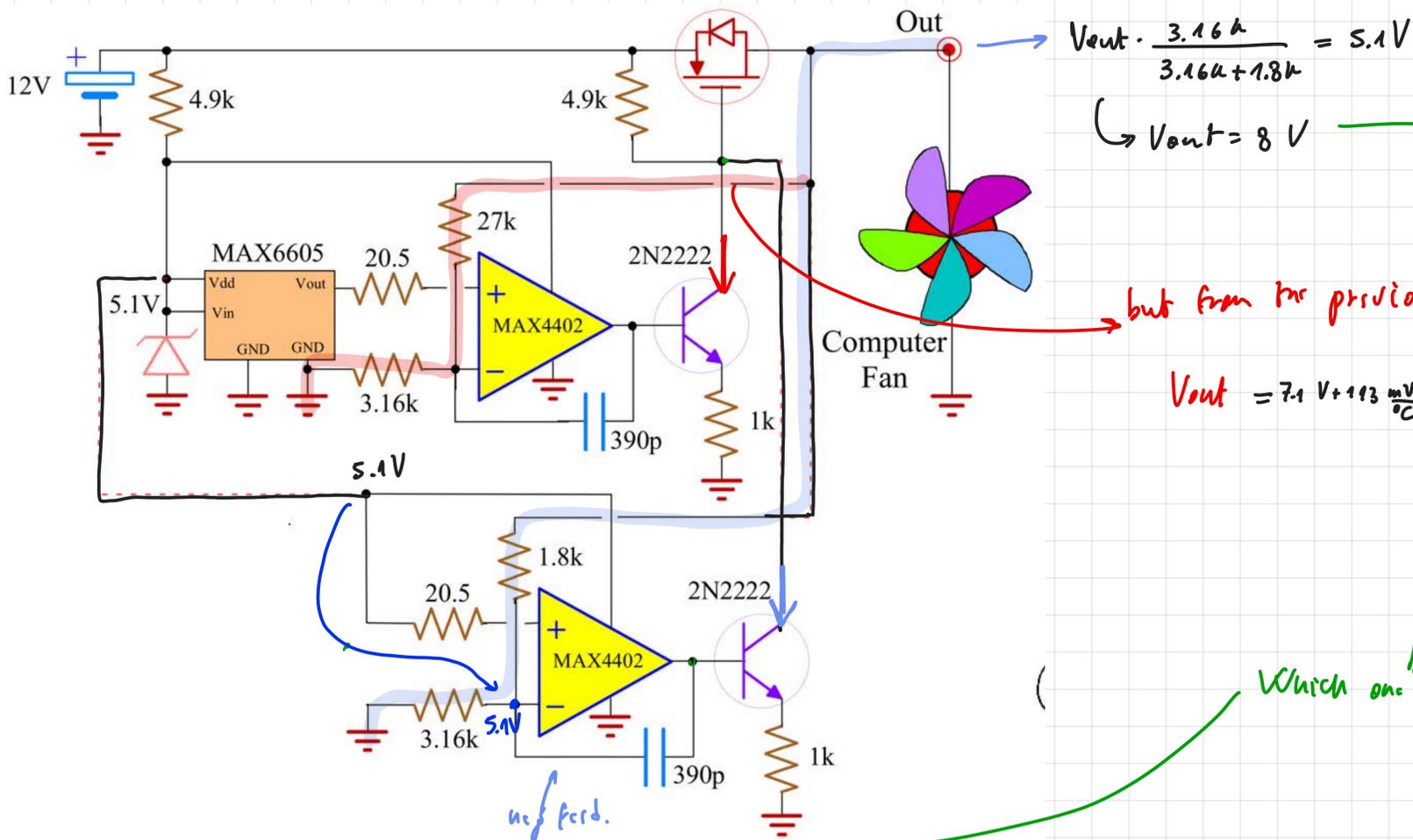
$$\rightarrow 12 - V_{omax} \cdot 4.9 + 3.43 \geq V_{omax} - 0.5$$

$$\rightarrow V_{omax} \leq \frac{16}{5.9} = 2.7V$$

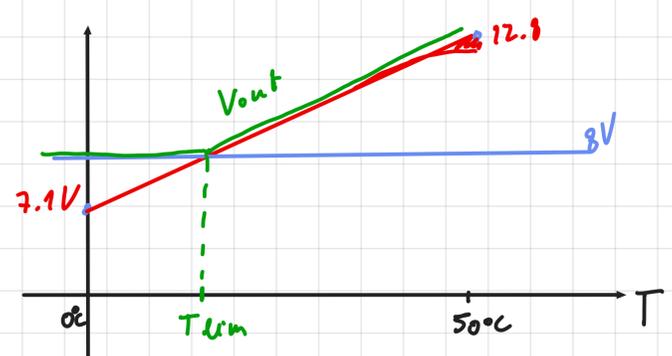
with INFO about the MOS we could have compute the  $V_{os}$  and  $i_{max}$  and the  $V_{outmax} \rightarrow$  that depends on  $T \rightarrow$  temp. range.

but not given in data  $\rightarrow$  so cannot compute it

c)

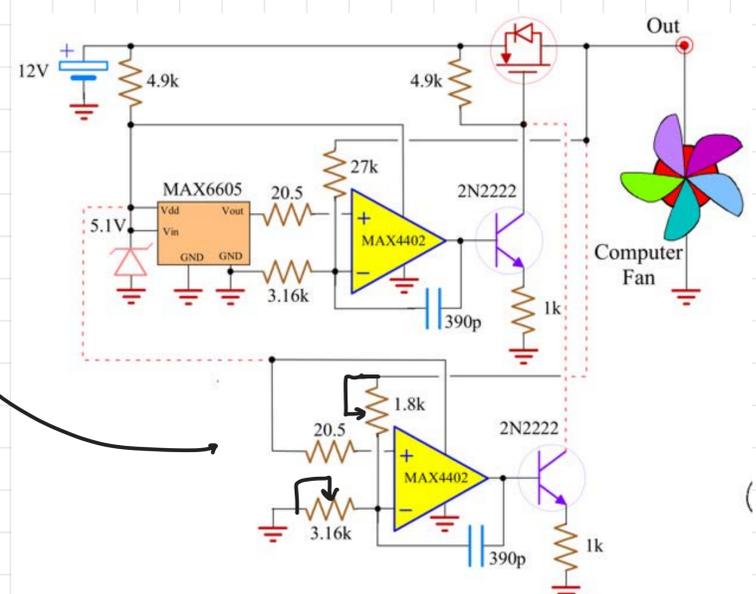


the 2 stages are in parallel



$\frac{(12.8 - 7.1V)}{50^\circ C} = \frac{(8 - 7.1)}{T_{lim}}$

we can put trimmers to make  $V_{out}$  dependent on  $T$



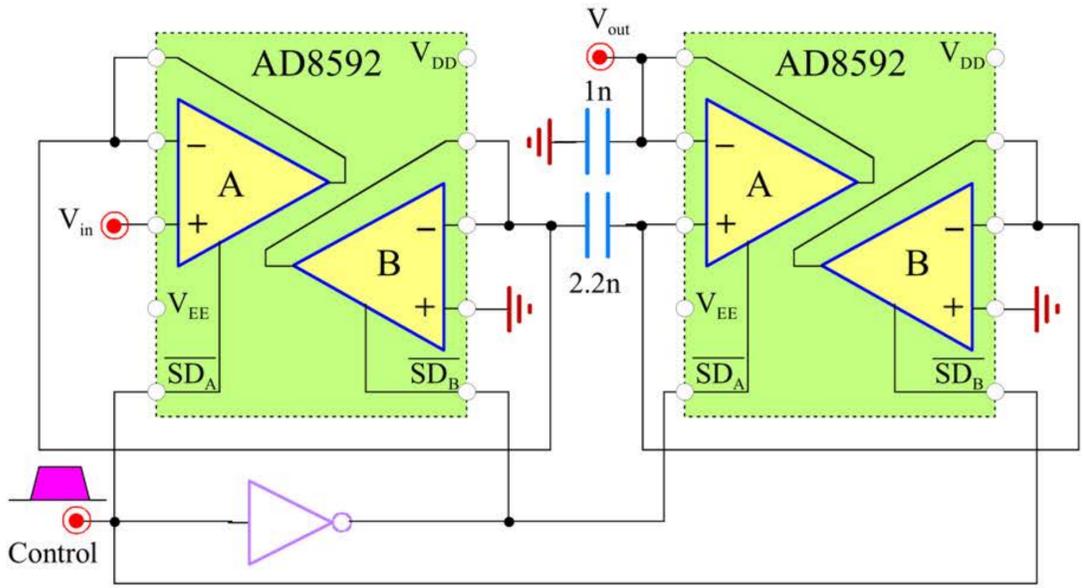
6

Es. 2

→ SD

I piedini di ShutDown *active-low* mandano in *three-state* la rispettiva uscita. Il segnale Control è a 1kHz.

- a) Determinare la funzione svolta dal circuito e il legame tra  $V_{out}$  e  $V_{in}$ . Si consiglia di studiare separatamente il caso  $Control=low$  e poi  $Control=high$ .
- b) Dire cosa accade quando  $Control=low$  per 1ms e si considerano tutte le  $I_B=100pA$ .



a)

